ANIMATED GAME THEORY

FOR FOOTBALL

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ANIMATED GAME THEORY

FOR FOOTBALL

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TABLE OF CONTENTS

	Page
ACKNOWLEDGMENTS	ii
LIST OF TABLES	v
LIST OF ILLUSTRATIONS	vi
SUMMARY	viii
Chapter	
I. INTRODUCTION	1
II. LITERATURE SURVEY AND STATEMENT OF PROBLEM	4
Literature Survey Statement of the Problem Scope and Limitations	
III. DESCRIPTION OF THE MODEL	13
Operational Constraints Kinematic Equations Physiological Constraints One-on-One M-on-N Blocking Model Tackling Model Capabilities of the Model Examples	
IV. VALIDATING THE MODEL	103
Source of Data Estimation of Parameters Model Verification	
V. DESCRIPTION OF PLAYS TO BE MODELED	121
Offense Defense Counter Dive Predetermined Fullback Play	

Page

VI. APPLICATIONS OF THE MODEL. 170

Play Selection
Evaluation of Plays and Players
Play Design

VII. RECOMMENDATIONS AND CONCLUSIONS. 191

Conclusions
Recommendations

APPENDIX A. 196

APPENDIX B. 198

BIBLIOGRAPHY 238

LIST OF TABLES

Table		Page
4-1.	Hill's Running Data.	109
4-2.	Game Film Running Data	110
4-3.	Average Error for Values of C_2 and C_3.	111
4-4.	Blocking Results for Values of C_2.	112
4-5.	Player Initial Positions and Attributes.	116
4-6.	Player Positions at 0.5 Seconds.	117
4-7.	Player Positions at 1.0 Second	118
4-8.	Player Positions at 1.5 Seconds.	119
4-9.	Fraction of Time H_0 is Accepted.	120
5-1.	Results of Counter Dive vs. Oklahoma-50.	126
5-2.	Results of Counter Dive vs. Split-Six.	127
5-3.	Results of Predetermined Fullback vs. Oklahoma-50.	151
5-4.	Results of Predetermined Fullback vs. Split-Six.	161

LIST OF ILLUSTRATIONS

Figure		Page
3-1.	Isaacs' Differential Football Game.	23
3-2.	Optimal Strategy When Players Are Circles of Diameter 1 .	24
3-3.	Geometric Interpretation of Equation (3.29)	30
3-4.	Defensive Game, "Optimal" Case.	32
3-5.	Defensive Game: "Non-Optimal" Case, Epoch 1.	33
3-6.	Defensive Game: "Non-Optimal" Case, Epoch 2.	33
3-7.	Defensive Game, Differential Case	34
3-8.	Offensive y^i as a Function of θ	44
3-9.	Offensive Blocking.	46
3-10.	Symmetry in Blockers' and Tacklers' Strategies.	49
3-11.	Rotation of Strategy Axes	50
3-12.	Distribution of Blocking Outcomes	57
3-13.	Torques and Momentums in the Blocking Interaction	58
3-14.	3-on-3: Example 1.	79
3-15.	3-on-3: Example 2.	92
4-1.	Relationship of σ_C to σ_{C_i}	114
5-1.	Wishbone Offensive Formation.	121
5-2.	Wishbone vs. Oklahoma-50.	123
5-3.	Wishbone vs. Split-Six.	123
5-4.	Counter Dive vs. Oklahoma-50.	124
5-5.	Counter Dive vs. Split-Six.	125

5-6. Counter-Dive vs. Oklahoma 50 (Simulation Results). . . . 129

5-7. Counter-Dive vs. Split-Six Defense (Simulation Results). 140

5-8. Predetermined Fullback vs. Oklahoma-50 149

5-9. Predetermined Fullback vs. Split-Six 149

5-10. Predetermined Fullback vs. 50 Defense (Simulation Results) 152

5-11. Predetermined Fullback vs. Split-Six Defense (Simulation Results) 162

6-1. Gain Distribution, Offense 1 vs. Defense 1 171

6-2. Gain Distribution, Offense 1 vs. Defense 2 172

6-3. Gain Distribution, Offense 2 vs. Defense 1 172

6-4. Gain Distribution, Offense 2 vs. Defense 2 172

6-5. Utility Function, Third Down and Three Yards to Go. . . 173

6-6. Utility Function, First and Ten on the Two Yard Line. . 174

6-7. Utility Function, First and Ten on the Twenty Yard Line 174

6-8. Graphical Solution to Game Theory Strategy Selection . . 179

SUMMARY

A Monte Carlo model is presented which simulates the time history of the players' positions and velocities. Passing plays are excluded. The play is broken into constant-interval epochs at which the players select their respective strategies. These in turn are used in conjunction with kinematic equations to update the players' state variables. Various objectives are formulated for the three classes of players (offensive ball-carrier, defensive tacklers and offensive blockers). Interactions between players (blocking and tackling) are postulated from considerations of particle dynamics. Sample results are given on unstructured 3-on-3 cases to show the workings of the model. The methods for the validation of the model are presented and the validation results listed. Four sample plays (two offenses vs. two defenses) are introduced and the model results shown. Uses for the model are demonstrated in the areas of play selection and strategy evaluation. Extensions are shown on applications in optimal blocking assignments.

CHAPTER I

INTRODUCTION

Football coaches as decision-makers are required to make long-range and short-range decisions just as surely as the management of a corporation. These decisions, in turn, have a definite effect on the health of that organization. And, just as in the case of the corporation, the coaches are accountable for the effect of those decisions. More and more frequently, the decision-maker has looked to mathematical models. In the cases where the system is even moderately complex, the methodology often used is simulation. This thesis develops a physical simulation model to describe the basic and fundamental interactions for the game of football.

It should not be surprising that an industrial engineer should do this. Horowitz, in his analysis of professional baseball, states in the Journal of Industrial Engineering [12]*

> . . . whether it is looked on as sport, business, or a combination of the two, organized baseball is run by managements that are attempting to maximize something, be it profit, utility, or number of first place finishes. . . . This model provides a framework for decision-making that can be extended beyond this particular 'industry'.

This combination of sports and business has been increasing in the last decades (indeed in professional sports they have been linked

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*page 170.

from their inception) to where the "sports industry" is a recognized and accepted part of the American culture. In this environment the Operations Researcher is comfortable--most of the techniques acquired for business and industry apply equally.

But this thesis is not primarily an attempt to model the decision-making process of the managers of football. The game itself, especially the play, offers a rich area for modeling, since by its nature it is competition-oriented, statistically documented, and provides an immediately available objective criterion (i.e. the yardage gained). Thus, Chapter III is the focus of the thesis. A model of 22 players acting at times as a cohesive unit and at times independently is staggeringly complex. Thus a number of simplifying assumptions are required, and these are discussed in the first three sections of the chapter. The chapter then develops, in sequential fashion, all the basic interactions between football players. The simplest isolated one defender versus one ball-carrier situation is studied, and then the concepts are generalized to eleven-player teams. Finally, the methods used to incorporate all the model's parts into a unified and flexible model are described.

Chapter IV takes the model from the previous chapter and shows the methods used to validate the model. Again, assumptions are needed and these are described. Data from game films are presented and compared to the model results. From this comparison, the hypothesis of the validity of the model is tested. Chapter V describes the offensive-defensive plays which were chosen for modeling. The chapter also lists the simulation results for these offensive-defensive pairs.

Chapters III, IV, and V deal with the development and validation of the model and the application of it to specific plays. With this as a basis, Chapter VI deals with how the results may be used to assist the decision-making process for the coaches. Both long-term and short-term planning is addressed, with an emphasis on frequently encountered (and highly publicized) decisions. Finally, Chapter VII reviews the research, states some conclusions, and lists some areas of possible further research.

In the immediately following pages, Chapter II addresses the literature relevant to the techniques required in Chapters III and VI. A survey is also presented of the modeling efforts in the sports field. The second half of Chapter II gives a statement of the problem and discusses the scope of the study.

CHAPTER II

LITERATURE SURVEY AND STATEMENT OF PROBLEM

This chapter provides a background for the topics addressed in the rest of the thesis. Section 2.1 presents a survey of the literature applicable to the thesis. Within this section, 2.1.1 describes the use of mathematical simulation as a modeling tool and 2.1.2 lists the literature of game theory. The various existing sports-related models are described in 2.1.3.

Section 2.2 presents a formal statement of the problem and the scope and limitations of the thesis.

2.1 Literature Survey

Although the literature abounds with the classical operations research techniques, relatively few apply these techniques to sports.

2.1.1 Mathematical Simulation

Thierauf and Grosse [21] describe Monte Carlo simulation in the following terms:

> Simulation involves the construction of some type of mathematical model that describes the system's operation in terms of individual events and components. The system is further divided into elements and the interrelationships of those elements with predictable behavior, at least in terms of a probability distribution, for each of the various possible states of the system and its inputs. . . .
>
> During World War II, psysicists at the Los Alamos Scientific Laboratory were puzzled by the behavior of neutrons. The two mathematicians [Von Neumann and Ulam] suggested a solution which amounted to submitting the problem to a roulette wheel. Step by

step, the probabilities of the separate events were merged into a total picture which gave an approximate but workable answer to a problem. Von Neumann gave it the code name "Monte Carlo" for the secret work of Los Alamos. The Monte Carlo method, which is actually the study of the laws of chance, was so successful on neutron diffusion problems that its popularity spread and included the area of operations research.*

In the post-World War II period, there are many published applications of the Monte Carlo simulation method. Most are straightforward applications of the procedure described above, differing only in the system to which the simulation is applied. Those studies involving sports are listed in Section 2.1.3.

2.1.2 Game Theory

The theory of games date to Von Neumann's first paper on the subject in 1928. The first comprehensive work on the subject, The Theory of Games and Economic Behavior [23], by Von Neumann and Morganstern appeared in 1944 and was hailed as "one of the major scientific achievements of the first half of the twentieth century." [6]** The theory provides a methodology to evaluate the various strategies available to two or more competivive players, relative to their respective payoffs for the resultant event. The concepts of pure and mixed strategies, value of the game, and the mini-max theorem which were first presented in this work now permeate game theory and mathematical programming.

Isaacs, in Differential Games [13], extended the theory to games

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*pages 471-472.

**pages 498-500.

where strategies were time dependent and differentiable. These games were motivated by, and possess a rich application in, pursuit and evasion games. The structuring of football as a pursuit-evasion game gives a natural application to a football model. Indeed, Isaacs gives several football examples to illustrate the theory; one of these is presented in Chapter III. Isaacs also introduces the use of kinematic equations as functions of the strategies of the players, a concept also used in Chapter III.

2.1.3 Mathematical Models of Sports

Baseball seems to be a topic inspiring the bulk of sports-related mathematical models. Cook presents a statistical analysis of professional baseball in Percentage Baseball [5]. Cook contends that many strategical decisions made by managers and universally accepted as "smart baseball" are in fact overrated and have no statistical justification. Horowitz presents in the Journal of Industrial Engineering [12] a decision-making model for coaches for baseball planning and trading decisions. Featherstone and Studenmund give in Research Quarterly [7] a regression model for baseball standings. The independent variables for this study are the team's earned-run average, batting average, number of home runs, and fielding average (the last term was found not statistically significant). The authors found that the model predicted the pennent winners and games wond remarkably well. Freeze [9] presents a Monte Carlo simulation of professional baseball and uses the model to analyze the possible batting order strategies. The model of baseball is taken from the Sports Illustrated Baseball Game. Freeze reports little difference (three extra games won or lost in a 162-game season) in the choices of the batting orders.

The sport of golfing is also the subject of mathematical models. The efforts in this area seem to concentrate on the problem of handicapping (i.e. making a fair match between two unequally skilled players). Scheid [20] fits a family of cubic equations to simulated data to derive a relationship between the probability of the stronger player winning, the handicap differential, and the handicap strokes allowed. Pollock [17] develops an analytical model which is interesting but requires a number of difficult assumptions to give manageable results.

The area of football has stimulated relatively fewer attempts at modeling. Carter (a former college and professional quarterback) and Machol [4] present a calculation of the expected point values of possession of the football with first-down and ten yards to go, based on National Football League data. Grouping the data into ten-yard strips, the authors arrive at a ten equations in ten unknowns system. By solving these equations, they conclude that a team which has first and ten behind its twenty yard line has a negative expected point value, a zero (approximately) value at the twenty yard line, four points at the opponent's twenty yard line and six points at the opponent's five yard line. Fitzgerald gives a game simulation in his Master's thesis [8]. The simulation concerns mainly an attempt to simulate strategic (run, pass, punt, or kick a field goal) decisions from game situations and translate these into game results.

The major practical use of computers in football at this time seems to be in scouting and game film data analysis. Wallace [24] describes a commercial computer package which reduces raw data inputted from game films to frequency data for the offense and defense of the team in question. This procedure is done manually in most cases anyway, and the package offers no modeling but is rather a more efficient data manipulation alternative. In the area of scouting, Zimmerman [26] reports three scouting syndicates for professional football, each of which use the computer to compile their scouting reports.

The modeling in the area of sports reported above are devoid of any analysis of the basic interactions of that sport. In the simulation of baseball, the strategy is concerned totally with decisions regarding players once they get on base--the relationships of balls and strikes to that probability of getting on base is not modeled. In golf, the model of the game consists of means and variances of the required shots for a hole (and some models eighteen holes), and not the modeling of club selection, the course layout, and means and variances of the various shots. In the football models, the game simulation is dependent on the distribution of gains of plays or classes of plays (runs, short passes, long passes, etc.) which are somehow estimated, without regard to who is playing and what specific play is called. In short, the kind of model to be presented in Chapter III does not appear in the literature--either from a lack of effort in this direction, or a lack of success.

Kinesiology, defined by Wells and Luttgens [25] as "the study of human movement from the point of view of the physical sciences,"* traces its origins to a remarkable series of lectures by Professor A. V. Hill, compiled in Living Machinery [11] in 1927. In this and following works, Hill introduces the application of physical laws to model human activity. The result most relevant to this work is his model of human running, which is identical in form to (3.8). Since that time, a considerable amount of effort has been expended to fit curves of a given form to world-record track times. These all are one-dimensional, at least implicitly in the model, even if the runners actually run around a track. Three of these models have enjoyed considerable success, and are mentioned below. Keller [14] gives a modification of Hill's equation to account for fatigue, which he calculates becomes a factor in his model at distances greater than 291 meters. Henry [10], with refinements by Purdy [18] give an equation relating average velocity as a summation of five exponential terms in the time run. The exponential form is motivated by models of physiological processes which are known to occur. The third model is an equation given by Ulbrich [22] relating time to a quadratic polynomial in the square root of distance. Despite the competing models, Hill's equation gives an adequate representation of runners at sprint distances, and is also flexible enough to model non-world record runners.

2.2 Statement of the Problem

A football player's position at a given time on the football

*page xiii.

field can be represented by a two-component vector $\vec{z}$.* This thesis treats the problem of describing, within the framework of a simulation model, the positions and velocities ($\vec{z}$ and $d\vec{z}/dt$, respectively) as functions of time for the twenty-two players involved in a running play. In the following development, passes are excluded in that they require a different philosophy for both the defense and offense.

Each individual has certain attributes which contribute to his effectiveness as a football player. These attributes, notably speed, strength and weight, are required to properly model the player's actions and reactions. This implies, in turn, some relationships between these attributes and the results on the playing field.

The first important aspect of the problem is to determine how individuals are able to move on the football field. To this end, mathematical representations of human motion must be applied in the model. The next aspect is to determine how a football player allocates his potential for movement. On the football field, this decision is dictated by the particular situation of each player and his relationship to other (team-mate and opposing) players. A justifiable decision-making criterion for each of these relationships must be found to model the possible situations which may arise. Finally, the major interactions--blocking and tackling--must be modeled.

The model specified is just that, a representation of what happens on a football field. The use of this model would come in drawing con-

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*$\vec{z}$ = (x , y) where the x-component is parallel to the yard markers and the y-component is parallel to the sidelines.

clusions about the game and the plays based on the outcomes of the simulation. The areas of these possible uses must therefore be investigated also.

2.3 Scope and Limitations

Some important aspects (to the players, coaches and fans) of football games will not be modeled here. These limitations, required either because of a dearth of supporting data or because of their second-order effects, include:

1. the effects the score or the down/yards-to-go have on the players
2. the effects of fatigue on the players
3. the effects of playing environment (weather, lighting conditions, playing surface, etc.) on the players
4. the mechanism whereby players determine distances and velocities of other players. (It is assumed each player knows these values for every other player.)

For convenience, the plays modeled assume that no fumbles take place. This could be changed with little difficulty, but then a new objective would have to be substituted for the ones in the model.

Penalty-free plays are the only ones considered. Again, these could be incorporated in the model, with the only added difficulty the minor problem of detecting the infractions.

Finally, it is implicitly assumed in the following development that a player's skill is completely represented by these attributes--his weight, his strength and his speed. These are important characteristics, and readily quantifiable ones, but no claim could be made that they are in fact the only ones. The problems of modeling what players and coaches consider important ("heart," determination, pride) are beyond the scope of the study, and perhaps the discipline, and are better left for future consideration.

CHAPTER III

DESCRIPTION OF THE MODEL

In Chapter II the statement of the problem was given. Chapter III develops the sub-models which describe the component parts of a play. These are then unified to obtain a complete model.

Section 3.1 addresses the football regulations which (under the specified scope of the thesis) represent constraints on the players.

In Section 3.2, Hill's equation is expanded into vector equations and generalized to include non-zero initial motions. These equations play a fundamental role in the subsequential analysis of the players' strategy, and thus considerable effort is taken to show the derivation.

Section 3.3 addresses some limitations placed on a player's means of actions due to his physical makeup. These physiological constraints are mathematically stated in terms of the formulas of Section 3.2. It must be stated that these constraints are in fact assumptions, and justifications of these assumptions, where possible, are included in the text.

With the first three sections of Chapter III as a foundation, Section 3.4 begins the analysis of determining the optimal strategies for the most simple one-on-one case. In 3.4.1 the analysis is for the offensive player's strategy, in 3.4.2 the defensive strategy is considered. Finally, in 3.4.3, the effect of using epochs with constant player strategies for the duration of those epochs is discussed. In 3.5 the one-on-one analyses are generalized and include the effects of team-mates blocking for the ball-carrier (3.5.1), the offensive blockers' strategies (3.5.2), and the M-on-N defensive reactions (3.5.3).

The blocking model is discussed next in Section 3.6. The simplest one-on-one block is considered in 3.6.1. The blocking model is broken into two component parts: 3.6.1.1 describes the delay distribution (the distribution of the length of time the block lasts) and 3.6.1.2 describes the players' resultant motion while the block continues. In 3.6.2, double-team blocking is considered, with 3.6.2.1 and 3.6.2.2 addressing the delay distributions and resultant motions, respectively. The tackling model is similar to the blocking model in many respects. Section 3.7 describes this model. It is perhaps important to reiterate that in Sections 3.6 and 3.7, whenever possible, physical laws are invoked.

Finally in Section 3.8, the model is unified. There are several techniques available to model plays realistically, and these are listed. Section 3.9 gives two examples.

3.1 Operational Constraints

Those football regulations regarding the players' positions and velocities will be considered as the operational constraints of the problem. These are not technically constraints (from the spectators' viewpoint) but rather infractions of the rules. The purpose here, however, is to model penalty-free plays; thus the consideration of the following as constraints is proper:

1. Eleven players on offense and eleven players on defense are required for each play.

2. Initial positions of the offensive and defensive players must be on their respective sides of the line of scrimmage. The initial velocities of the defensive players are arbitrary. All offensive players, except for one pass-elligible receiver or back, must be stationary at the beginning of each play. The movement of the one player who is allowed a non-stationary initial motion must be away from or parallel to the line of scrimmage.

3. If any player leaves the playing field (i.e. moves outside the sidelines) he is prohibited from having any further effect on the play. If the player leaving the field is the ball-carrier, the play is terminated with the gain the point the ball-carrier leaves the field of play.

4. If the ball-carrier's forward velocity is so impaired by the effects of a tackle that he ceases to gain yardage, and no probable occurance will change this situation, the play is terminated with the gain as the most forward point of the ball-carrier's position.

3.2 Kinematic Equations

The kinematic equations describe the relationship between a player's desire to move (to be called his strategy) and his resultant motion. These are fundamental relationships and all subsequent derivations will in some way depend on these results.

Let the vector $\vec{z}(t) = (\ x(t),\ y(t)\ )$ be a representation of a player's position from the assigned origin* at time t. Then $d\vec{z}(t)/dt$ and $d^2\vec{z}(t)/dt^2$ are the player's velocity and acceleration at time t, respectively. The goal in this section is to formulate a relationship

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*The origin is chosen to be midfield along the x-direction and on the line of scrimmage along the y-direction.

between the player's strategy and his resultant motion. The obvious first step is to apply Newton's Second Law:

$$\vec{F}(t) = m \frac{d^2\vec{z}(t)}{dt^2} \tag{3.1}$$

where $F(\vec{t})$ is the force the player generates (in the horizontal plane) and m is the player's mass. Assume $F(\vec{t})$ takes the form

$$\vec{F}(t) = m\vec{C}(t) - k_1 \frac{d\vec{z}(t)}{dt} \tag{3.2}$$

then

$$m \frac{d^2\vec{z}(t)}{dt^2} + k_1 \frac{d\vec{z}(t)}{dt} = m\vec{C}(t) \quad * \tag{3.3}$$

Setting $C_1 = k_1/m > 0$

---

*This equation is mathematically equivalent to that of pushing a mass along a horizontal surface with a given friction coefficient.

$$\frac{d^2\vec{z}(t)}{dt^2} + C_1 \frac{d\vec{z}(t)}{dt} = \vec{C}(t)$$

$$\frac{d^2\vec{z}(t)}{dt^2} e^{C_1 t} + C_1 \frac{d\vec{z}(t)}{dt} e^{C_1 t} = \vec{C}(t) e^{C_1 t} \tag{3.4}$$

$$\frac{d}{dt}\left[\frac{d\vec{z}(t)}{dt} e^{C_1 t}\right] = \vec{C}(t) e^{C_1 t}$$

Integrating (3.4) requires the form of $\vec{C}(t)$. Two cases will be considered.

<u>Case A</u>: $\vec{C}(t) = \vec{C} = \text{constant}$

Thus (3.4) becomes

$$\frac{d}{dt}\left[\frac{d\vec{z}(t)}{dt} e^{C_1 t}\right] = \vec{C}e^{C_1 t}$$

$$\frac{d\vec{z}(t)}{dt} e^{C_1 t} = \frac{C}{C_1} e^{\vec{C}_1 t} + \vec{K}_1 \tag{3.5}$$

$$\frac{d\vec{z}(t)}{dt} = \frac{\vec{C}}{C_1} + \vec{K}_1 e^{-C_1 t}$$

Applying initial condition at $t = 0$

$$\left.\frac{d\vec{z}(t)}{dt}\right|_{t=0} = \frac{\vec{C}}{C_1} + \vec{K}_1 \quad *$$

$$\vec{K}_1 = \frac{d\vec{z}(0)}{dt} - \frac{\vec{C}}{C_1}$$

and

$$\frac{d\vec{z}(t)}{dt} = \frac{\vec{C}}{C_1}\,(1-e^{-C_1 t}) + \frac{d\vec{z}(0)}{dt}\,e^{-C_1 t} \qquad (3.6)$$

Integrating once more

$$\vec{z}(t) = \frac{\vec{C}}{C_1}\,t + \left[\frac{\vec{C}-C_1(d\vec{z}(0)/dt)}{C_1^2}\right] + \vec{K}_2$$

Applying initial conditions

$$\vec{z}(0) = \left[\frac{\vec{C}-C_1(d\vec{z}(0)/dt)}{C_1^2}\right] + \vec{K}_2$$

$$\vec{K}_2 = \vec{z}(0) - \left[\frac{\vec{C}-C_1(d\vec{z}(0)/dt)}{C_1^2}\right]$$

---

*Let $\left.\frac{d\vec{z}(t)}{dt}\right|_{t=0} \equiv \frac{d\vec{z}(0)}{dt}$

and

$$\vec{z}(t) = \vec{z}(0) + \frac{\vec{C}}{C_1}t - \left[\frac{\vec{C}-C_1(d\vec{z}(0)/dt)}{C_1^2} \right] (1-e^{-C_1 t}) \tag{3.7}$$

For convenience later, this equation is broken into its coordinate parts:

$$x(t) = x(0) + \frac{C_x}{C_1}t - \left[\frac{Cx-C_1\dot{x}(0)}{C_1^2} \right] (1-e^{-C_1 t}) \tag{3.8}$$

$$y(t) = y(0) + \frac{C_y}{C_1}t - \left[\frac{Cy-C_1\dot{y}(0)}{C_1^2} \right] (1-e^{-C_1 t}) \tag{3.9}$$

Hill's equation is the one-dimensional equivalent for $x(0) = \dot{x}(0) = 0$ and C_x a constant for all time. The author's extension is to expand the equations to planar motion, allow initial positions and velocities different from zero, and to treat C_x (and C_y) as constrained variables (see the Physiological Constraints) rather than constants.

Case B: $\vec{C}(t) - \vec{C} + \vec{\gamma}t$; where $\vec{C}$ and $\vec{\gamma}$ are constants

Using a similar technique as that above, the integration gives

$$\frac{d\vec{z}(t)}{dt} = \frac{\vec{C}}{C_1} - \frac{\vec{\gamma}}{C_1^2} + \frac{\vec{\gamma}}{C_1}t$$

$$+ \left[\frac{d\vec{z}(0)}{dt} - \frac{\vec{C}}{C_1} + \frac{\vec{\gamma}}{C_1^2} \right] e^{-C_1 t} \tag{3.10}$$

and

$$\vec{z}(t) = \vec{z}(0) + \left[\frac{\vec{C}}{C_1} - \frac{\vec{\gamma}}{C_1^2} \right] t + \frac{\vec{\gamma}}{2C_1} t^2$$

$$- \left[\frac{\vec{C}}{C_1^2} - \frac{\vec{\gamma}}{C_1^3} - \frac{d\vec{z}(0)/dt}{C_1} \right] (1-e^{-C_1 t}) \tag{3.11}$$

Note that in the above equations

$[\vec{C}]$ = yards/second^2

$[C_1]$ = second^{-1}

$[\vec{\gamma}]$ = yards/second^3

$[t]$ = second

$[z(t)]$ = yards

$[d\vec{z}(t)/dt]$ = yards/second

The model treats the $\vec{C}$ and $\vec{\gamma}$ as variables to be determined by the process of optimizing various objectives. The following sections, in addition to various other topics, will deal with certain constraints that define the feasible values for these variables.

3.3 Physiological Constraints

Physiological constraints are those limitations placed on a player's actions due to either his particular physical make-up or physical characteristics common to all players. The constraints used in the model are:

1. No direct interaction (i.e. blocking or tackling) can occur at distances greater than one yard. Indirect interactions (influencing an individual's strategy and thus his position or velocity by one's presence) may occur at any distance.

2. $||\vec{C}|| \leq C$, where C is a constant suitably chosen. For an heuristic justification of this, note the case $\dot{x}(0) = \dot{y}(0) = x(0) = y(0) = 0$. Then (3.8) and (3.9) become

$$\dot{x}(t) = \frac{C_x}{C_1}(1-e^{-C_1 t}) \leq \frac{C_x}{C_1} \tag{3.12}$$

$$\dot{y}(t) = \frac{C_y}{C_1}(1-e^{-C_1 t}) \leq \frac{C_y}{C_1} \tag{3.13}$$

and

$$||\frac{d\vec{z}(t)}{dt}|| \leq \frac{1}{C_1} \sqrt{C_x^{\,2} + C_y^{\,2}} \tag{3.14}$$

Also, as $t \to \infty$, $\dot{x}(t) \to C_x/C_1$, $\dot{y}(t) \to C_y/C_1$, and $||\frac{d\vec{z}(t)}{dt}|| \to \frac{\sqrt{C_x^2 + C_y^2}}{C_1}$.

Experience dictates that a runner starting from a standstill and sprinting in a straight line should increase his velocity until it reaches a maximum (or asymtotically approaches it). The velocity will then stay relatively constant until fatigue sets in--usually after one hundred yards or so. A reasonable constraint for this individual running in a straight line (say the y-direction) is $|C_y| \leq C$. The generalization suggested by (3.14) is

$$\sqrt{C_x^2 + C_y^2} \leq C \qquad (3.15)$$

or

$$||\vec{C}|| \leq C \qquad (3.16)$$

3. C_1 does not depend on the individual player but rather is constant for all players. There is really only one justification for this assumption: there is no data available to determine what the value should be for every player on a team. The remainder of the formulation would not require this assumption; if the various values of C_1 were available, the formulas could be modified to reflect this refinement.

4. A player can make his choice of $\vec{C}(t)$ only at the beginning of an epoch. The value of $\vec{C}(t)$ then remains constant until the next epoch when he is allowed a new choice. There is a physiological justification for this criterion in that it is physically difficult to change directions except at the instant that one's feet hit the ground. Taking each epoch as that instant, the above assumption is that a close approximation results if it is assumed that all the players take steps (of equal duration) simultaneously.

3.4 One-On-One

The case of the offensive ball-carrier isolated on one defensive defender rarely occurs in the course of a play. This one-on-one case, however, is the simplest of all indirect interactions, and the analysis of this restricted case is necessary prior to the more general formulations.

Consider two players: player E is an offensive ball-carrier with a given C^E, player P is a defensive player with a given C^P, and their respective positions and velocities are known to each other. (The P and E are for pursuer and evader as in the literature of pursuer-evader games).

How then, should $\vec{C}^P$ and $\vec{C}^E$ be reasonably chosen? Isaacs [13] presents a differential game example* where the initial velocities are zero and C^P and C^E are equal.

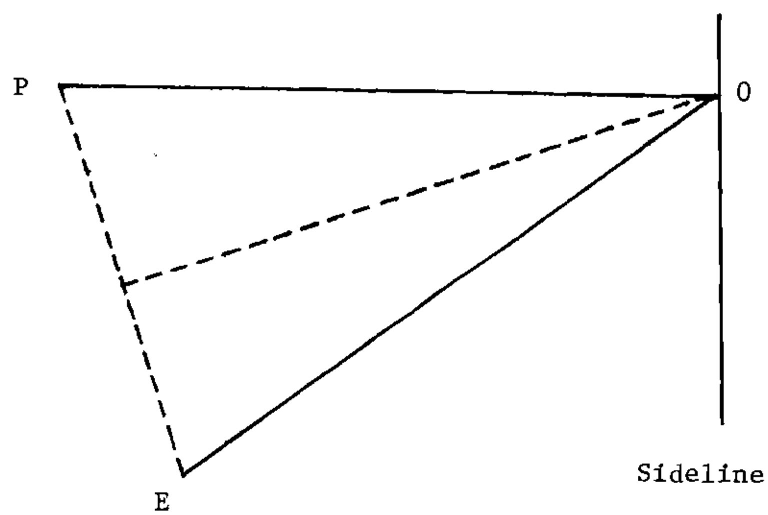

Figure 3-1. Isaacs' Differential Football Game

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*p. 146.

Using symmetry arguments, Isaacs reasons the optimal strategies are for both players to run to O. By doing so E will maximize the y-component of the intercept point while P simultaneously minimizes the y-component of the intercept point. Figure 3-1 can be easily modified to take into account that P and E are not points but rather (in two dimensions) circles of diameter 1.

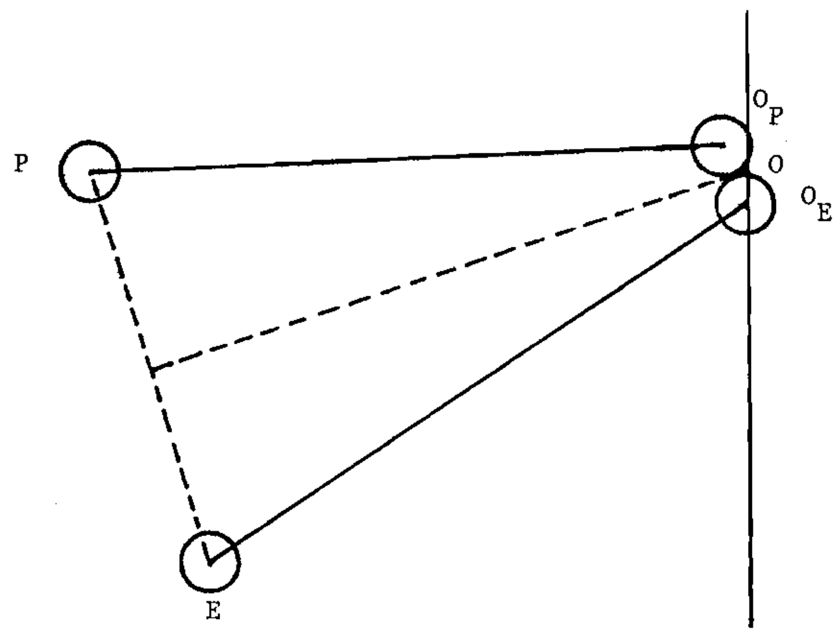

Figure 3-2. Optimal Strategy When Players Are Circles of Diameter 1

In this case P should run toward O_P and E should run toward O_E. The concept of this mini-max criterion will be used to develop E's objective function.

3.4.1 Offense: Ball-Carrier

The mini-max criterion is applied as follows: E chooses his $\vec{C}^E$ in such a way that the y-component of the worst-case intercept point is maximized. A more precise description of E's decision process is

$$\text{maximize } y^E[t^*(\vec{C}^E)] \tag{3.17}$$

$$\text{s.t.} \qquad t^*(\vec{C}^E) = \inf_{\{\vec{C}^P\}} [T(\vec{C}^P, \vec{C}^E)] \tag{3.18}$$

$$T(\vec{C}^P, \vec{C}^E) = \{t: \ ||\vec{z}^E(t) - \vec{z}^P(t)|| \leq 1 \text{ or } |y^E(t)| \geq 26.67\} \tag{3.19}$$

$$t^*(\vec{C}^E) \geq 0 \tag{3.20}$$

$$0 \leq t \leq t^*(\vec{C}^E) \tag{3.21}$$

$$\frac{d\vec{C}^E}{dt} = \frac{d\vec{C}^P}{dt} = 0 \tag{3.22}$$

$$|y^E(t)| \leq 26.67 \tag{3.23}$$

$$|y^P(t)| \leq 26.67 \tag{3.24}$$

$$||\vec{C}^E|| \leq c^E \tag{3.25}$$

$$||\vec{C}^P|| \leq c^P \qquad (3.26)$$

The objective function (3.17) seeks to maximize the y-component of the intercept point, where the time of interception is $t*(\vec{C}^E)$. Constraint (3.19) defines the intercept requirements, and (3.18) is a constraint which forces the ball-carrier to assume P's motion is such that the time of interception is minimized. This is obviously a valid requirement if $dy^E(t)/dt \geq 0$ for all $0 \leq t \leq t*(\vec{C}^E)$ and all $\vec{C}^E$. If $dy^E(t)/dt < 0$, or "backwards" motion is to be allowed, the problem becomes more difficult (it is in the defender's interest to allow the ball-carrier more time to run--but only if doing such does not eventually result in a larger gain). These difficulties can be circumvented if

1. only $C_y^E \geq 0$ are allowed
2. $dy^E(0)/dt < 0$ only in cases when $dy^E(t)/dt \geq 0$ for all $t \in T(\vec{C}^P, \vec{C}^E)$

With these restrictions, (3.18) becomes a logical mini-max criterion for E to follow. Constraints (3.20) and (3.21) force considerations to be in the future and prior to (or at) the end of the play. Constraint (3.22) dictates the Case A equations are to be used. Constraints (3.23) and (3.24) require both players to remain on the playing field during the course of the play. Finally, (3.25) and (3.26) require the $\vec{C}^E$ and $\vec{C}^P$ that are chosen meet the physiological velocity constraint.

If $\vec{C}^E \neq (0,C_y^E)$, then $T(\vec{C}^P,\vec{C}^E)$ will not be null. If $\vec{C}^E = (0,C_y^E)$, $T(\vec{C}^P,\vec{C}^E)$ could conceivably be null (i.e. E would never move out-of-bounds, and P is located in such a position and has such a C^P that no intercept is possible). In this case, $\vec{C}^E$ is set to $(0,C^E)$.*

It is important to note that the calculations above imply that both $\vec{C}^E$ and $\vec{C}^P$ are constant for the duration of the play. In fact, this is not the case in the simulation. Rather, $\vec{C}^E$ is held constant only for the duration of the epoch. Then the process of generating a new $\vec{C}^E$ is repeated based on new (updated) inital conditions.

The crux of solving the problem is finding a way to find $T(\vec{C}^P,\vec{C}^E)$ and $t^*(\vec{C}^E)$. The method chosen is to solve for T and t* for each given value of $\vec{C}^E$. Rewriting (3.19)

$$T(\vec{C}^P,\vec{C}^E) = \left\{t: \ ||\vec{z}^E(t) - \vec{z}^P(t)|| \leq 1\right\} \cup \left\{t: \ |y^E(t)| \geq 26.67\right\} \tag{3.28}$$

$$\equiv T_1(\vec{C}^E,\vec{C}^P) \cup T_2(\vec{C}^E)$$

and using (3.18)

$$t^*(\vec{C}^E) = \min \left[\inf_{\{\vec{C}^P\}} T_1(\vec{C}^E,\vec{C}^P)\right], \left[\inf T_2(\vec{C}^E)\right]$$

$$= \min [t_1,t_2]$$

---

*In every other case the problem could be simplified greatly by noting that for the optimal $\vec{C}^P$, $x^P(t^*(\vec{C}^E)) = 26.67$. This does not generalize to the M-on-N case, however, so this aspect will not be utilized.

where $t_1 = \inf T_1$ and $t_2 = \inf T_2$. Assuming $T_2(\vec{C}^E)$ is non-empty ($C_x^E \neq 0$ is sufficient), and using arguments on the continuity of $y^E(t)$ and $|y^E(0)| \leq 26.67$, then t_2 must be the smallest value which satisfies $|y^E(t_2)| = 26.67$. Thus t_2 is found by increasing t from 0.1 seconds by 0.1 second increments and checking to see if $|y^E(t)| \geq 26.67$. If this has not occurred by t = 10.0 seconds, then $T_2(\vec{C}^E)$ is assumed null, and t_2 set to infinity. If a t is found so that $|y^E(t)| \geq 26.67$, then a value for which $|y^E(t)| = 26.67 \pm 0.1$ is found, and set to t_2.

Assuming T_1 non-empty, and again arguing on the continuity of $\vec{z}^E(t)$ and $\vec{z}^P(t)$ and the fact that $||\vec{z}^E(0) - \vec{z}^P(0)|| > 1$, it is clear that t_1 is the smallest value of t that satisfies

$$||\vec{z}^E(t) - \vec{z}^P(t)|| = 1$$

$$||\vec{z}^E(t) - \vec{z}^P(t)||^2 = 1 \tag{3.29}$$

$$[x^E(t) - x^P(t)]^2 + [y^E(t) - y^P(t)]^2 = 1$$

Since $\vec{C}^E$ is specified, $x^E(t)$ and $y^E(t)$ are known for all $t \geq 0$. For any value of t, then, the only variables to be determined are C_x^P and C_y^P. Applying (3.8) and (3.9)

$$k_1^2[C_x^P - k_2]^2 + k_1^2[C_y^P - k_3]^2 = 1 \tag{3.30}$$

where

$$k_1 = \frac{t}{C_1} - \frac{(1-e^{-C_1 t})}{C_1^2} \tag{3.31}$$

$$k_2 = C_x^E - \frac{[x^P(0) - x^E(0)]}{k_1} - \frac{[\dot{x}^P(0) - \dot{x}^E(0)]}{C_1 k_1}(1-e^{-C_1 t}) \tag{3.32}$$

$$k_3 = C_y^E - \frac{[y^P(0) - y^E(0)]}{k_1} - \frac{[\dot{y}^P(0) - \dot{y}^E(0)]}{C_1 k_1}(1-e^{-C_1 t}) \tag{3.33}$$

Applying constraint (3.26)

$$\sqrt{(C_x^P)^2 + (C_y^P)^2} \leq C^P$$

$$(C_x^P)^2 + (C_y^P)^2 \leq (C^P)^2 \tag{3.34}$$

Figure 3-3 below shows graphically (3.30) and (3.34):

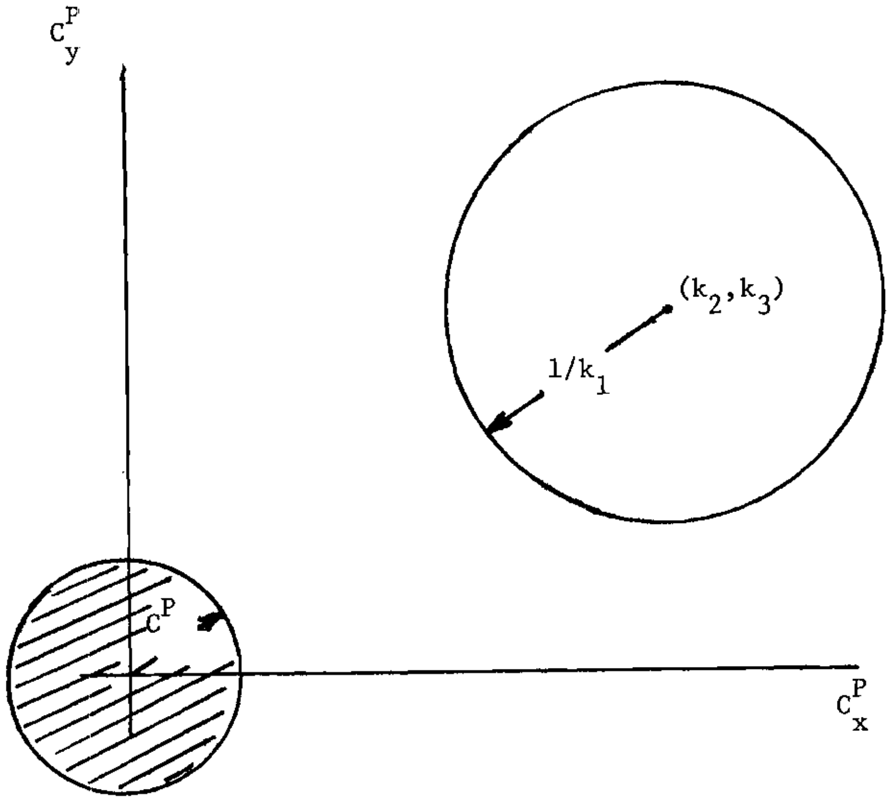

Figure 3-3. Geometric Interpretation of Equation (3.29)

Equation (3.34) describes a disc centered at the origin of the C_x^P - C_y^P plane, and with a radius of C^P. Equation (3.30) describes a circle centered at (k_2,k_3) with radius $1/k_1$. For $t = 0$, the circle is ill-defined, but for all $t > 0$, it is possible to determine if the disc and circle intersect. The value for t_1 is precisely the smallest value for which they do, or equivalently

$$\sqrt{k_2^2 + k_3^2} \leq C^P + \frac{1}{k_1} \tag{3.35}$$

This is solved by allowing t to vary between 0.01 seconds and 10.0

seconds by 0.05 second increments. If at $t = 10.0$ seconds, no value has satisfied (2.35), T_1 is assumed null and t_1 is set to infinity. If, however, some $t < 10.0$ seconds is found that satisfies (3.35), a value of t is found so that $\sqrt{k_2^2 + k_3^2} = C^P + 1/k_1 \pm 0.05$ and t_2 is set to this value.

Once t_1 and t_2 are found, it is a simple matter to use (3.28) to determine $t^*(\vec{C}^E)$, which is of course dependent on which value of $\vec{C}^E$ is used. (If both t_1 and t_2 are infinite, this implies that $T(\vec{C}^P, \vec{C}^E)$ is null.) This value of $t^*(\vec{C}^E)$ can be used to evaluate the objective function. A closed form representation of (3.17) as a function of $\vec{C}^E$ is difficult. Thus, instead of evaluating analytically all possible $\vec{C}^E$, a representative (finite) sample is selected by letting

$$C_x^E = C^E \cos\theta \tag{3.36}$$

$$C_y^E = C^E \sin\theta \tag{3.37}$$

for θ ranging between 0° and 180° in increments of 2°. If a more accurate determination were required, smaller increments or an appropriate search scheme could be utilized.

3.4.2 Defense

The mini-max criterion for a defensive player is appealing--much of the analysis for the defensive ball-carrier would apply. But consider the Isaacs example of before. If after an epoch a player moves a certain distance, say Δ, then Figure 3-4 shows the optimal case:

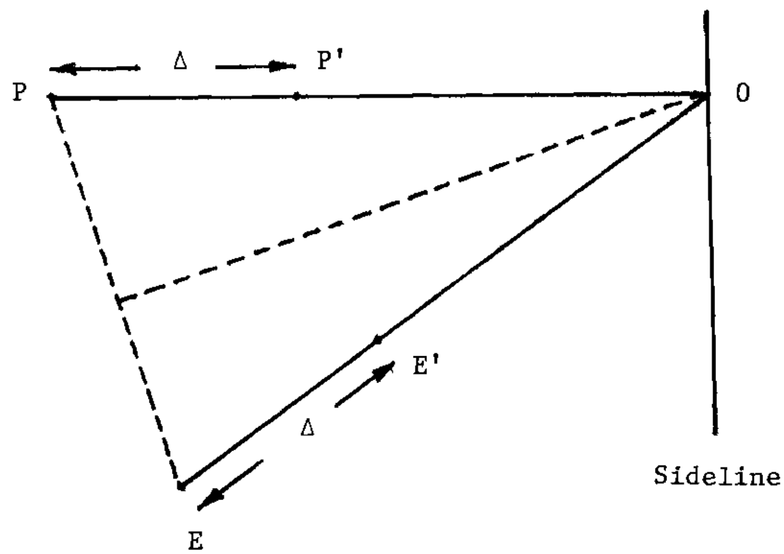

Figure 3-4. Defensive Game, "Optimal" Case

P goes to P' along the path PO and E goes to E' along EO. The problem of determining optimal strategies for the next epoch would give identical strategies as this epoch for both P and E. Thus O would seem to be the optimal gain.

Consider the results if E should select a "non-optimal" strategy:

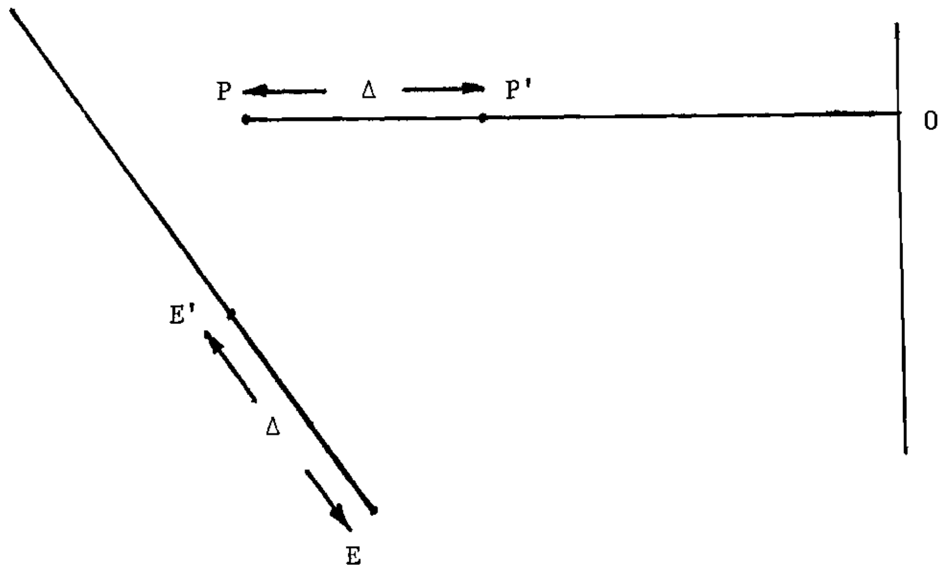

Figure 3-5. Defensive Game: "Non-Optimal" Case, Epoch 1

The situation is then illustrated in Figure 3.6 for the next eopch:

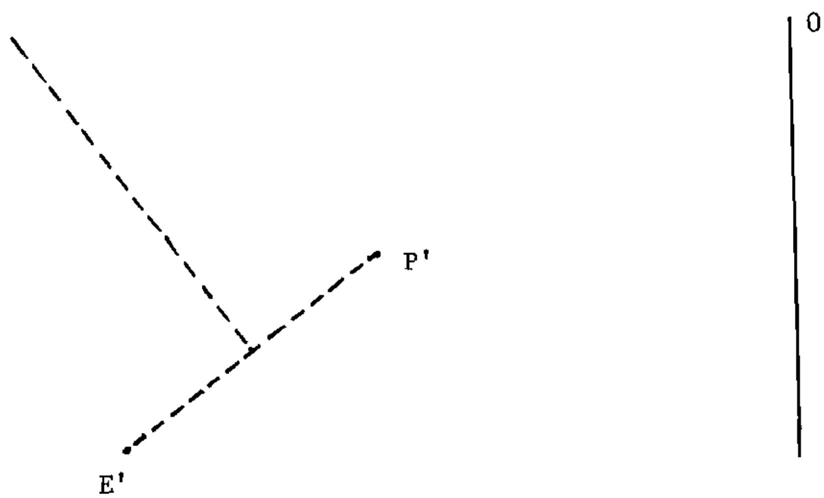

Figure 3-6. Defensive Game: "Non-Optimal" Case, Epoch 2

It is obvious that P is in a considerably worse position at the second epoch (not even taking into account P's velocity vector is toward the near sideline while E's velocity is toward the far sideline, making the situation favor E further). What happened to the "optimal strategy" and "optimal gain"?

The answer is that an assumption made in elementary differential games is that all contestants have instantaneous response. But this assumption violates the fourth physiological constraint: the simulation requires constant $\vec{C}^E$ and $\vec{C}^P$ over the span of an epoch while the differential game requires them to be differentiable. The differentiable game counterpart to E's non-optimal strategy of Figure 3-6 would be as below:

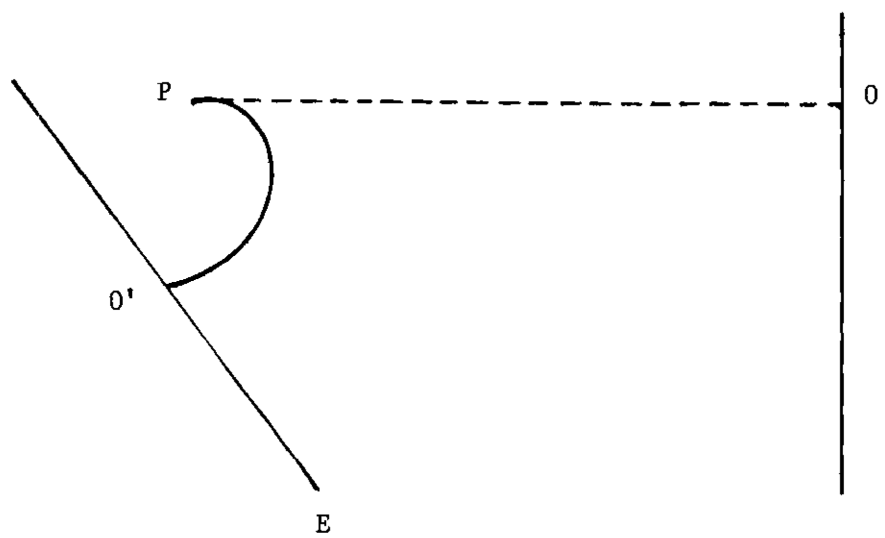

Figure 3-7. Defensive Game, Differential Case

Since 0 is further downfield than 0', the E0 strategy is to be preferred by E.

Although the paradox is resolved, it does no good in enlightening an adequate strategy for P. Some points which may help in the formulation of P's objective function below:

1. P cannot be as aggressive as E. It is P who is on the spot. He must allow E to commit himself but retain the option of being able to react to any movement.*

2. P does not want to allow E to "beat" him (i.e. E attains P's y-coordinate position without a direct interaction occurring) if he can help it. P can control this (except in extreme cases) by how aggressive he is.

3. P is most vulnerable to abrupt changes in E's x-coordinate velocity over the span of one or two epochs. This is so because of P's minimal capacity for reaction over short time periods. In the language of the game, E "knows where he's going" while P does not.

P's decision process is divided into two sections in the following description: the choice of C_y^P (governed by point 2 above) and the choice of C_x^P (governed by point 3). The two parts are related, of course, by $\sqrt{(C_x^P)^2 + (C_y^P)^2} \leq C^P$. Unlike the decision process for E, situations may arise when the physiological velocity constraint will not be tight. This is directly attributable to the first point above.

Assuming $y^P(0) > y^E(0)$ for the time being, P's decision process for selecting C_y^P is to choose the minimum value for which E cannot beat P:

$$\text{minimize } C_y^P \tag{3.38}$$

$$\begin{aligned} \text{s.t. } & x^{P-}(t'(\vec{C}^E)) - 1 \leq x^E(t'(\vec{C}^E)) \leq x^{P+}(t'(\vec{C}^E)) + 1 \\ & \text{or } |x^E(t'(\vec{C}^E))| \geq 26.67 \end{aligned} \tag{3.39}$$

---

*It is heartening to know that the edge in one-on-one interactions is considered to be decidedly on the side of the offensive player.

$$t'(\vec{C}^E) = \inf\{t:\ \ y^P(t) = y^E(t)\} \tag{3.40}$$

$$||\vec{C}^E|| \leq C^E \tag{3.41}$$

$$||\vec{C}^P|| \leq C^P \tag{3.42}$$

where $x^{P-}(t^*(\vec{C}^E))$ and $x^{P+}(t^*(\vec{C}^E))$ are evaluated using $C_x^P = -\sqrt{(C^P)^2 - (C_y^P)^2}$ and $C_x^P = +\sqrt{(C^P)^2 - (C_y^P)^2}$ respectively. It may be that there is no C_y^P satisfying both (3.39) and (3.42). In this case, $C_y^P = C^P$ and $C_x^P = 0$.

The determination of C_y^P from the requirements of (3.28) to (3.41) is fairly straight-forward. A trial value for C_y^P (0 initially) is chosen and using this value constraint (3.39) is checked. This is done similarly to the offensive ball-carrier portion (letting $C_x^E = C^E \cos\theta$ and $C_y^E = C^E \sin\theta$, and θ ranging from 0° to 180° by increments of 2°) and solving (3.40) iteratively for $t'(\vec{C}^E)$

$$y^P(t') = y^E(t') \tag{3.43}$$

$$\begin{aligned}&\left[y^P(0) - y^E(0)\right] + \frac{1}{C_1}\left[C_y^P - C_y^E\right]\\ &\quad - \frac{1}{C_1^2}(1-e^{-C_1t'})\left[(C_y^P - C_y^E) - C_1(\dot{y}^P(0) - \dot{y}^E(0))\right] = 0\end{aligned} \tag{3.44}$$

$$k_4 + k_5t' = k_6(1-e^{-C_1t'}) \tag{3.45}$$

where

$$k_4 = y^P(0) - y^E(0) \tag{3.46}$$

$$k_5 = \frac{C_y^P - C_y^E}{C_1} \tag{3.47}$$

$$k_6 = \frac{-k_5}{C_1} + \frac{[\dot{y}^P(0) - \dot{y}^E(0)]}{C_1} \tag{3.48}$$

Assume $k_5 \neq 0$. Letting the subscripts of t' denote the iteration number, set

$$t_0' = -\frac{k_4 + k_6}{k_5} \tag{3.49}$$

Letting

$$E_k = k_4 + k_5\, t_k' + k_6(1-e^{-C_1 t_k'}) \tag{3.50}$$

$$E_k' = k_5 + C_1 k_6 e^{-C_1 t'_k} \tag{3.51}$$

and

$$t_{k+1}' = t_k' - \frac{E_k}{E_k'} \tag{3.52}$$

The procedure stops when $|E_k/E_k'| \leq 0.001$.

If $k_5 = 0$, then (2.49) is not defined, but (2.45) becomes

$$k_4 + k_6(1-e^{-C_1 t'}) = 0 \qquad (3.53)$$

$$t' = -\frac{1}{C_1} \ln (1 + \frac{k_6}{k_4}) \qquad (3.54)$$

which is defined only if $k_6/k_4 \geq -1$ and feasible only if $k_6/k_4 \leq 0$.

Once the value of $t'(\vec{C}^E)$ is obtained (there is a different value for each θ), constraint (3.40) is checked. If (3.40) is satisfied, the next trial value is decreased. If (3.40) is not satisfied, the next trial value is increased. Eventually, two values of C_y^P are produced: a larger value for which E cannot beat P and a smaller value for which he can. The optimum is between these, and a search is employed to find it. Although this procedure is straight-forward, it is also time-consuming for there are 90 computations of $t'(\vec{C}^E)$ for each value of C_y^P--of which there may be as many as 50.

When the value of C_y^P has been determined, the feasible C_x^P is $|C_x^P| \leq \sqrt{(C^P)^2 - (C_y^P)^2}$. The choice of C_x^P will be such that

$$x^P(t') = x^{E^o}(t') \qquad (3.55)$$

$$\dot{x}^P(t') = \dot{x}^{E^o}(t') \qquad (3.56)$$

where

$$t' = \inf_{\vec{C}^E} [t'(\vec{C}^E)] \tag{3.57}$$

and $x^{E^o}(t')$ is evaluated with $C_x^E = 0$. This is justified on the grounds that P has no prior knowledge of which C_x^E will be chosen and the reaction based on $C_x^E = 0$ does not commit P unnecessarily.

The first thing of interest to note about (3.55) and (3.56) is that if the Case A kinematic equations are used, the problem is over-determined. Yet the third point above is negated if either the x-coordinate position or velocity constraints are relaxed. Thus the Case B equations will be used for P's x-coordinate motion in the determination of C_x^P. (Note that the Case B assumption as to the nature of $\vec{C}^P(t)$ is not allowed by the requirement of constant $\vec{C}^P$ within an epoch--more about this later.) Applying the Case B equations to (3.55)

$$\begin{aligned} x^P(0) &+ \left(\frac{C_x^P}{C_1} - \frac{\gamma_x^P}{C_1^2} \right) t' + \frac{\gamma_x^P}{2C_1} (t')^2 \\ &- \left(\frac{C_x^P}{C_1^2} - \frac{\gamma_x^P}{C_1^3} - \frac{\dot{x}^P(0)}{C_1} \right) \left(1-e^{-C_1 t'} \right) \\ &= x^E(0) + \frac{\dot{x}^E(0)}{C_1} \left(1-e^{-C_1 t'} \right) \end{aligned} \tag{3.58}$$

or

$$k_7 C_x^P + k_8 \gamma_x^P = k_9 \tag{3.59}$$

where

$$k_7 = -\frac{t'}{C_1} - \frac{(1-e^{-C_1 t'})}{C_1^2} \tag{3.60}$$

$$k_8 = -\frac{k_7}{C_1} + \frac{(t')^2}{2C_1} \tag{3.61}$$

$$k_9 = [x^P(0) - x^E(0)] + [\dot{x}^P(0) - \dot{x}^E(0)]\,\frac{(1-e^{-C_1 t'})}{C_1} \tag{3.62}$$

Likewise applying the Case B equations to (3.56)

$$\frac{C_x^P}{C_1} - \frac{\gamma_x^P}{C_1^2} + \frac{\gamma_x^P}{C_1}\,t' + \left[\dot{x}^P(0) - \frac{C_x^P}{C_1} + \frac{\gamma_x^P}{C_1^2}\right] e^{-C_1 t'}$$

$$= \dot{x}^E(0)\ e^{-C_1 t'} \tag{3.63}$$

or

$$k_{10} C_x^P + k_7 \gamma_x^P = k_{11} \tag{3.64}$$

where

$$k_{10} = \frac{(1-e^{-C_1 t'})}{C_1} \tag{3.65}$$

$$k_{11} = [\dot{x}^P(0) - \dot{x}^E(0)]\, e^{-C_1 t'} \tag{3.66}$$

Thus

$$C_x^P = \frac{k_7 k_9 - k_8 k_{11}}{k_7^2 - k_8 k_{10}} \tag{3.67}$$

$$\gamma_x^P = \frac{k_7 k_{11} - k_9 k_{10}}{k_7^2 - k_8 k_{10}} \tag{3.68}$$

If $C_x^P(t) = C_x^P + \gamma_x^P t$ then (3.55) and (3.56) would be satisfied. The fourth physiological constraint forbids this, however. As a compromise, then $\frac{1}{2}\gamma_x^P t_e$ (where t_e is the length of an epoch) is added to (3.67) to obtain the chosen value for C_x^P. It may happen that $|C_x^P| > \sqrt{(C^P)^2 - (C_y^P)^2}$, resulting in infeasibility. C_x^P is then set to its extreme limit. The nonoptimality of the strategy should not be cause for alarm, since the constraints on the selection of C_y^P insures an interaction at the worst-case $\vec{C}^E$.

If C_x^P is chosen as above, it may occur that $|x^P(t')| > 25.67$. This is obviously not a good strategy. Instead, if a C_x^P were chosen so that $|x^P(t')| = 25.67$, an interaction is guaranteed on the outside, while P is also better able to guard the inside of the field. Thus solving for C_x^P in (3.8)

$$x^P(0) + \frac{C_x^P}{C_1} t' - \left(\frac{C_x^P - C_1\dot{x}^P(0)}{C_1^2}\right)\left(1-e^{-C_1 t'}\right) = \pm 25.67 \quad (3.69)$$

$$C_x^P = \frac{[\pm 25.67 - x^P(0)] - \frac{1}{C_1}\dot{x}^P(0)(1-e^{-C_1 t'})}{k_7} \quad (3.70)$$

where the applicable sign for 25.67 is chosen.

If $y^P(0) \leq y^E(0)$, P has already been beaten. Here $\vec{C}^P$ is $C^P\vec{d}_{PE}$ where $\vec{d}_{PE}$ is a unit vector from P to E.

3.4.3 The Use of Epochs

It is instructive to consider the implications of the use of epochs and the requirement of a constant $\vec{C}$ within the epochs. If this were not assumed, and the relaxed assumptions of differential game theory (i.e. $\vec{C}(t)$ differentiable) were used, then even for the one-on-one case, the solution for the "best" strategy is very difficult. For example by basing the choice of $\vec{C}^E$ on the proposition that the choice will remain constant until the end of the play (thus allowing the Case A equations to be used in the analysis), the situation is revised to an infinite game. Further restrictions on the allowable θ change it to a finite game. From the geometry of the situation, it is also clear that the game possesses a saddle point. (Note that the above game is only being played optimally by the offensive ball-carrier.

3.5 M-on-N

This section expands the previous one-on-one development to include the effects of multiple defenders and offensive blockers. Of the M offensive players, one is the ball-carrier (denoted by E_o) and M-1 are offensive blockers (denoted by E_i, i = 1, 2, ..., M-1*).

3.5.1 Offense: Ball-Carriers

If the same technique is used as developed above, for each value of θ, there are associated y^i, i = 1, 2, ..., N where the defenders can first interact with the ball-carrier. Also associated with the θ (except possibly $\theta = 90^o$) is a y^o corresponding to the first time the ball-carrier goes out-of-bounds.

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*As shall be shown, the concept of blockers as "evaders" is something of a misnomer, but the terminology is kept to avoid confusion regarding the identification of team-mates.

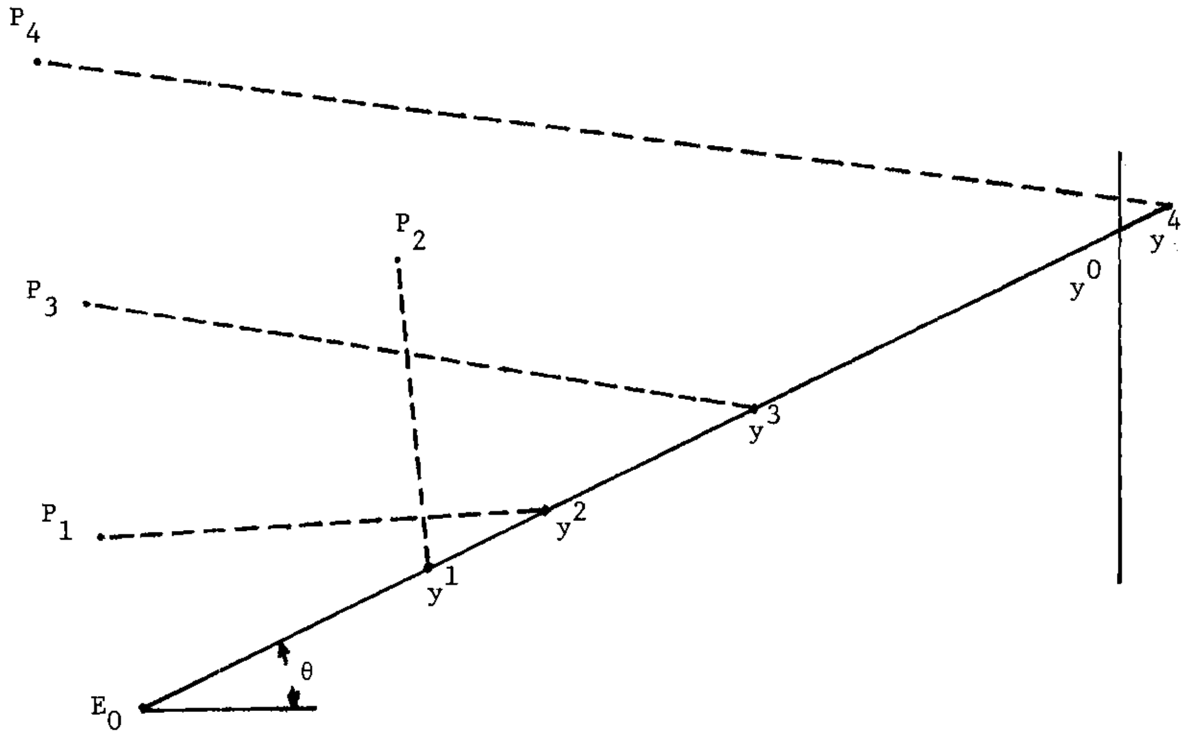

Figure 3-8. Offensive y^i as a Function of θ

The locus of E_o's position for a given θ will be a line only if $d\vec{z}^o(0)/dt$ is parallel to $(\cos\theta, \sin\theta)$ or is zero. In the general case $\vec{z}^o(t)$ is a curve.

The analysis in the one-on-one case centered on the location of the first interaction. This clearly generalizes the one-on-N case. In the figure above, y^1 would be the value corresponding to θ. The complications of the M-on-N case arise in evaluating the offensive blockers' potential for making a successful block and thereby reducing the capacity for the opponents to achieve a direct interaction.

Let t_1^* denote the first possible time of interaction with P_1, t_2^* the first time of interaction with P_2, etc. E_i has the capability to achieve a block of P_i if there exists a feasible strategy such that

$\vec{z}^{E_i}(t_i^*) = \vec{z}^{o}(t_i^*)$. Modifying (3.30) to

$$k_1^2[C_x^{E_i} - k_2]^2 + k_1^2[C_y^{E_i} - k_3] = 0 \tag{3.71}$$

or

$$C_x^{E_i} = C_x^{P_i} - \left[\frac{x^{E_i}(0) - x^{P_i}(0)}{k_1}\right] - \left[\frac{\dot{x}^{E_i}(0) - \dot{x}^{P_i}(0)}{C_1 k_1}\right](1-e^{-C_1 t_i^*}) \tag{3.72}$$

$$C_y^{E_i} = C_y^{P_i} - \left[\frac{y^{E_i}(0) - y^{P_i}(0)}{k_1}\right] - \left[\frac{\dot{y}^{E_i}(0) - y^{P_i}(0)}{C_1 k_1}\right](1-e^{-C_1 t_i^*}) \tag{3.73}$$

where

$$k_1 = \frac{t_i^*}{C_1} - \frac{(1-e^{-C_1 t_i^*})}{C_1^2} \tag{3.74}$$

If

$$\sqrt{(C_x^{E_i})^2 + (C_y^{E_i})^2} \leq C^{E_i} \tag{3.75}$$

then E_i has the capability of successfully blocking P_i. Such is the case in Figure 3-9 below.

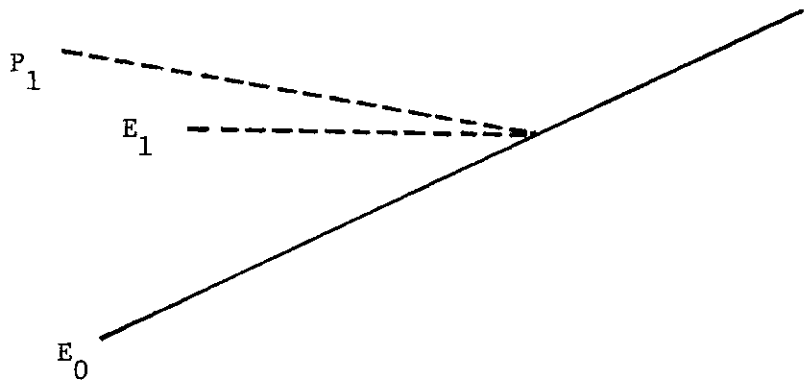

Figure 3-9. Offensive Blocking

Two obvious decision rules for E_o present themselves. The first seeks to maximize the y-coordinate of the first possible interaction, regardless of the blocking potentials. This criterion is predicted on the assumption that even though the possibility for a block may exist, no block will take place. The second decision rule is to maximize the y-coordinate of the first possible interaction for which an offensive blocker cannot make a successful block. This presupposes that any player who can position himself for the block will be successful. The decision rule which will be used is to place a probability for a successful block on each interaction, and then maximize the expected y-coordinate of those resultant interactions. If the probability of a successful block is zero, the first criterion results. If the probability is one, the second criterion is obtained.

The ordering of the defensive players (P_j, j = 1, 2, ..., N) is done so that $t^*_1 \leq t^*_2 \leq . . .$ (and thus $y^1 \leq y^2 \leq . . .$). If the probability of E_j successfully blocking P_j is Pr_j, then

$$\text{Prob (gain} = y^1) = 1 - Pr_1 \qquad (3.76)$$

$$\text{Prob (gain} = y^2) = Pr_1 \ (1 - Pr_2) \qquad (3.77)$$

$$\text{Prob (gain} = y^3) = Pr_1 \ Pr_2 \ (1 - Pr_3) \qquad (3.78)$$

etc.

if the independence of the Pr_j is assumed. Thus the expected gain is

$$E(\text{gain}) = y^1(1-Pr_1) + y^2Pr_1(1-Pr_2) + y^3Pr_1Pr_2(1-Pr_3) + \ldots \qquad (3.79)$$

The value of the probability of a successful block when no successful block is possible (or there is no one assigned to the defensiveman) is clearly zero. Likewise if the y-coordinate represents the point where E_o goes out-of-bounds (i.e. y^0), the associated probability is also zero. For the case where a successful block is possible, the assumed form is

$$Pr_j = k_{12} + (1-k_{12})\left(\frac{||\vec{C}^{P_j}||}{C^{P_j}} - \frac{||\vec{C}^{E_j}||}{C^{E_j}}\right) \tag{3.80}$$

but since t_j^* is defined as the first possible time of interaction, $||\vec{C}^{P_j}||/C^{P_j} = 1$ so (3.80) becomes

$$Pr_j = k_{12} + (1-k_{12})\left(1 - \frac{\sqrt{(C_x^{E_j})^2 + (C_y^{E_j})^2}}{C_j^E}\right) \tag{3.81}$$

where $C_x^{E_j}$ and $C_y^{E_j}$ are determined by (3.72) and (3.73). The value for k_{12} used is 0.5.

The effect of using the third decision rule rather than the first two is to force the offensive ball-carrier to "follow his blockers" while also not completely ignoring the defensive players.

3.5.2 Offense: Blockers

As mentioned before, each offensive blocker has a man whom he is to block. The offensive blocker desires to keep himself between the assigned defensive man and the ball-carrier. An interesting symmetry results between the defensive man and the offensive blocker:

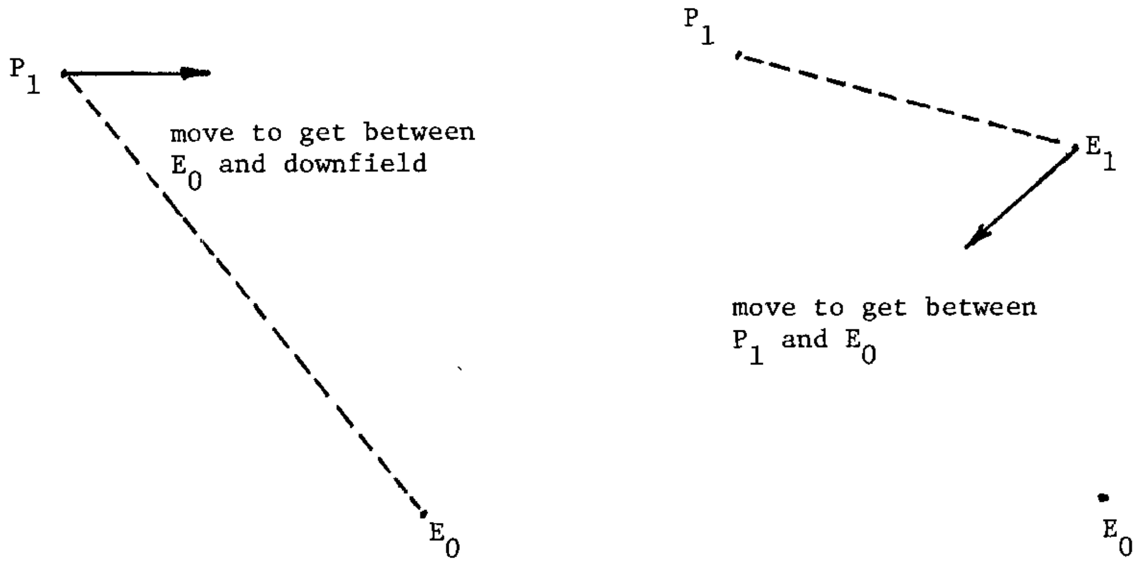

Figure 3-10. Symmetry in Blockers' and Tacklers' Strategies

E_1's strategic interests are similar to P_1's, except for two differences:

1. P_1 is guarding against a y-coordinate motion while E_1 is guarding along a direction from P_1 to E_o.
2. E_1 can be considerably more aggressive than P_1.

These two characteristics can be handled easily. The first is accomplished by a rotation of the axes:

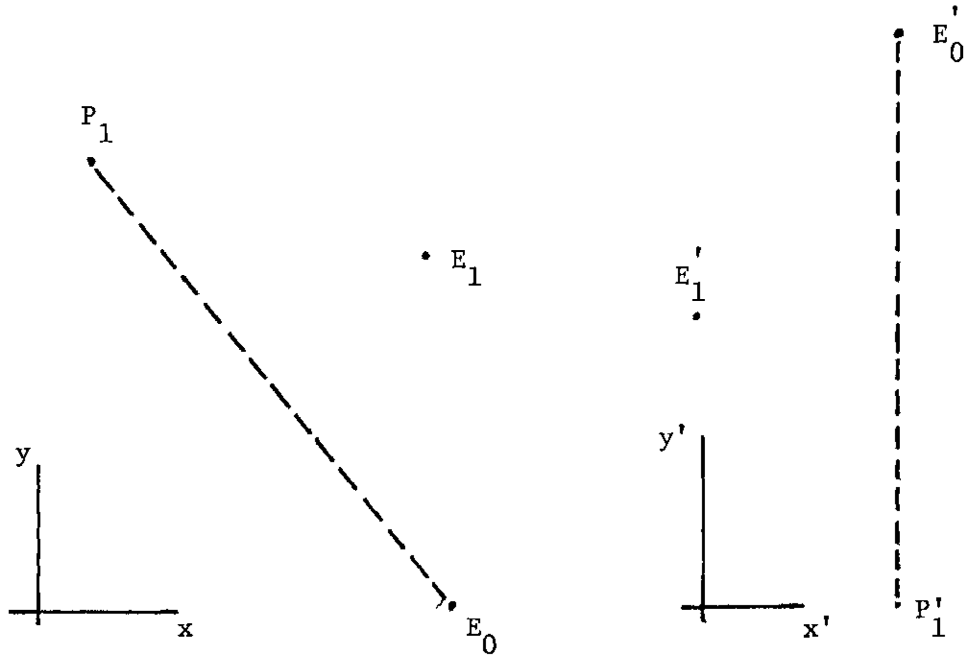

Figure 3-11. Rotation of Strategy Axes

Then $\vec{C}^{E_1'}$ can be found in the primed axes using the same logic as for the defense. Once the strategy is found, $\vec{C}^{E_1}$ can be found by derotating $\vec{C}^{E_1'}$ to the original coordinate system.

The second characteristic of the offensive blockers is modeled by noting that it is C^{P_1} which determines E_1's aggressiveness (in the primed coordinates, the $C_y^{E_1'}$ value). If E_1 is close to E_o, movement directly toward P_1 guaranteed by a small C^{P_1}) is required. If E_1 is close to P_1, then it is imperative that E_1 interacts with P_1. This suggests a more prudent, larger value of C^{P_1}. Thus a C^{P_1*} is generated by

$$C^{P_1*} = k_{13}C^{P_1} \tag{3.82}$$

where

$$k_{13} = 1 - \frac{d_{E_1P_1}}{d_{E_0P_1}} \tag{3.83}$$

where $d_{E_1P_1}$ is the distance between E_1 and P_1 and $d_{E_0P_1}$ is the distance between E_0 and P_1. The allowable values are $0.1 \leq k_{13} \leq 1.0$.

Direct application of this strategy results in the offensive blocker lagging the defenseman by one epoch. This in turn tends to cause the blocker to follow the defenseman around the field. This is corrected by using

$$\vec{z}^{P_1}(0) + t_e \frac{d\vec{z}^{P_1}(0)}{dt} \tag{3.84}$$

as the defender's position, where t_e is the length of an Epoch. Also,

$$\vec{z}^{E_1}(0) + t_e \frac{d\vec{z}^{E_1}(0)}{dt} \tag{3.85}$$

is used for the ball-carrier's position. The anticipation of the motion of E_1 and P_1 is allowable here where before it was unwise because of the more aggressive nature of the blocker's strategy.

3.5.3 Defense

The formulation of the one-on-one defensive reactions did not take into account the possible presence of any blockers. The assumption for the M-on-N case will be identical: defensemen pay no attention to blockers until the onset of the block requires it. A second assumption is that defensemen's actions have no effect on each other. Thus each defender acts as though he were the only defender. These two assumptions make the one-on-one strategy completely applicable to the M-on-N case.

The above assumptions are difficult ones. They seem most reasonable when there is no set structure to a play or portion of a play. This is encouraging, since the structure of the chosen defense will constrain the defensive players (at least initially) and thus force a structure where normally there would be none.

3.6 Blocking Model

A block occurs when a defensive player and offensive players (excluding the ball-carrier) first come within one yard of each other. The blocking model seeks to describe the motion of the offensive and defensive players throughout the course of a block and to determine the time of termination of that block. One-on-one blocks are the simplest blocking interactions and thus are considered first. Later, double-team blocking is described.

3.6.1 One-on-One Blocks

A block may be considered to be a "delay" to the defensive player caused by the offensive player. Thus the probability of a block ending in any given epoch is called the delay distribution.

3.6.1.1. Delay Distribution. A player wins a block by forcing the opposing player off his feet, resulting in a zero velocity vector for the opposing player. There are four possible circumstances at the end of a block:

1. the offensive player has won
2. the defensive player has won
3. both players are forced off their feet
4. neither player has won

The first assumption of the blocking model is that a block can terminate only at the beginning of an epoch or the instant that an offensive player moves within one yard of the defensive player for the first time.*

The following points dictate the nature of the probabilities which govern the delay distribution:

1. The offensive player is less likely to win a block than a defensive player since he is not allowed to use his hands.

2. The stronger player is more likely to win a block.

3. The player with a higher velocity is more likely to win a block.

4. The angle between the two players is critical to the determination of who wins the block. The straight-ahead (shoulder) block is designed not to bring a defensive man off his feet but to move him from an area or to impede

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*For one-on-one blocking the second criterion corresponds to the beginning of the block. For double-team blocking it corresponds to the beginning of the block and the first instant of double-team blocking (i.e. both offensive players are within one yard of the defensive player).

his motion to an area. Blocks from the side (cross-body, rolling, and cross-check) increase the likelihood of the offensive blocker winning.

In a block between P and E, define Pr_P (the probability P wins on any given epoch) as

$$Pr_P = k_{14}\left[\frac{S^P}{S^E} + \frac{1 + ||d\vec{z}^P(0)/dt||}{1 + ||d\vec{z}^E(0)/dt||} + \left(1 - \frac{(d\vec{z}^P(0)/dt) \cdot (d\vec{z}^E(0)/dt)}{||d\vec{z}^P(0)/dt||\ ||d\vec{z}^E(0)/dt||}\right)\right] \tag{3.86}$$

where S^P and S^E are P's and E's respective strength. Likewise, Pr_E (the probability of E winning on a given epoch) is given by

$$Pr_E = k_{15}\left[\frac{S^E}{S^P} + \frac{1 + ||d\vec{z}^E(0)/dt||}{1 + ||d\vec{z}^P(0)/dt||} + \left(1 + \frac{(d\vec{z}^P(0)/dt) \cdot (d\vec{z}^E(0)/dt)}{||d\vec{z}^P(0)/dt||\ ||d\vec{z}^E(0)/dt||}\right)\right] \tag{3.87}$$

The first point is assured by the selection of $k_{14} > k_{15}$ (the model uses $k_{14} = 0.04$ and $k_{15} = 0.015$). The second point is modeled by the terms S^P/S^E and S^E/S^P in (3.86) and (3.87) respectively. The terms

$$\frac{1 + ||d\vec{z}^P(0)/dt||}{1 + ||d\vec{z}^E(0)/dt||}$$

and

$$\frac{1 + ||d\vec{z}^E(0)/dt||}{1 + ||d\vec{z}^P(0)/dt||}$$

reflect the third point. Finally

$$\frac{(d\vec{z}^P(0)/dt) \cdot (d\vec{z}^E(0)/dt)}{||d\vec{z}^P(0)/dt|| \; ||d\vec{z}^E(0)/dt||} = \cos \gamma \tag{3.88}$$

where γ is the angle between the velocities of the players. As γ goes to 180^o (indicating a head-on block) $\cos \gamma$ goes to -1, increasing P's probability of winning the block and decreasing E's probability. As γ nears 90^o, $\cos \gamma$ nears 0 and the term does not affect the probability of either player winning. As γ nears 0^o (a blind-side block), $\cos \gamma$ goes to 1 and this increases E's probability of winning while decreasing P's.

In the simulation, for each epoch while the block is occurring, a random uniformly distributed number (say x) between 0 and 1 is generated. If $x \leq Pr_E$ then E wins the block. If $x \geq 1 - Pr_P$ then P wins the block. If $Pr_E < x < 1 - Pr_P$ then the interaction continues.

Figure 3-12 below shows the cumulative probability that P wins, E wins, or no one wins for the selected values of k_{14} and k_{15}, and for $S^P = S^E$, $||d\vec{z}^E(t)/dt|| = ||d\vec{z}^P(t)/dt||$* and $\gamma = 0^o$, 90^o, and 180^o.

Finally, a further constraint is placed on the continuation of a block. At the beginning of an epoch, it is determined if the defensive player is closer to the ball-carrier than the offensive blocker. If he is, then the block is terminated and the defensive player is assumed to have won. This is reasonable, since the blocking angle for the offensive man is so poor as to render the block useless.

3.6.1.2 Players' Motion. From the inception of the block until the end of the delay, the players undergo a change in motion due to the interaction. This motion is the result of the players' initial velocity and the direction the players wish to move.

The model treats the two players' motion virtually equivalent to rigid body motion. The momentum of the players and the applied forces result in a translation motion and rotational motion around the center of motion:

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*The following section develops the equations determining $\vec{z}^E(t)$ and $\vec{z}^P(t)$. In general $||d\vec{z}^E(t)/dt|| \neq ||d\vec{z}^P(t)/dt||$. These examples are given only to show approximate results.

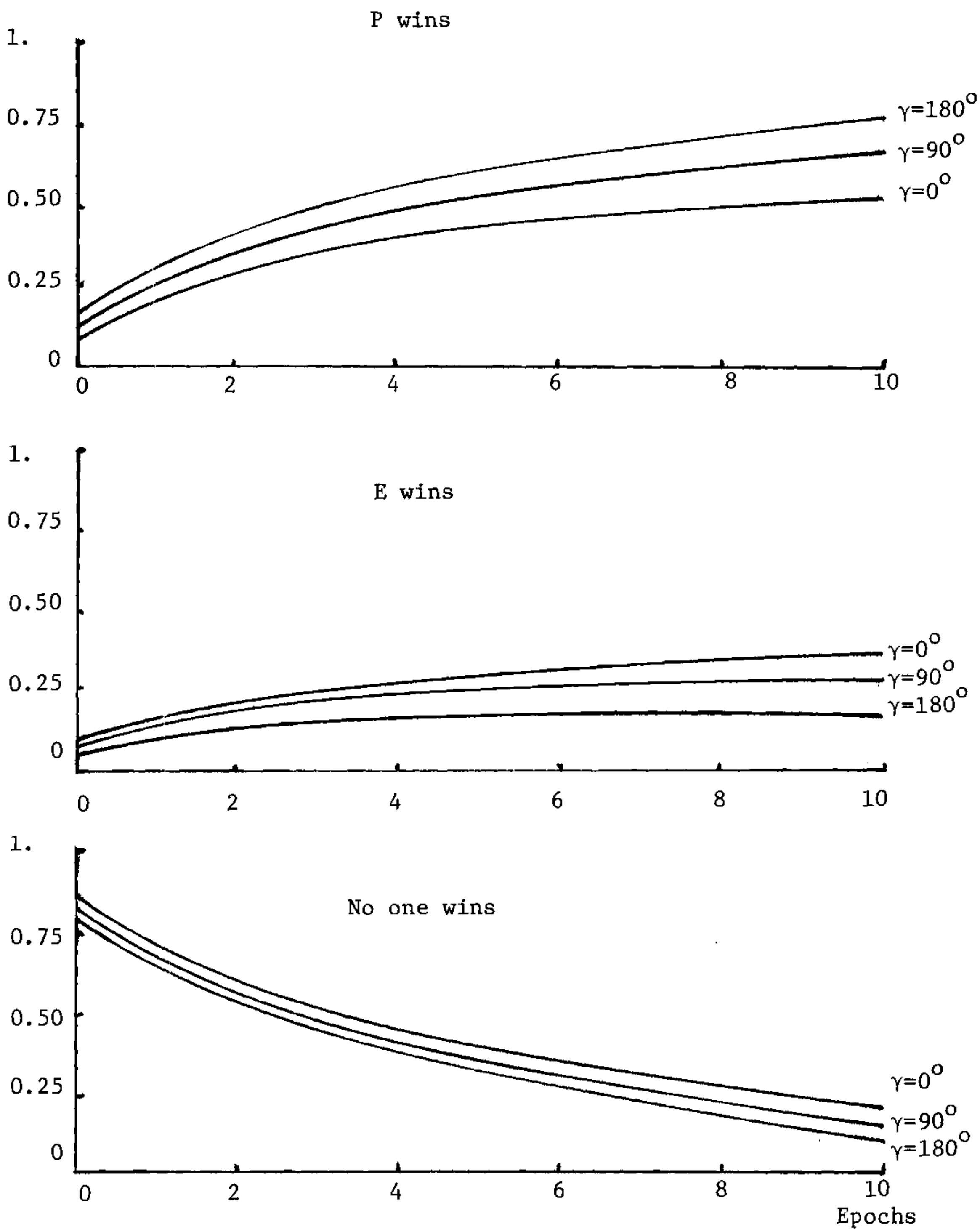

Figure 3-12. Distribution of Blocking Outcomes

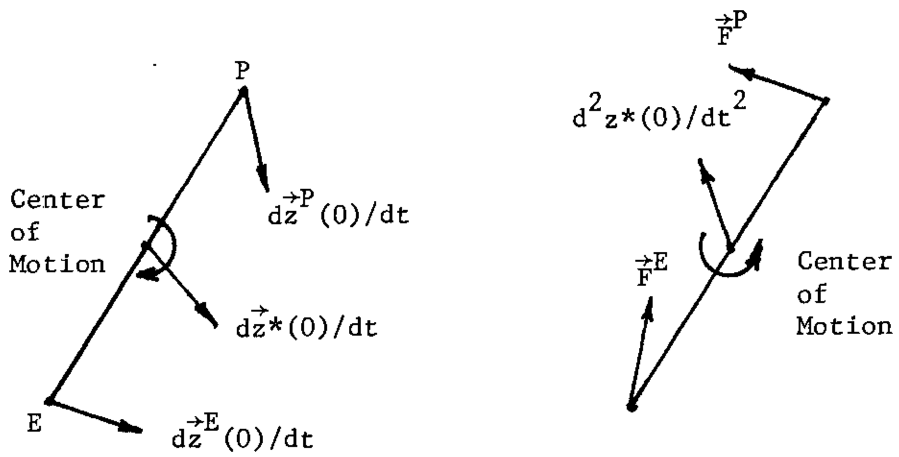

Figure 3-13. Torques and Momentums in the Blocking Interaction

The center of motion, $\vec{z}^*(0)$, at the beginning of the epoch (or at the inception of the block) is defined as the average of the two players position, i.e.

$$x^*(0) = \tfrac{1}{2}\,[x^P(0) + x^E(0)] \tag{3.89}$$

$$y^*(0) = \tfrac{1}{2}\,[y^P(0) + y^E(0)] \tag{3.90}$$

Strict rigid body motion demands that

$$(W^P+W^E)\ \vec{z}^*(0) = W^P\vec{z}^P(0) + W^E\vec{z}^E(0) \tag{3.91}$$

where W^P and W^E are the respective players' weights. This is equivalent to (3.90) and (3.91) only if $W^P = W^E$, which in general is not true.

The reason (3.92) is not used to define $\vec{z}^*(0)$ is that it penalizes the heavier player in generating torque to affect the rotation of the block, since his distance to the center of motion is reduced. This result is contrary to expectations based on experience. Thus (3.89) and (3.90) are used to allow a fair block. Since these definitions are not equivalent to the usual center of mass definitions, the term center of motion is used instead. It should be noted that this relaxation of rigid body dynamics does not influence the translational motion, although it does affect the rotational results.

Using conservation of linear momentum,

$$(W^P+W^E)\ \frac{d\vec{z}^*(0)}{dt} = W^P\ \frac{d\vec{z}^P(0)}{dt} + W^E\ \frac{d\vec{z}^E(0)}{dt} \tag{3.92}$$

or

$$\frac{d\vec{z}^*(0)}{dt} = \frac{W^P}{W^P+W^E}\ \frac{d\vec{z}^P(0)}{dt} + \frac{W^t}{W^P+W^E}\ \frac{d\vec{z}^E(0)}{dt} \tag{3.93}$$

Likewise summing forces

$$\frac{W^P+W^E}{g}\ \frac{d^2\vec{z}^*(0)}{dt^2} = \vec{F}^P + \vec{F}^E \tag{3.94}$$

where g is the gravitational acceleration due to gravity and $\vec{F}^P$ and

$\vec{F}^E$ are the forces P and E generate. Introducing S^P and S^E, the strength of P and E (actually the weight a player can bench-press), then $||\vec{F}^P||$ and $||\vec{F}^E||$ are assumed proportional to S^P and S^E, respectively. Likewise, the direction of $\vec{F}^P$ and $\vec{F}^E$ are taken to be parallel to the directions of $\vec{C}^P$ and $\vec{C}^E$ as generated above. Incorporating g into the proportionality constant, C_2:

$$\frac{d^2\vec{z}*(0)}{dt^2} = \frac{C_2}{W^P+W^E}\left[\frac{S^P\vec{C}^P}{C^P} + \frac{S^E\vec{C}^E}{C^E}\right] \tag{3.95}$$

Writing a second-order Maclauren series for $\vec{z}*(t)$,

$$\vec{z}*(t) = \vec{z}*(0) + t\,\frac{d\vec{z}*(0)}{dt} + \tfrac{1}{2}t^2\,\frac{d^2\vec{z}*(0)}{dt^2} \tag{3.96}$$

which describes the position of the center of motion throughout the duration of an epoch.

Using the principle of conservation of angular momentum:

$$\vec{H} = I\,\frac{d\vec{\phi}(0)}{dt} \tag{3.97}$$

where $\vec{H}$ is the angular momentum, I is the moment of inertia and $d\vec{\phi}(0)/dt$ is the angular velocity at the beginning of the epoch. H_z is given by the z-component of $\vec{H}$

$$\vec{H} = \left[\vec{z}^P(0) - \vec{z}*(0)\right] X \frac{W^P}{g} \frac{d\vec{z}^P(0)}{dt}$$

$$+ \left[\vec{z}^E(0) - \vec{z}*(0)\right] X \frac{W^E}{g} \frac{d\vec{z}^E(0)}{dt} \tag{3.98}$$

which is perpendicular to the playing field. Thus $||\vec{H}|| = H_z$. And also $||d\vec{\phi}(0)/dt|| = \dot{\phi}(0)$. H_z is given by

$$H_z = \frac{W^P}{g} \left[(x^P(0) - x*(0)) \frac{dy^P(0)}{dt} - (y^P(0) - y*(0)) \frac{dx^P(0)}{dt}\right]$$

$$+ \frac{W^E}{g} \left[(x^E(0) - x*(0)) \frac{dy^E(0)}{dt} - (y^E(0) - y*(0)) \frac{dy^E(0)}{dt}\right] \tag{3.99}$$

Likewise, I is given by

$$I = \frac{W^P}{g} \left[(x^P(0) - x*(0))^2 + (y^P(0) - y*(0))^2\right]$$

$$+ \frac{W^E}{g} \left[(x^E(0) - x*(0))^2 + (y^E(0) - y*(0))^2\right] \tag{3.100}$$

$$= \frac{W^P+W^E}{2g} \left[(x^P(0) - x^E(0))^2 (y^P(0) - y^E(0))^2\right] \tag{3.101}$$

Equation (3.97) gives results which are much too large for $\dot{\phi}(0)$.

To counteract this, I is divided by a constant $C_3/2$ to impede the initial angular velocity:

$$\dot{\phi}(0) = C_3 \left\{ \frac{W^P}{W^P+W^E} \left[\frac{x^P(0) - x^*(0)}{d^2} \frac{dy^P(0)}{dt} - \frac{y^P(0) - y^*(0)}{d^2} \frac{dx^P(0)}{dt} \right] \right.$$

$$\left. + \frac{W^E}{W^P+W^E} \left[\frac{x^E(0) - x^*(0)}{d^2} \frac{dy^E(0)}{dt} - \frac{y^E(0) - y^*(0)}{d^2} \frac{dx^E(0)}{dt} \right] \right\} \tag{3.102}$$

where $d^2 = [x^P(0) - x^E(0)]^2 + [y^P(0) - y^E(0)]^2$.

Angular acceleration is governed by

$$\vec{T} = I \frac{d^2\vec{\phi}(0)}{dt^2} \tag{3.103}$$

where $\vec{T}$ is the torque and $\frac{d^2\vec{\phi}(0)}{dt^2}$ is the angular acceleration (again these quantities are vectors normal to the playing field and thus they may also be considered scalar quantities). $\vec{T}$ is given by

$$\vec{T} = [\vec{z}^P(0) - \vec{z}*(0)] \times \vec{F}^P$$

$$+ [\vec{z}^E(0) - \vec{z}*(0)] \times \vec{F}^E \tag{3.104}$$

Allowing $\vec{F}^P$ and $\vec{F}^E$ to take the same form as above,

$$\ddot{\phi}(0) = 2\ C_2 \left\{ \frac{S^P}{W^P + W^E} \left[\frac{x^P(0) - x^*(0)}{d^2} \left(\frac{C_y^P}{C^P} \right) - \frac{y^P(0) - y^*(0)}{d^2} \left(\frac{C_x^P}{C^P} \right) \right] \right.$$

$$\left. + \frac{S^E}{W^P + W^E} \left[\frac{x^E(0) - x^*(0)}{d^2} \left(\frac{C_y^E}{C^E} \right) - \frac{y^E(0) - y^*(0)}{d^2} \left(\frac{C_x^E}{C^E} \right) \right] \right\} \tag{3.105}$$

As before

$$\phi^P(t) = \phi^P(0) + \alpha(t) \tag{3.106}$$

$$\phi^E(t) = \phi^E(0) + \alpha(t) \tag{3.107}$$

where $\alpha(t) = t\dot{\phi}(0) + \frac{1}{2}\, t^2\, \ddot{\phi}(0)$. Clearly $\phi^P(0)$ is defined by

$$x^P(0) = x^*(0) + d^* \cos\, [\phi^P(0)] \tag{3.108}$$

$$y^P(0) = y^*(0) + d^* \sin\, [\phi^P(0)] \tag{3.109}$$

where $d^* = \sqrt{[x^P(0) - x^*(0)]^2 + [y^P(0) - y^*(0)]^2}$, and $\phi^E(0)$ is defined by

$$x^E(0) = x^*(0) + d^* \cos\, [\phi^E(0)] \tag{3.110}$$

$$y^E(0) = y^*(0) + d^* \sin\, [\phi^E(0)] \tag{3.111}$$

Since this is "rigid body motion,"

$$x^P(t) = x^*(t) + d^* \cos [\phi^E(t)] \tag{3.112}$$

$$y^P(t) = y^*(t) + d^* \sin [\phi^E(t)] \tag{3.113}$$

and

$$x^E(t) = x^*(t) + d^* \cos [\phi^E(t)] \tag{3.114}$$

$$y^E(t) = y^*(t) + d^* \sin [\phi^E(t)] \tag{3.115}$$

or

$$\begin{aligned} x^P(t) = x^*(t) &+ [x^P(0) - x^*(0)] \cos [\alpha(t)] \\ &- [y^P(0) - y^*(0)] \sin [\alpha(t)] \end{aligned} \tag{3.116}$$

$$\begin{aligned} y^P(t) = y^*(t) &+ [x^P(0) - x^*(0)] \sin [\alpha(t)] \\ &- [y^P(0) - y^*(0)] \cos [\alpha(t)] \end{aligned} \tag{3.117}$$

and

$$x^E(t) = x^*(t) + [x^E(0) - x^*(0)] \cos [\alpha(t)]$$
$$- [y^E(0) - y^*(0)] \sin [\alpha(t)] \tag{3.118}$$

$$y^E(t) = y^*(t) + [x^E(0) - x^*(0)] \sin [\alpha(t)]$$
$$+ [y^E(0) - y^*(0)] \cos [\alpha(t)] \tag{3.119}$$

This completes the description of the position of the players during an epoch. The velocity is calculated similarly:

$$\frac{d\vec{z}^P(t)}{dt} = \frac{d\vec{z}^*(t)}{dt} + \frac{d\vec{\phi}(t)}{dt} \times [\vec{z}^P(t) - \vec{z}^*(0)] \tag{3.120}$$

$$\frac{d\vec{z}^E(t)}{dt} = \frac{d\vec{z}^*(t)}{dt} + \frac{d\vec{\phi}(t)}{dt} \times [\vec{z}^E(t) - \vec{z}^*(0)] \tag{3.121}$$

Again invoking rigid-body motion

$$\frac{dx^P(t)}{dt} = \frac{dx^*(t)}{dt} - \dot{\phi}(t) \Big\{ [x^P(0) - x^*(0)] \sin [\alpha(t)]$$
$$+ [y^P(0) - y^*(0)] \cos [\alpha(t)] \Big\} \tag{3.122}$$

$$\frac{dy^P(t)}{dt} = \frac{dy^*(t)}{dt} - \dot{\phi}(t) \left\{ [x^P(0) - x^*(0)] \cos [\alpha(t)] \right.$$
$$\left. - [y^P(0) - y^*(0)] \sin [\alpha(t)] \right\} \tag{3.123}$$

and

$$\frac{dx^E(t)}{dt} = \frac{dx^*(t)}{dt} - \dot{\phi}(t) \left\{ [x^E(0) - x^*(0)] \sin [\alpha(t)] \right.$$
$$\left. + [y^E(0) - y^*(0)] \cos [\alpha(t)] \right\} \tag{3.124}$$

$$\frac{dy^E(t)}{dt} = \frac{dy^*(t)}{dt} - \dot{\phi}(t) \left\{ [x^E(0) - x^*(0)] \cos [\alpha(t)] \right.$$
$$\left. - [y^E(0) - y^*(0)] \sin [\alpha(t)] \right\} \tag{3.125}$$

where $\dot{\phi}(t) = \dot{\phi}(0) + t\ddot{\phi}(0)$ and $\frac{dx^*(t)}{dt} = x^*(0) + t\,\frac{d^2x^*(0)}{dt^2}$.

3.6.2 Double-Teaming

Double-teaming occurs when two offensive blockers assigned to block the same defensive man come into direct interaction with that defensive player. As will be seen, one-on-one blocking results generalize well to double-teaming interactions. It will be assumed (in the development and also in the computer model) that no triple- (or more) teaming takes place, an assumption well-justified in practice.

3.6.2.1 Delay Distribution. The differences in the delay distribution from the one-on-one block are caused by the fact that there are now two offensive blockers, E_1 and E_2. On any given epoch, one of the following must occur:

1. E_1, E_2, or both have won
2. The defensive player has won over E_1 or E_2
3. All of the players are forced off their feet
4. None of the players have won

To model all of these possibilities, four probabilities are generated. Two of the probabilities Pr_{P_1} and Pr_{P_2} are equivalent to the probabilities generated in (3.86) with the values associated with E_1 and E_2, respectively. The value for k_{14} is 0.02, indicating the fact that it is considerably more difficult to beat a double-team block than a one-on-one block. Likewise, two probabilities Pr_{E_1} and Pr_{E_2} equivalent to that generated in (3.86) are generated, with a value of $k_{15} = 0.06$. Two random numbers are chosen from a uniform distribution (say x and y) similar to the one-on-one case. For each, three possibilities exist:

1. a. $x \leq Pr_{E_1}$

   b. $x \geq 1 - Pr_{P_1}$

   c. $Pr_{E_1} < x < Pr_{P_1}$

2. a. $y \leq Pr_{E_2}$

   b. $y \geq 1 - Pr_{P_2}$

   c. $Pr_{E_2} < y < Pr_{P_2}$

If 1.a. or 2.a. or 1.a. and 2.a. occurs, then the block terminates (P loses the block). If 1.b. occurs, the block reverts to a one-on-one block with P and E_1. If both 1.b. and 2.b. occur, the block terminates with P winning. If both 1.c. and 2.c. occur, then the block continues.

Like the one-on-one case, an offensive player who is further away from the ball-carrier than the defensive blocker is considered to have lost the block. Unlike the one-on-one case, however, the block is not necessarily terminated under these conditions since there is more than one participant from the offensive team.

3.6.2.2 Players' Motion. The concept of the center of motion of the one-on-one block was to insure a fair block, i.e. the distance from each player to the center of motion was equal. This concept could be expanded by defining $\vec{z}^*(0)$ by

$$\begin{aligned} ||\vec{z}^{P}(0) - \vec{z}^*(0)|| &= ||\vec{z}^{E_1}(0) - \vec{z}^*(0)|| \\ &= ||z^{E_2}(0) - \vec{z}^*(0)|| \end{aligned} \tag{3.126}$$

There are two problems with this definition, however. The first is that the algebra needed to determine $x^*(0)$ and $y^*(0)$ is somewhat messy. The second, more important, reason is that the point $\vec{z}^*(0)$ may be so far removed from the players that the angular motion around the center of motion would cause unreasonable velocities. Thus the definition for $\vec{z}^*(0)$ is chosen so

$$x^*(0) = \frac{1}{3}\left[x^P(0) + x^{E_1}(0) + x^{E_2}(0)\right] \tag{3.127}$$

$$y^*(0) = \frac{1}{3}\left[y^P(0) + y^{E_1}(0) + y^{E_2}(0)\right] \tag{3.128}$$

Likewise

$$\frac{d\vec{z}^*(0)}{dt} = \frac{W^P}{W^P+W^{E_1}+W^{E_2}} \frac{d\vec{z}^P(0)}{dt} + \frac{W^{E_1}}{W^P+W^{E_1}+W^{E_2}} \frac{d\vec{z}^{E_1}(0)}{dt} + \frac{W^{E_2}}{W^P+W^{E_1}+W^{E_2}} \frac{d\vec{z}^{E_2}(0)}{dt} \tag{3.129}$$

and

$$\frac{d^2\vec{z}^*(0)}{dt^2} = \frac{C_2}{W^P+W^{E_1}+W^{E_2}} \left[\frac{S^P\vec{C}^P}{C^P} + \frac{S^{E_1}\vec{C}^{E_1}}{C^{E_1}} + \frac{S^{E_2}\vec{C}^{E_2}}{C^{E_2}}\right] \tag{3.130}$$

Equation (3.96) is again used to determine $\vec{z}^*(t)$.

The expressions of the angular motion are somewhat more complex, since the distance from each player to the center of motion is no longer the same for each player. Equation (3.99) becomes

$$H_z = \frac{W^P}{g}\left[\left(x^P(0) - x^*(0)\right)\frac{dy^P(0)}{dt} - \left(y^P(0) - y^*(0)\right)\frac{dx^P(0)}{dt}\right] \tag{3.131}$$

$$+ \frac{W^{E_1}}{g} \left[\left(x^{E_1}(0) - x^*(0) \right) \frac{dy^{E_1}(0)}{dt} - \left(y^{E_1}(0) - y^*(0) \right) \frac{dx^{E_1}(0)}{dt} \right]$$

(3.131)
(Cont'd.)

$$+ \frac{W^{E_2}}{g} \left[\left(x^{E_2}(0) - x^*(0) \right) \frac{dy^{E_2}(0)}{dt} - \left(y^{E_2}(0) - y^*(0) \right) \frac{dx^{E_2}(0)}{dt} \right]$$

and (3.100) becomes

$$I = \frac{W^P}{g} \left[\left(x^P(0) - x^*(0) \right)^2 + \left(y^P(0) - y^*(0) \right)^2 \right]$$

$$+ \frac{W^{E_1}}{g} \left[\left(x^{E_1}(0) - x^*(0) \right)^2 + \left(y^{E_1}(0) - y^*(0) \right)^2 \right] \tag{3.132}$$

$$+ \frac{W^{E_2}}{g} \left[\left(x^{E_2}(0) - x^*(0) \right)^2 + \left(y^{E_2}(0) - y^*(0) \right)^2 \right]$$

and again

$$\dot{\phi}(0) = \frac{C_3 H_z}{I} \tag{3.133}$$

Also (3.104) generalizes to

$$T = C_2 \left\{ \frac{S^P}{C^P} \left[\left(x^P(0) - x^*(0) \right) C_y^P - \left(y^P(0) - y^*(0) \right) C_x^P \right] \right.$$

(3.134)

$$+ \frac{S^{E_1}}{C^{E_1}} \left[\left(x^{E_1}(0) - x^*(0) \right) C_y^{E_1} - \left(y^{E_1}(0) - y^*(0) \right) C_x^{E_1} \right]$$

$$+ \frac{S^{E_2}}{C^{E_2}} \left[\left(x^{E_2}(0) - x^*(0) \right) C_y^{E_2} - \left(y^{E_2}(0) - y^*(0) \right) C_x^{E_2} \right] \Bigg\} \qquad (3.134) \text{ (Cont'd.)}$$

and again

$$\ddot{\phi}(0) = T/I \qquad (3.135)$$

The updating of the players' position and velocity is equivalent to the one-on-one case (3.116) through (3.119) and (3.122) through (3.125), with the subscripts E_1 and E_2 used in the place of E.

3.7 Tackling Model

The tackling model closely resembles the blocking model. Both the concepts of delay distribution and player motion are retained in the tackling model.

A tackling interaction starts when a defenseman who is not being blocked comes within a yard of the ball-carrier. The requirement that the defensiveman not be in a blocking interaction at first may seem too restrictive. It should be remembered, however, that a block terminates if at the beginning of an epoch there is no offensive blocker involved in the interaction closer to the ball-carrier than the defensive blocker. A tackle is completed when the ball-carrier is forced off his feet or his y-component velocity is significantly impeded*. A tackle is discontinued if all the defensive tacklers are forced off

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*The term "significantly impeded" will be defined later.

their feet.

Unlike a block, any number of defensive players can be involved in a tackle. For convenience, assume k players are participating in the tackle ($1 \leq k \leq N$). Assume these k players are the members of a set {K}.

3.7.1 Delay Distribution

Define $Pr_{\Sigma P_i}$, the probability that E is tackled by one or more of the k defensive players, as

$$Pr_{\Sigma P_i} = k_{16} \left[\frac{\Sigma S^{P_i}}{S^E} + \frac{1 + \left|\left| \Sigma \frac{d\vec{z}^{P_i}(0)}{dt} \right|\right|}{1 + \left|\left| \frac{d\vec{z}^{E}(0)}{dt} \right|\right|} \right] \tag{3.136}$$

where $k_{16} = 0.02$ and the summation is done on $i \in \{K\}$. A random number is chosen from a uniformly distributed variable between 0 and 1 as before (call the number x). If $x \leq Pr_{P_i}$ then E is considered tackled and the play ends. If $x > Pr_{\Sigma P_i}$ then k values of Pr_E^i are calculated by

$$Pr_E^i = k_{17} \left[\frac{S^E}{S^{P_i}} + \frac{1 + ||d\vec{z}^E(0)/dt||}{1 + ||d\vec{z}^{P_i}(0)/dt||} \right] \tag{3.137}$$

with $k_{17} = 0.04$. Then k random numbers y^i are generated as above.

For each $i \varepsilon \{K\}$, if $y^i \leq Pr^i_E$ then player i is considered to be forced off his feet and no longer a participant in the tackle.

If the ball-carrier's y-component velocity is non-positive at the end of two epochs in a row, then the ball-carrier's motion is considered impeded significantly enough to end the play.

3.7.2 Players' Motion

The players' motion is a further extention of the one-on-one and double-team blocking equations. As before,

$$x^*(0) = \frac{1}{k+1} [x^E(0) + \Sigma x^{P_i}(0)] \quad (3.138)$$

$$y^*(0) = \frac{1}{k+1} [y^E(0) + \Sigma y^{P_i}(0)] \quad (3.139)$$

were again the summation is on $i \varepsilon \{K\}$.* The equations of motion are then obvious extensions of those used in the blocking model.

3.8 Capabilities of the Model

The model integrates the methods of the previous sections to simulate a play. For each epoch, the $\vec{C}^{E_i}$ and $\vec{C}^{P_i}$ are selected and the position and velocity of each player are updated and used as the input values for the next epoch. Hence a play is a sequential decision-making process for each of the twenty-two participant players.

There are special techniques which can be used to model plays more realistically. Some of these are mentioned below:

*{K} and k are reduced by defensive players being forced off their feet.

1. Beginning of the play. The majority of plays start out with the players motion completely determined for the first few epochs. After the play develops, then the ball-carrier, blockers and defenders can be "released" to react according to their respective objectives.

2. Area blocking. Offensive linemen normally block according to where the ball-carrier is to move through the line, rather than where the ball-carrier is at that instant (or the next epoch, as is discussed in Section 3.5.2). This can be modelled by the use of a "dummy" ball-carrier at the position in the line where the ball-carrier is to go. If the lineman blocks according to the dummy ball-carrier's position, then the desired effect is achieved.

3. Defensive aggressiveness. Without any defensive structure, each player assumes that he alone must stop the ball-carrier, and there are two effects of this. First, each player reacts to his worst-case offensive direction, and the resultance $C_y^{P_i}$ are too positive. Secondly, the desire of each player to match his x-coordinate position and velocity with the ball-carrier's results in the players collapsing to the middle. Sometimes doing this results in a weakness on the flank of the defense (a situation, indidentally, which the model's ball-carrier is quite capable of capitalizing on). These two problems can be handled by placing areas of responsibility of each defender. In the previous development, the defender was concerned with values of θ from 0^o to 180^o. This refinement specifies two corresponding values, θ_1 and θ_2. The determination of $C_y^{P_i}$ is then identical as before, and the equation governing $C_x^{P_i}$ (3.66) becomes

$$C_x^P = \frac{k_7 k_9 - k_8 k_{11}}{k_7^2 - k_8 k_{10}} + C^E \cos \frac{\theta_1 + \theta_2}{2} \qquad (3.140)$$

4. Gang tackling. As the ball-carrier first comes into direct interaction with a defender, he may not be tackled immediately. As he is slowed up, however, he no longer presents the same threat. Thus, the C^E is observed as $0.1\ C^E$ by the defensive players. This leads to more aggressive

reactions by the defense, resulting in more individuals tackling the ball-carrier and more effort in forcing him in the negative y-direction and thus ending the play.

The delay distributions used in the blocking and tackling models are probabilistic in nature and would by themselves account for a non-degenerate gain distribution. Another factor, however, should be considered probabilistically. Human beings are by nature non-deterministic; this is also true in their running. The method used to model this is to add a normal and independently distributed random component to each component of $\vec{C}^{P_i}$ and $\vec{C}^{E_i}$. The method used to generate the normal variate is as follows*: first a uniformly distributed variable on [0,1], x is selected. Then x_n is calculated by

$$t_1 = \begin{cases} \sqrt{\ln 1/x^2} & x \leq 1/2 \\ \sqrt{\ln 1/(1-x^2)} & x > 1/2 \end{cases} \tag{3.141}$$

$$t_2 = \begin{cases} 1 & x \leq 1/2 \\ -1 & x > 1/2 \end{cases} \tag{3.142}$$

$$x_n = t_2 \left[t_1 - \frac{2.30753 + 0.27061\ t_1}{1 + 0.99229\ t_1 + 0.04481\ t_1^2} \right] \tag{3.143}$$

Then $\sigma_C x_n$ is added to each component of the $\vec{C}^{P_i}$ and $\vec{C}^{E_i}$ at the start

*[1], page 933, Section 26.2.22.

of each epoch. The procedure used to determine the proper value for σ_C is addressed in Section 4.2.3.

3.9 Examples

Two three-on-three examples are given below. The examples have been selected so that they are illustrative of the factors developed above and yet not so cumbersome as a complete eleven-on-eleven play. In all cases, the players are not constrained to run in given directions, but are "released" to the model to determine the optimal strategies.

The three offensive players are numbered 1 through 3 by the program, although player 3 is labeled by a *, since the program denotes the ball-carrier by this symbol. The defensive players are lettered A through C.

The following parameters were used in both examples:

a. all strengths (S^{P_i} and S^{E_i}) were 200 pounds

b. all weights (W^{P_i} and W^{E_i}) were 200 pounds

c. $C_1 = 2.45$, $C_2 = 1.5$ and $C_3 = 0.2$. The choice of these parameters is addressed in Sections 4.2.2 and 4.2.3.

d. $\sigma_c = 0$. This allowed viewing the strategies as they were actually calculated by the model rather than strategies distorted by the random component.

e. $C^{E_i} = C^{P_i} = 17$

f. all initial velocities are zero

g. player 1 attempts to block player A and player 2 attempts to block player B

3.9.1 Example 1

Example 1 is shown in Figure 3-14. Figure 3-14a (Epoch = 0.00 seconds) shows the initial positions of the six players. Player A's responsibility is for strategies between 45° and 135° ($\theta_1 = 45$ and $\theta_2 = 135$ in equation (3.140)), B's is for strategies between 0° and 90° and C's is for strategies between 90° and 180°. An explanation of the computer printout is made in Appendix B.

With the matchups of player 1 vs. player A and player 2 vs. player B, player C appears to be the biggest threat to the ball carrier (assuming the blocks are made). Thus one suspects the ball carrier should be going to the right, and this is confirmed by the strategy (9.01,14.42) selected. Both players 1 and 2 head toward their respective assignments, player 2 being somewhat more aggressive than player 1 (his relatively more favorable distance away from the ball carrier accounts for this). Player A has the same x-coordinate as the ball carrier, his θ_1 is 45° and θ_2 is 135° (symmetrical about the y-coordinate direction), so his strategy in the x-coordinate is zero. The -14.17 is carefully chosen so that he cannot be beaten by the ball carrier if his strategy is within the prescribed limits. Players B and C both collapse to the center, seeing a $\theta = 90^{\circ}$ as the most potent threat. Note that B moves more to the center, despite the fact that he has longer to react, since he also has the sideline.

In Figure 3-14c (Epoch = 0.50), the ball carrier has moved to the right and upfield. His positive x-coordinate velocity makes $\theta = 90^{\circ}$ about the only threat for C, and so he moves accordingly. Player A is lagging an epoch in his motion and so is accelerating in the positive x-coordinate direction. Player 1 is about to make his block, and player 2 is only two epochs from it.

In Figure 3-14e (Epoch = 1.00 second) both players 1 and 2 have made their blocks, and the ball carrier has become very aggressive. Note that his x-coordinate velocity will carry him around the two blocks if they stay stationary, and so the important player is C. Player C is moving back and to the right, accordingly.

In Figure 3-14g (Epoch = 1.50 seconds) the ball carrier has passed players A and 1, and A is now closer to the ball carrier than player 1. For player 1 to continue the block would require holding, so player 1 is forced on the ground for the duration of this epoch (it doesn't show up on the output until the next epoch). Player C has gained such a high x-coordinate velocity that he fears the ball carrier can beat him to the left (his area of responsibility). The ball carrier continues forward, however, and player C will miss the tackle. This may be considered inept play on player C's part (no doubt football fans in the stands would see it as such), but one must remember that player C's area of responsibility was defined as to the left of the ball carrier, and the ball carrier ran to the right. That player C came so close (within 0.3 yards) of making the tackle is due to the fact that the ball carrier came close to the boundary of player C's responsibility in the few epochs prior to the 1.50 seconds epoch.

In Figure 3-14i (Epoch - 2.00 seconds), player B is now in the same situation as player A two epochs previously. Thus in the next epoch (Figure 3-14j), players 1 and 2 are on the ground, player B is tackling the ball carrier, and players A and C moving to tackle the ball carrier as well. The play ends on the next epoch, however, before they can reach the ball carrier.

EPOCH = 0.00 SECONDS

EPOCH = 0.00 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	9.00	-2.00	0.00	0.00	0	11.08	12.89	A
	2	15.00	0.00	0.00	0.00	0	2.23	16.73	B
	*	12.00	-6.00	0.00	0.00	0	9.01	14.42	
DEFENSE									
	A	12.00	2.00	0.00	0.00	0	.00	-14.17	
	B	16.00	4.00	0.00	0.00	0	-7.90	-15.05	
	C	8.00	0.00	0.00	0.00	0	7.31	-15.35	

Figure 3-14a. 3-on-3: Example 1

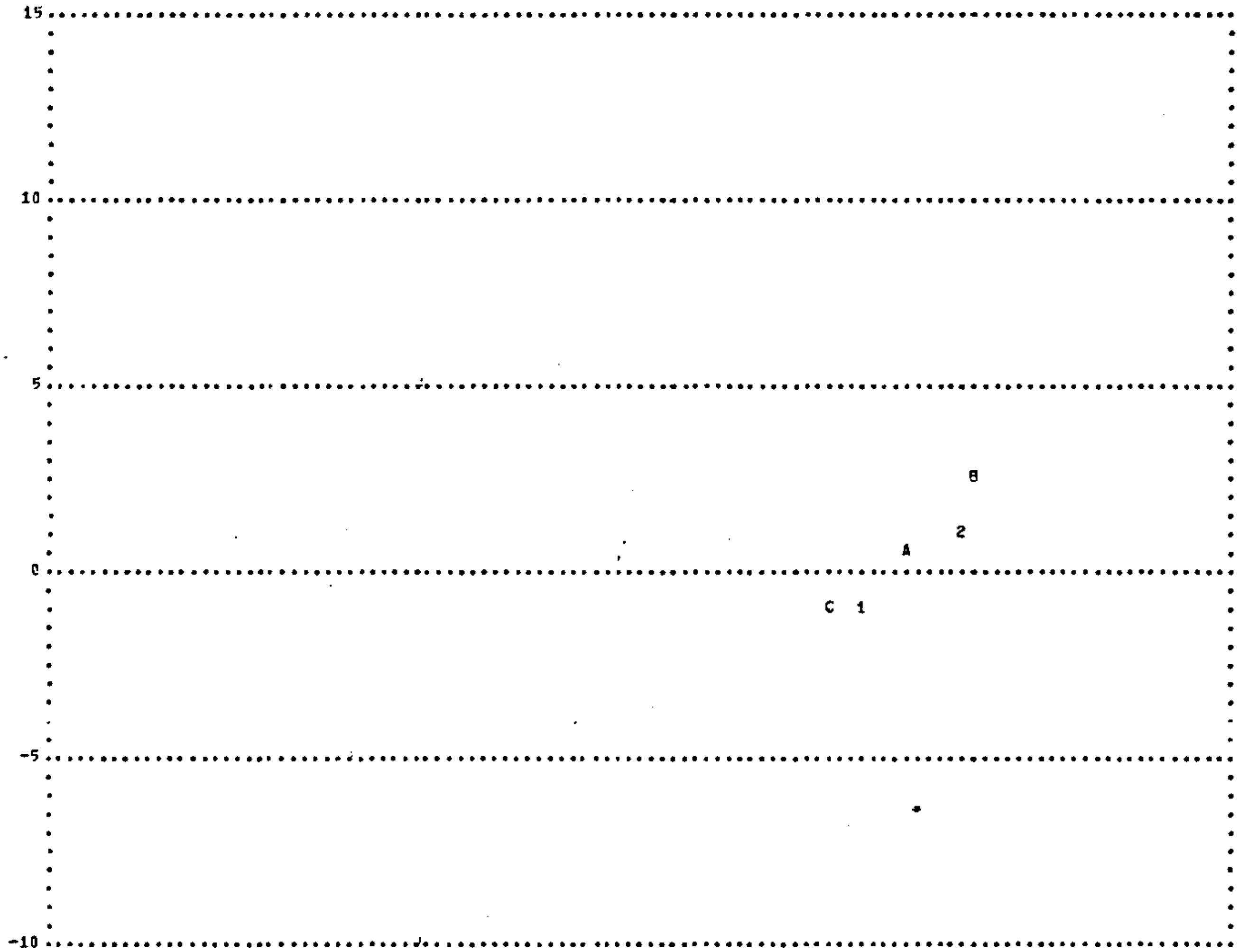
EPOCH = .25 SECONDS

EPOCH = .25 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	9.29	-1.67	2.07	2.41	0	16.63	-3.54	A
	2	15.06	.43	.42	3.13	0	-7.36	15.32	B
	*	12.23	-7.63	1.58	2.70	0	6.37	15.76	
DEFENSE									
	A	12.00	1.64	.00	-2.65	0	.65	-15.83	
	B	15.80	3.61	-1.48	-2.81	0	-12.73	-11.20	
	C	8.19	-.40	1.37	-2.87	0	14.96	-8.07	

Figure 3-14b. 3-on-3: Example 1

EPOCH = .50 SECONDS

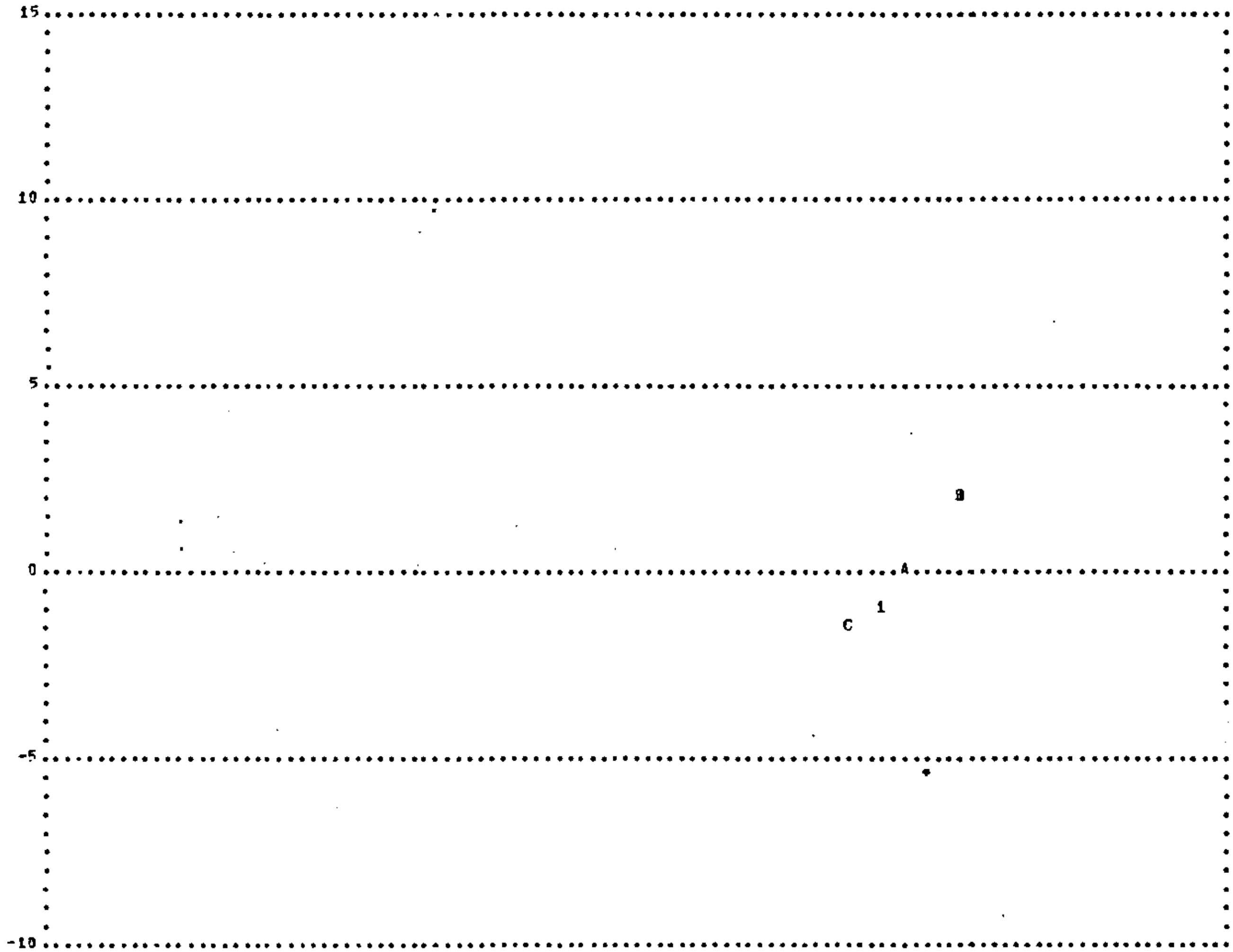


EPOCH = .50 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	10.10	-1.31	4.23	.65	1	16.91	1.73	A
	2	14.95	1.41	-1.15	4.56	0	16.68	-3.26	B
	*	12.71	-6.72	2.10	4.41	0	12.63	11.38	
DEFENSE									
	A	12.02	.73	.16	-4.39	0	8.20	-11.96	
	B	15.19	2.80	-3.19	-3.62	1	-10.98	-8.00	
	C	8.83	-1.14	3.54	-3.06	0	17.00	0.00	

Figure 3-14c. 3-on-3: Example 1

EPOCH = .75 SECONDS

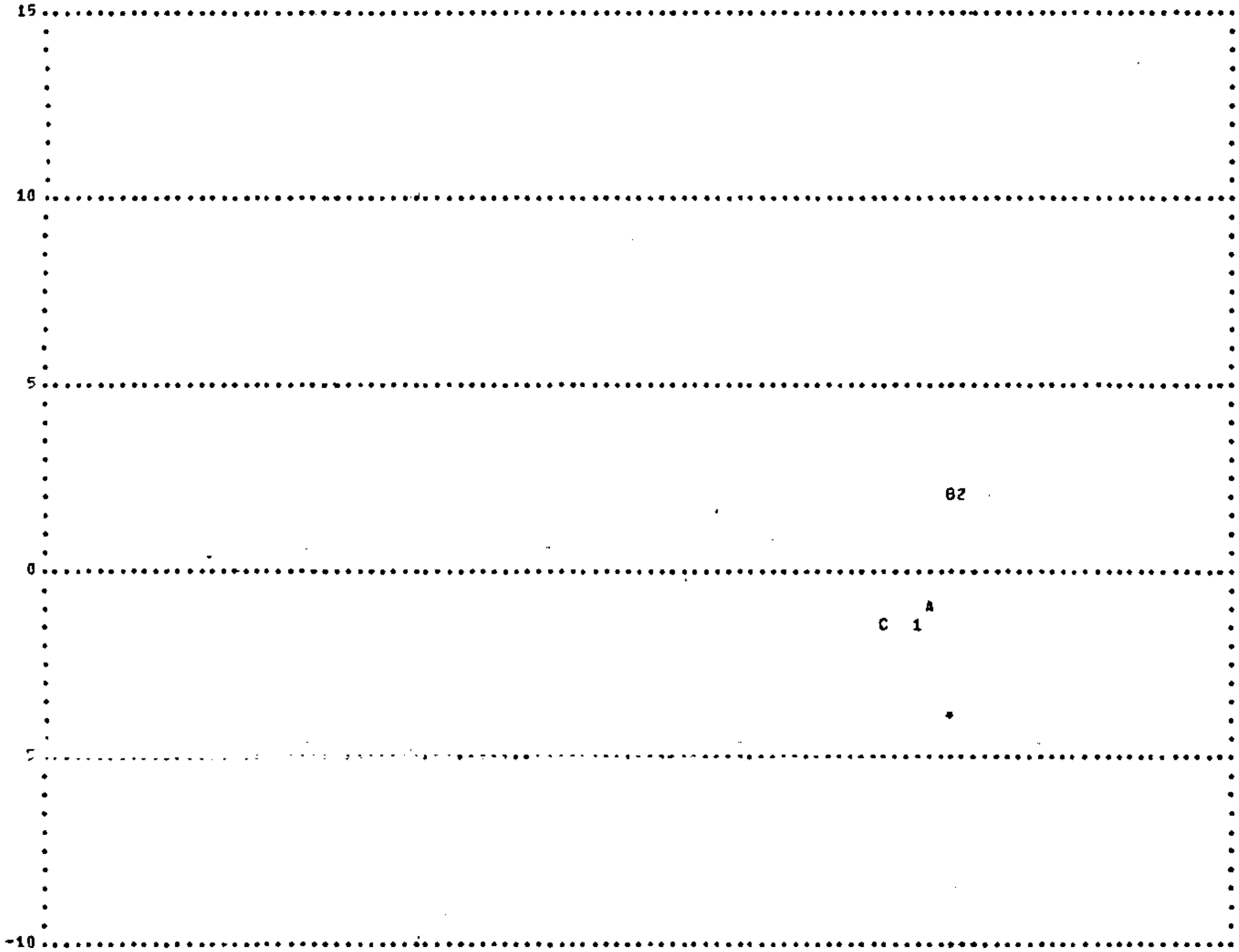


EPOCH = .75 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	11.33	-1.14	5.45	.67	0	16.27	-4.94	A
	2	14.84	2.37	-.85	.14	1	-15.76	-2.85	B
	*	13.43	-5.60	3.50	4.52	0	3.53	16.63	
DEFENSE									
	A	12.26	-.40	1.62	-4.62	0	16.22	-5.09	
	B	14.56	2.40	-.80	-.44	1	-.26	-5.91	
	C	9.93	-1.71	5.10	-1.66	0	16.41	4.43	

Figure 3-14d. 3-on-3: Example 1

EPOCH = 1.00 SECONDS



EPOCH = 1.00 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	12.86	-1.72	4.38	-2.10	1	-16.56	-3.86	A
	2	14.66	2.44	-.42	-.37	1	-13.11	-10.82	B
	*	14.17	-4.33	2.56	5.56	0	.59	16.99	
DEFENSE									
	A	13.19	-1.66	4.08	-1.25	1	14.54	-8.81	
	B	14.44	2.28	-.52	-.50	1	8.10	-4.20	
	C	11.30	-1.91	5.83	-.07	0	9.81	13.89	

Figure 3-14e. 3-on-3: Example 1

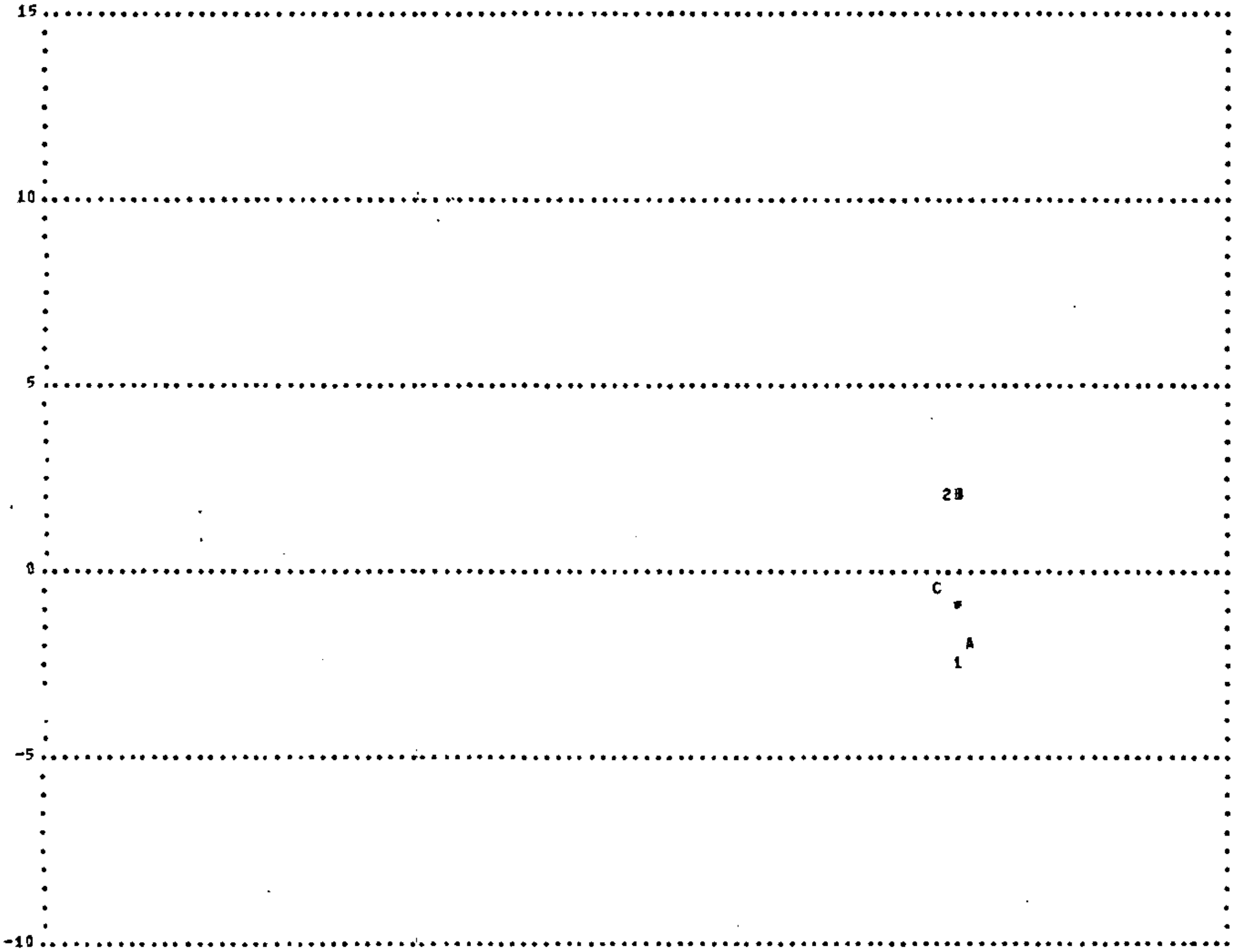
EPOCH = 1.25 SECONDS

EPOCH = 1.25 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	13.84	-2.23	4.45	-2.11	1	6.28	15.80	A
	2	14.48	2.41	.03	-.49	1	-4.31	-16.44	B
	*	14.67	-2.85	1.50	6.19	0	1.19	16.96	
DEFENSE									
	A	14.27	-1.64	3.95	-2.48	1	-17.00	4.40	
	B	14.42	2.16	-.17	-.54	1	15.93	-5.93	
	C	12.64	-1.56	4.99	2.56	0	2.27	16.39	

Figure 3-14f. 3-on-3: Example 1

EPOCH = 1.50 SECONDS



EPOCH = 1.50 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	14.85	-2.76	3.99	-2.20	1	8.14	-14.92	A
	2	14.40	2.24	.51	-.78	1	7.67	-15.17	B
	*	14.98	-1.26	1.03	6.52	0	.00	17.00	
DEFENSE									
	A	15.29	-2.18	4.18	-2.05	1	-5.51	16.03	
	B	14.53	2.03	.17	-.56	1	17.00	0.00	
	C	13.64	-.66	3.13	4.45	0	-16.57	3.79	

Figure 3-14g. 3-on-3: Example 1

EPOCH = 1.75 SECONDS

EPOCH = 1.75 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	14.85	-2.76	0.00	0.00	2	13.43	10.42	A
	2	14.46	2.02	.64	-.57	1	16.14	-5.33	B
	*	15.17	.40	.56	6.71	0	.00	17.00	
DEFENSE									
	A	15.93	-2.15	1.24	1.89	0	-4.87	16.29	
	B	14.70	1.96	.55	-.25	1	10.77	-13.15	
	C	13.80	.26	-1.40	3.12	0	16.92	1.61	

Figure 3-14h. 3-on-3: Example 1

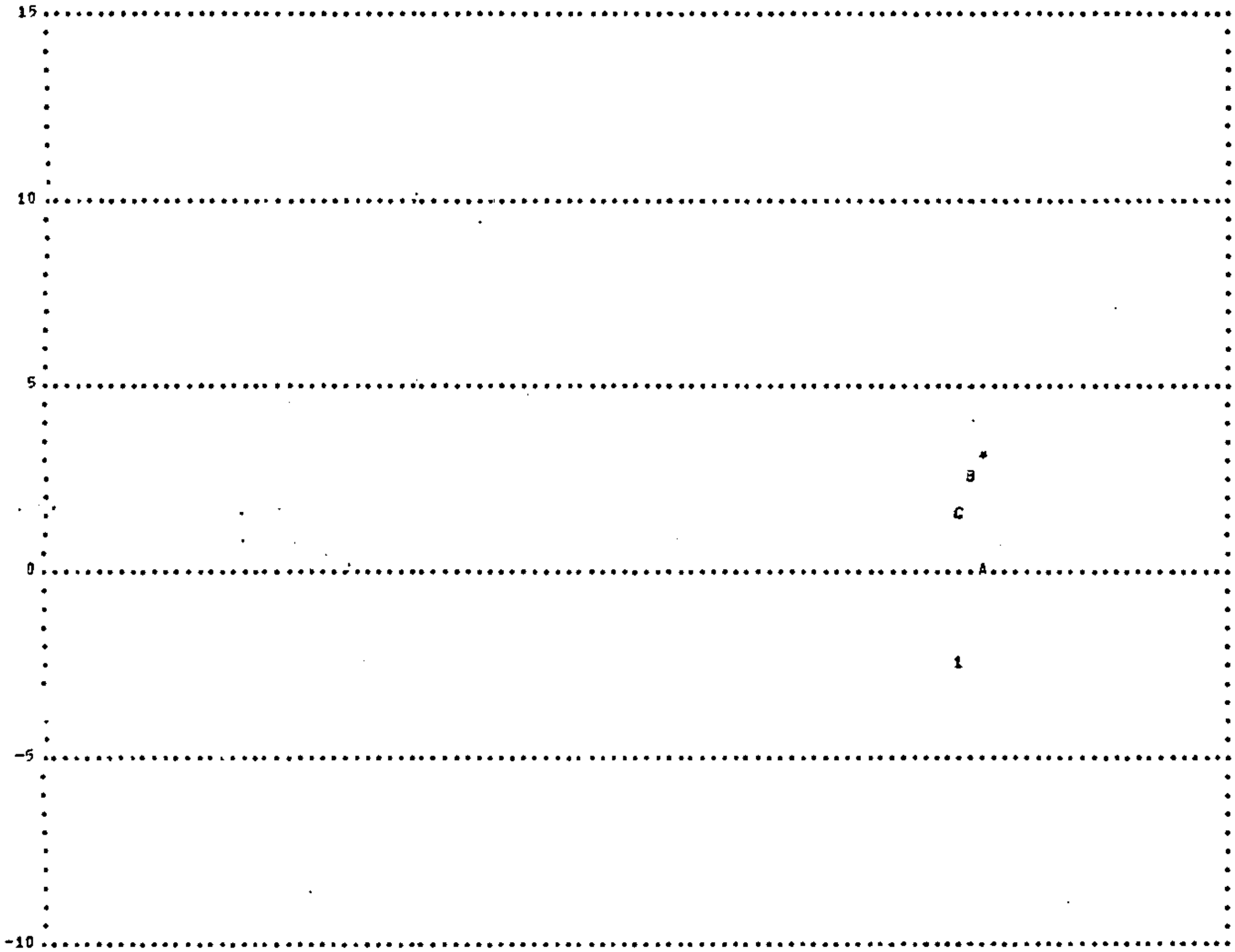
EPOCH = 2.00 SECONDS

EPOCH = 2.00 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	14.85	-2.76	0.00	0.00	2	7.25	15.37	A
	2	14.62	1.77	.83	-.49	1	16.24	-5.02	B
	*	15.28	2.09	.30	6.82	0	.00	17.00	
DEFENSE									
	A	16.04	-1.38	-.24	4.07	0	-3.65	16.60	
	B	14.85	1.81	.79	-.70	1	14.22	9.31	
	C	13.97	.89	2.40	1.99	0	12.53	11.49	

Figure 3-14i. 3-on-3: Example 1

EPOCH = 2.25 SECONDS

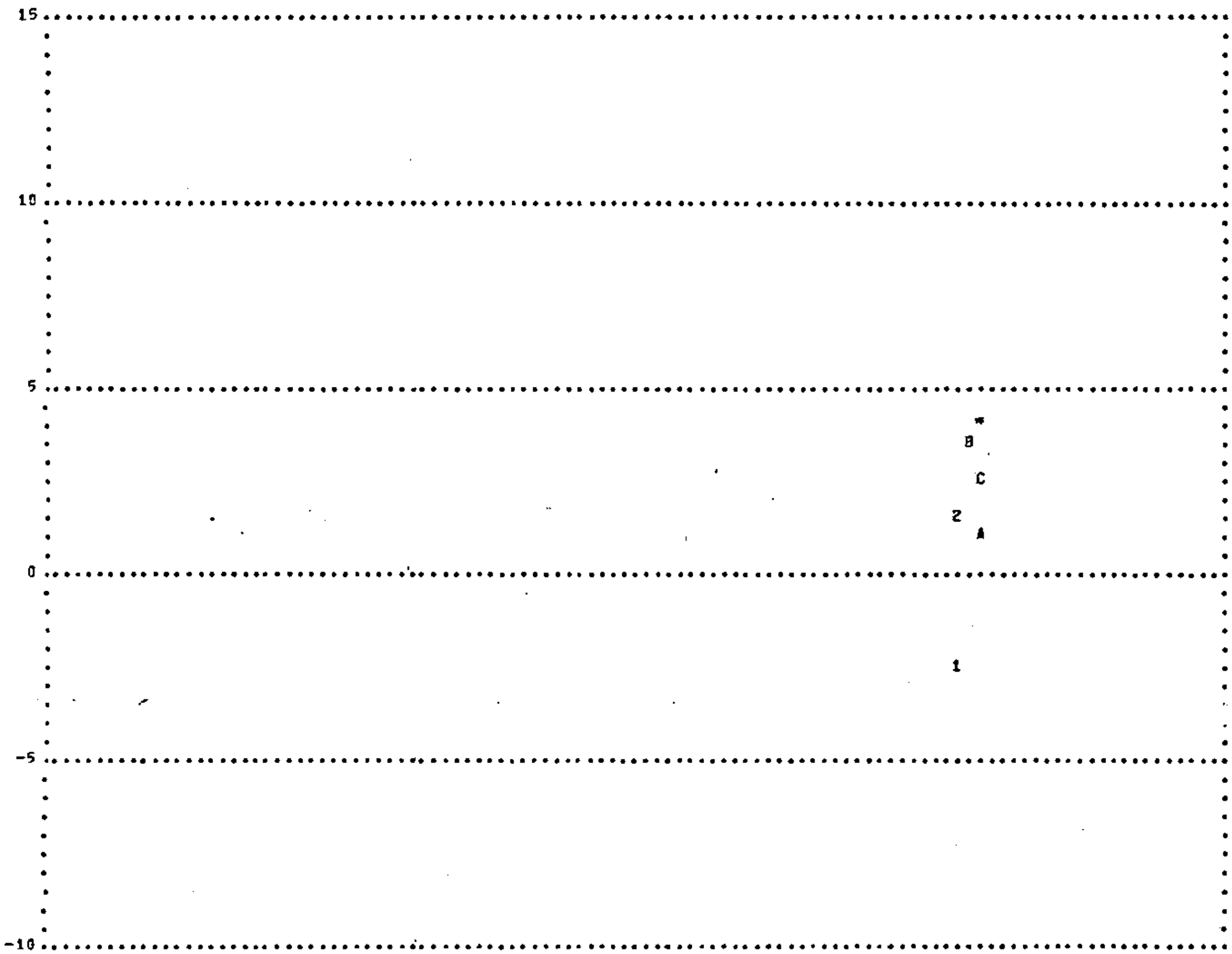


EPOCH = 2.25 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	14.85	-2.76	0.00	0.00	2	3.62	16.61	A
	2	14.62	1.77	0.00	0.00	2	-16.74	-2.97	B
	*	15.53	3.28	1.50	3.97	3	.00	17.00	
DEFENSE									
	A	15.90	-.19	-.81	5.31	0	-1.78	16.91	
	B	15.20	2.61	.35	3.41	3	0.00	-17.00	
	C	14.74	1.56	3.65	3.23	0	7.09	15.45	

Figure 3-14j. 3-on-3: Example 1

EPOCH = 2.50 SECONDS



EPOCH = 2.50 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	14.85	-2.76	0.00	0.00	2	2.03	16.88	A
	2	14.62	1.77	0.00	0.00	2	-16.90	1.85	B
	*	15.79	4.19	1.00	3.73	3	.00	17.00	
DEFENSE									
	A	15.70	1.24	-.77	6.04	0	.51	16.99	
	B	15.43	3.53	.85	3.65	3	0.00	-17.00	
	C	15.60	2.56	3.30	4.64	0	1.93	16.89	

Figure 3-14k. 3-on-3: Example 1

3.9.2 Example 2

Example 2 is shown in Figure 3-15. Note that this example is not excessively different from the previous one. The only difference is that the blocking assignments have been changed so that player 1 now blocks the player at (8,0) and player 2 blocks the player at (12,2). Player C, at (16,4) is left unblocked. The defensive areas of responsibilities are the same (A has to the left, B has the middle and C has to the right). Note the players have interchanged letters.

In this example, the ball carrier runs to the left, since C is the unblocked defensiveman, and his position is to the right. The ball carrier cannot, however, run too far to the left, since this makes the block player 2 must make on player B impossible (in the sense of the methodology).

As the play develops, player 1 makes his block (Figure 3-15c) as does player 2 (Figure 3-15d) and the outcome hinges on a duel between the ball carrier and player C. Again the ball carrier is on the boundary of player C's responsibility, but this time player C initiates the interaction. The tackle is unsuccessful, however (Figure 3-15k), and the ball carrier is free to run downfield unchecked. (The play continues from this point but is uninteresting).

The differences in the inputs between the two examples are not large and yet the whole nature of the play is changed. If a distribution of the gains were made by rerunning each play a number of times, it would be possible to determine which of the two blocking assignments (or any of the others, for that matter) is optimal in the sense of maximizing the expected gain. This is a tedious process, however, for plays involving a larger number of players. A more efficient method for optimal blocking assignments is given in Section 6.3.

EPOCH = 0.00 SECONDS

EPOCH = 0.00 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	9.00	-2.00	0.00	0.00	0	-7.65	14.75	A
	2	15.00	0.00	0.00	0.00	0	-17.00	0.00	B
	*	12.00	-8.00	0.00	0.00	0	-6.91	15.53	
DEFENSE									
	A	8.00	0.00	0.00	0.00	0	7.31	-15.35	
	B	12.00	2.00	0.00	0.00	0	.00	-14.17	
	C	16.00	4.00	0.00	0.00	0	-7.90	-15.05	

Figure 3-15a. 3-on-3: Example 2

EPOCH = .25 SECONDS

EPOCH = .25 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	8.82	-1.62	-1.32	2.76	0	15.16	7.69	A
	2	14.56	0.00	-3.18	0.00	0	-16.52	-4.03	B
	*	11.82	-7.60	-1.29	2.90	0	-16.17	5.25	
DEFENSE									
	A	8.19	-.40	1.37	-2.87	0	14.60	-8.71	
	B	12.00	1.64	.00	-2.65	0	-.48	-15.38	
	C	15.80	3.61	-1.48	-2.81	0	-9.47	-14.12	

Figure 3-15b. 3-on-3: Example 2

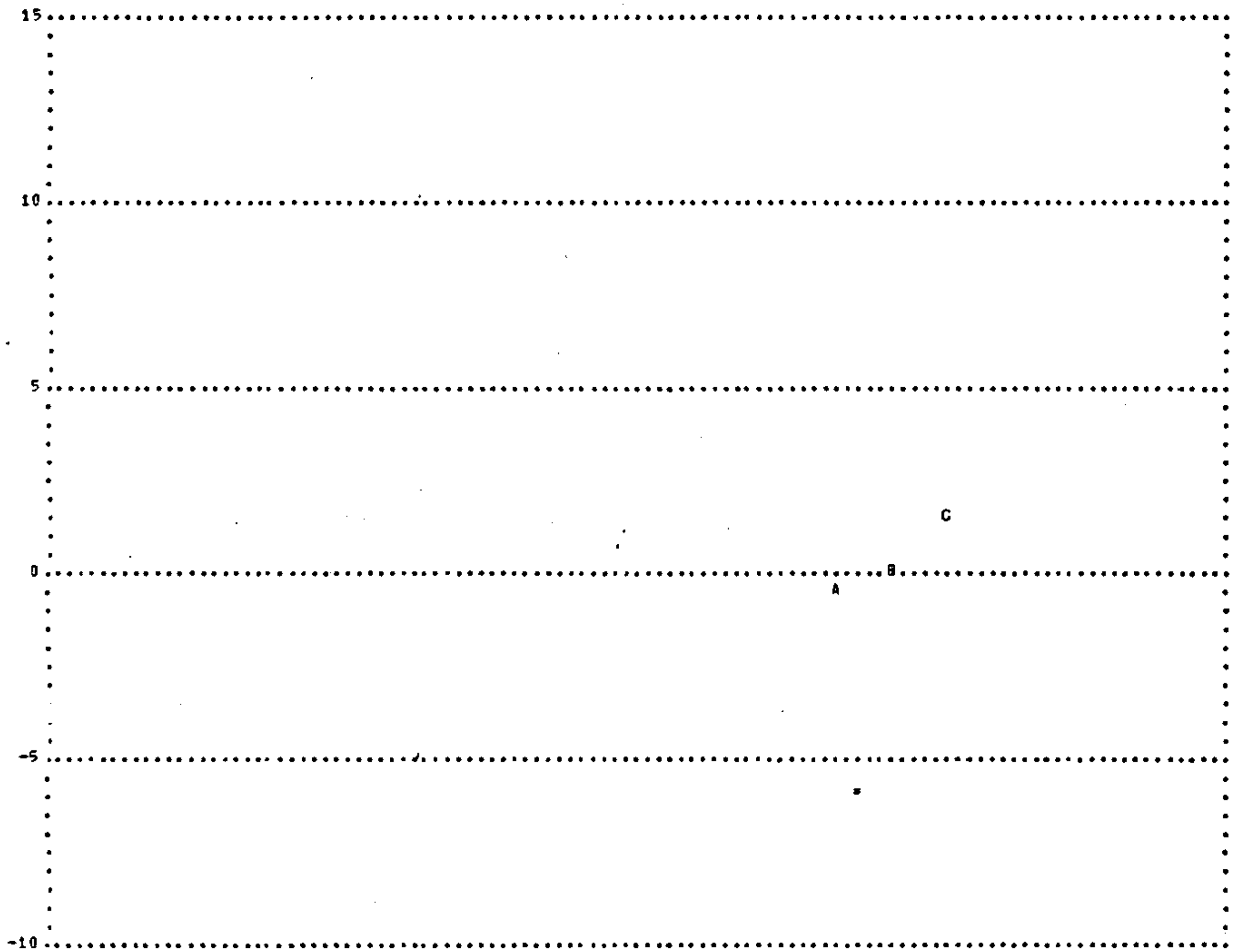
EPOCH = .50 SECONDS

EPOCH = .50 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	8.85	-.69	1.72	-.07	1	13.48	-10.36	A
	2	13.54	-.13	-4.81	-.75	0	-16.82	-2.45	B
	*	11.16	-6.92	-3.72	2.56	0	-13.40	10.47	
DEFENSE									
	A	8.57	-.72	1.59	-.20	1	10.46	1.02	
	B	11.99	.74	-.39	-4.31	0	-6.65	-10.33	
	C	15.28	2.72	-2.57	-4.16	0	-12.85	-11.13	

Figure 3-15c. 3-on-3: Example 2

EPOCH = .75 SECONDS

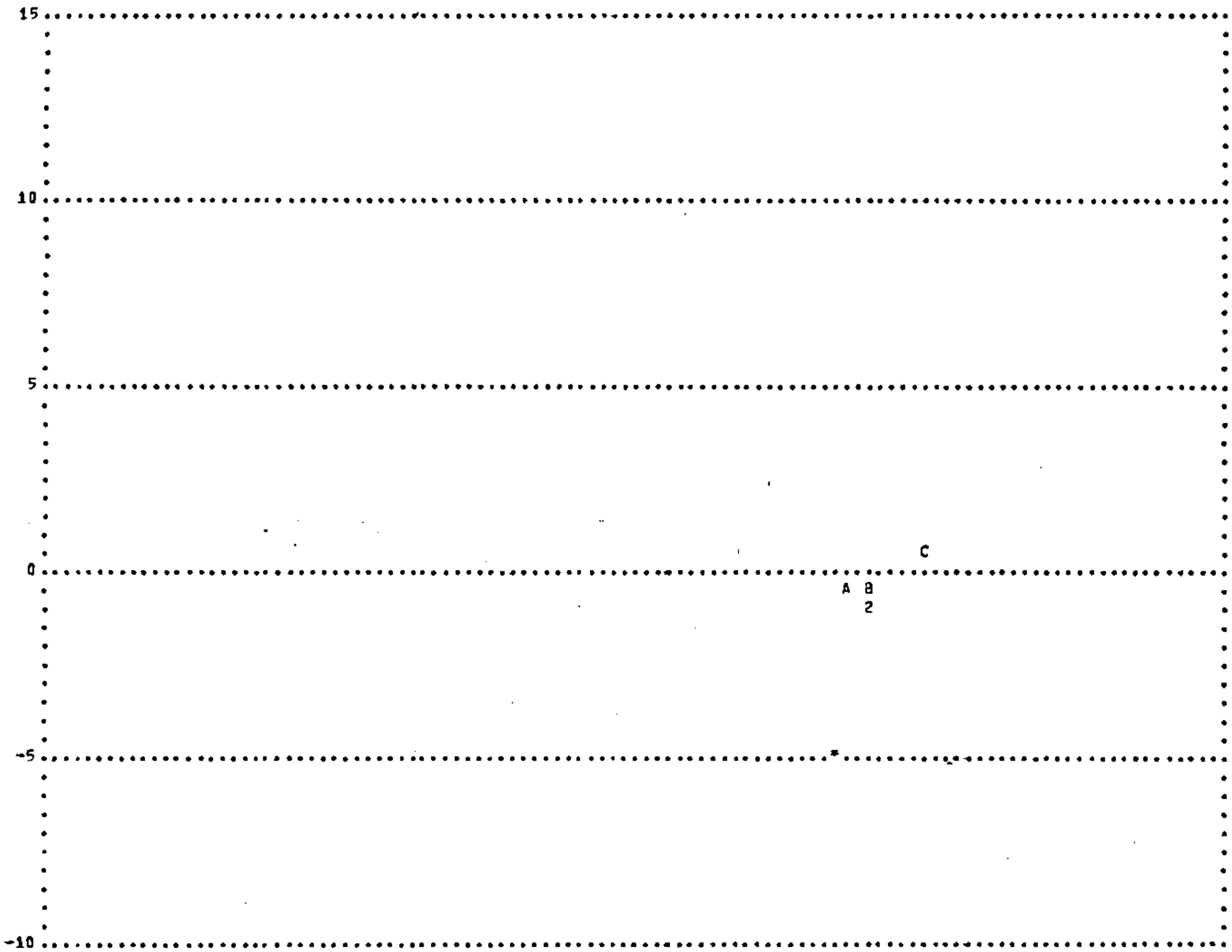


EPOCH = .75 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	9.31	-.73	1.97	-.36	1	16.61	-3.62	A
	2	11.57	-.42	-3.31	-2.79	1	-14.79	-8.39	B
	*	10.12	-6.18	-4.52	3.34	0	-9.51	14.09	
DEFENSE									
	A	9.03	-.75	1.99	-.12	1	7.93	14.19	
	B	11.58	-.17	-3.22	-2.78	1	-15.40	-7.20	
	C	14.47	1.66	-3.80	-4.34	0	-15.78	-6.32	

Figure 3-15d. 3-on-3: Example 2

EPOCH = 1.00 SECONDS

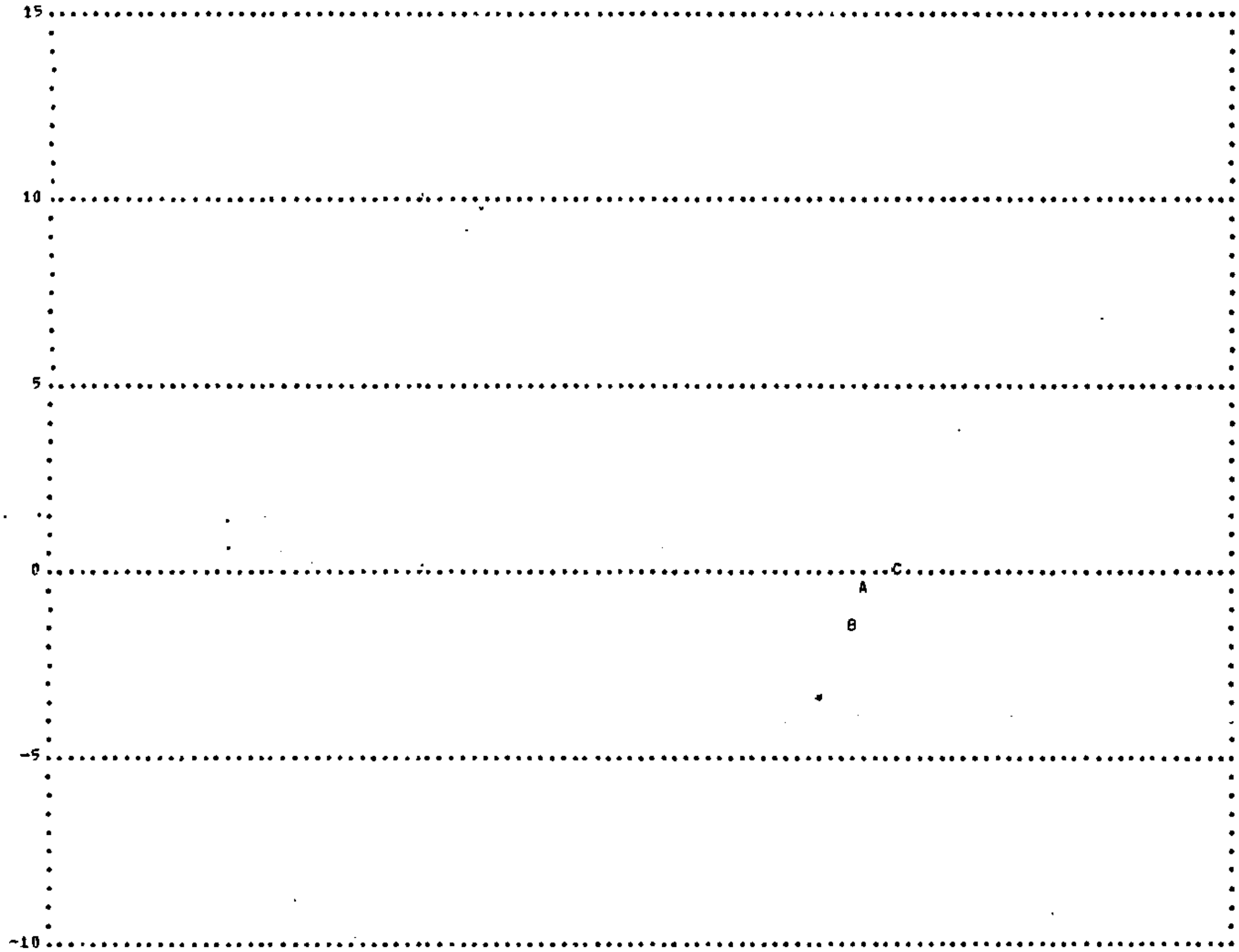


EPOCH = 1.00 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	9.23	-.82	2.31	-.34	1	16.52	-4.01	A
	2	10.70	-1.14	-3.60	-2.96	1	-14.76	-8.40	B
	*	9.03	-5.19	-4.23	4.45	0	-5.81	15.97	
DEFENSE									
	A	9.56	-.76	2.21	.10	1	1.35	16.89	
	B	10.73	-.89	-3.60	-2.96	1	-17.00	0.00	
	C	13.35	.69	-5.01	-3.53	0	-16.99	.55	

Figure 3-15e. 3-on-3: Example 2

EPOCH = 1.25 SECONDS

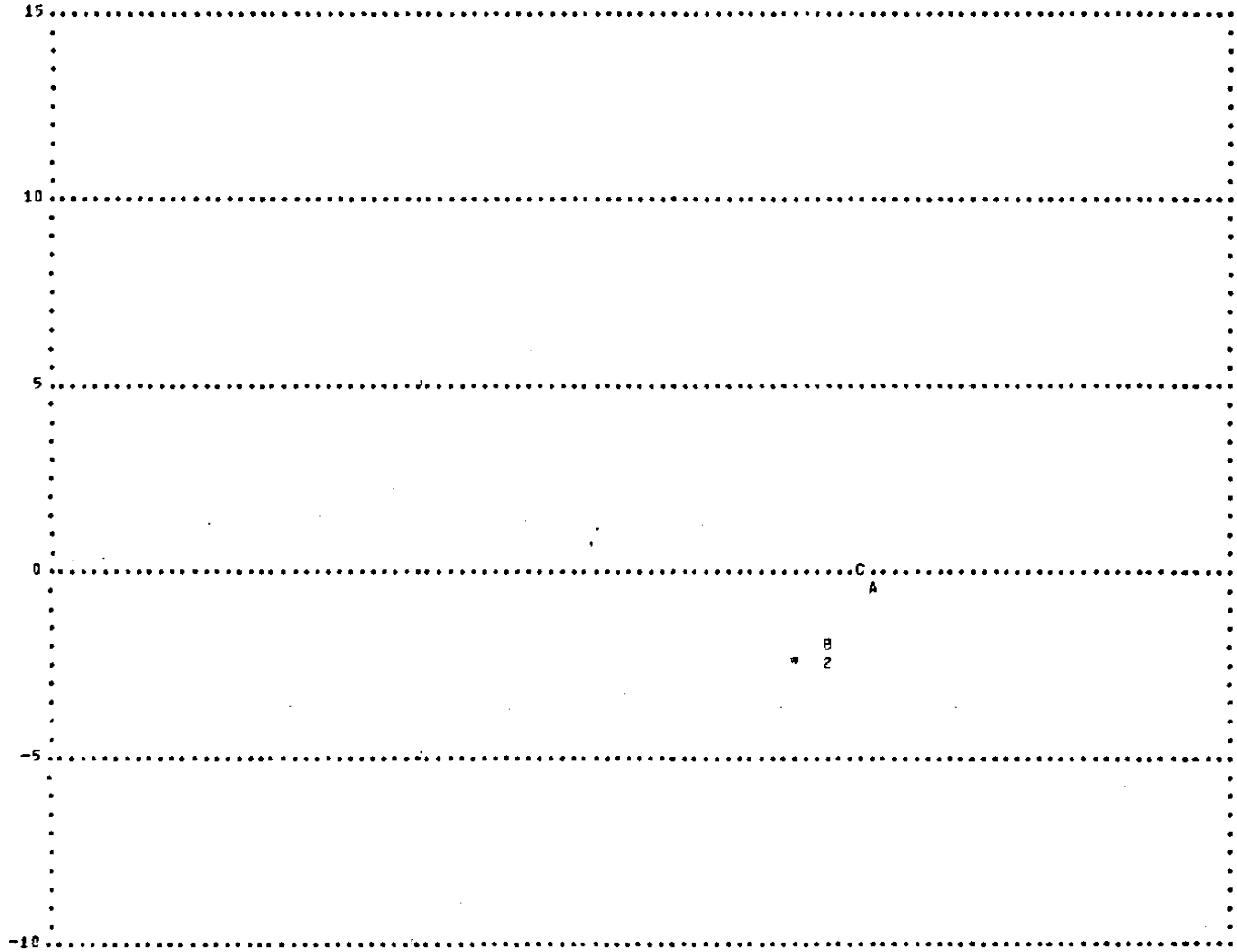


EPOCH = 1.25 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	10.41	-.87	2.58	-.18	1	15.97	5.84	A
	2	9.74	-1.89	-3.98	-3.06	1	-14.89	-8.20	B
	*	8.09	-3.95	-3.38	5.40	0	-2.37	16.83	
DEFENSE									
	A	10.16	-.74	2.36	.23	1	1.12	16.96	
	B	9.78	-1.64	-3.91	-3.05	1	-1.90	3.75	
	C	11.98	.04	-5.89	-1.81	0	-14.79	8.38	

Figure 3-15f. 3-on-3: Example 2

EPOCH = 1.50 SECONDS



EPOCH = 1.50 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	10.67	-.64	1.17	.52	1	10.99	12.98	A
	2	8.70	-2.64	-4.02	-2.99	1	-10.69	-13.22	B
	*	7.40	-2.50	-2.27	6.07	0	.00	17.00	
DEFENSE									
	A	10.54	-.57	1.04	.76	1	0.00	17.00	
	B	8.78	-2.41	-4.25	-3.07	1	7.25	-15.38	
	C	10.49	-.08	-5.96	.59	0	-10.00	13.74	

Figure 3-15g. 3-on-3: Example 2

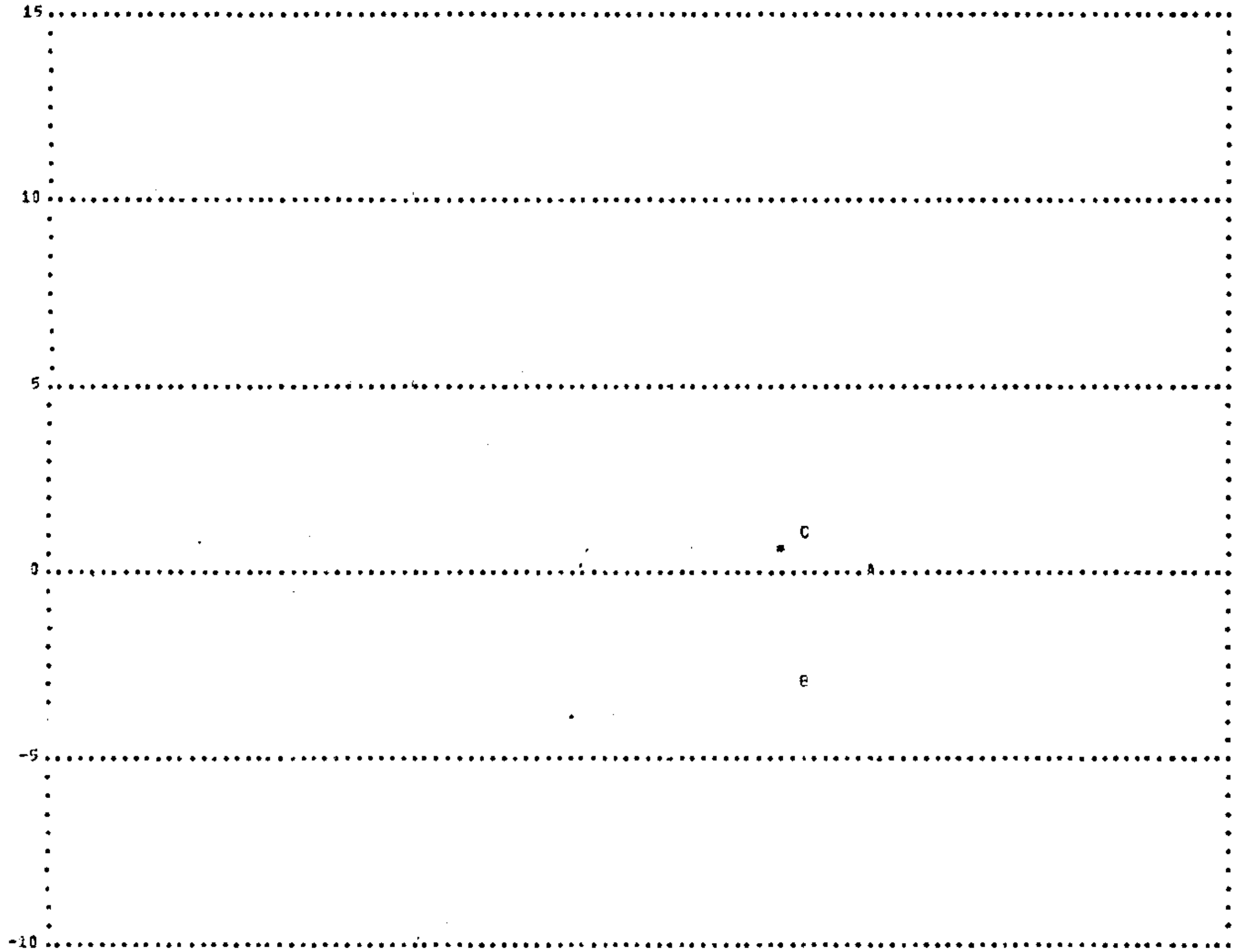
EPOCH = 1.75 SECONDS

EPOCH = 1.75 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	10.81	-.39	.54	.91	1	12.31	-11.72	A
	2	8.35	-2.94	-1.52	-1.63	1	-16.32	-4.76	B
	*	6.98	-.93	-1.23	6.47	0	.00	17.00	
DEFENSE									
	A	10.71	-.33	.46	1.02	1	0.00	17.00	
	B	8.26	-3.02	-2.03	-2.26	1	-8.89	14.49	
	C	9.12	.38	-5.10	2.89	0	-8.59	14.67	

Figure 3-15h. 3-on-3: Example 2

EPOCH = 2.00 SECONDS

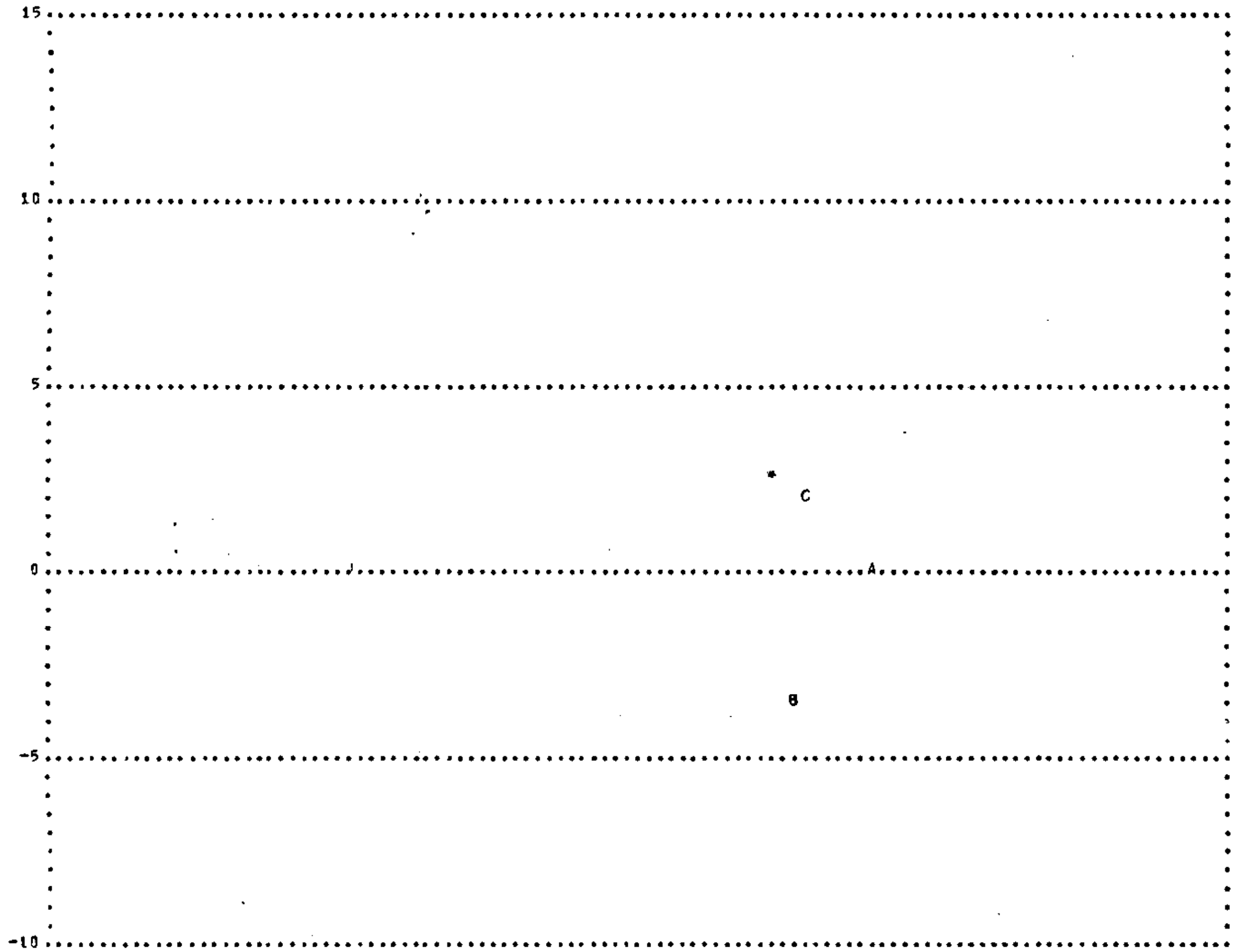


EPOCH = 2.00 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	10.89	-.15	.31	.75	1	-2.26	16.85	A
	2	7.89	-3.43	-2.08	-1.93	1	-16.89	-1.92	B
	*	6.75	.72	-.67	6.63	0	.00	17.00	
DEFENSE									
	A	10.81	-.08	.17	.92	1	-16.68	3.27	
	B	7.77	-3.48	-2.01	-1.75	1	-4.04	16.51	
	C	7.95	1.30	-4.37	4.31	0	17.00	0.00	

Figure 3-15i. 3-on-3: Example 2

EPOCH = 2.25 SECONDS

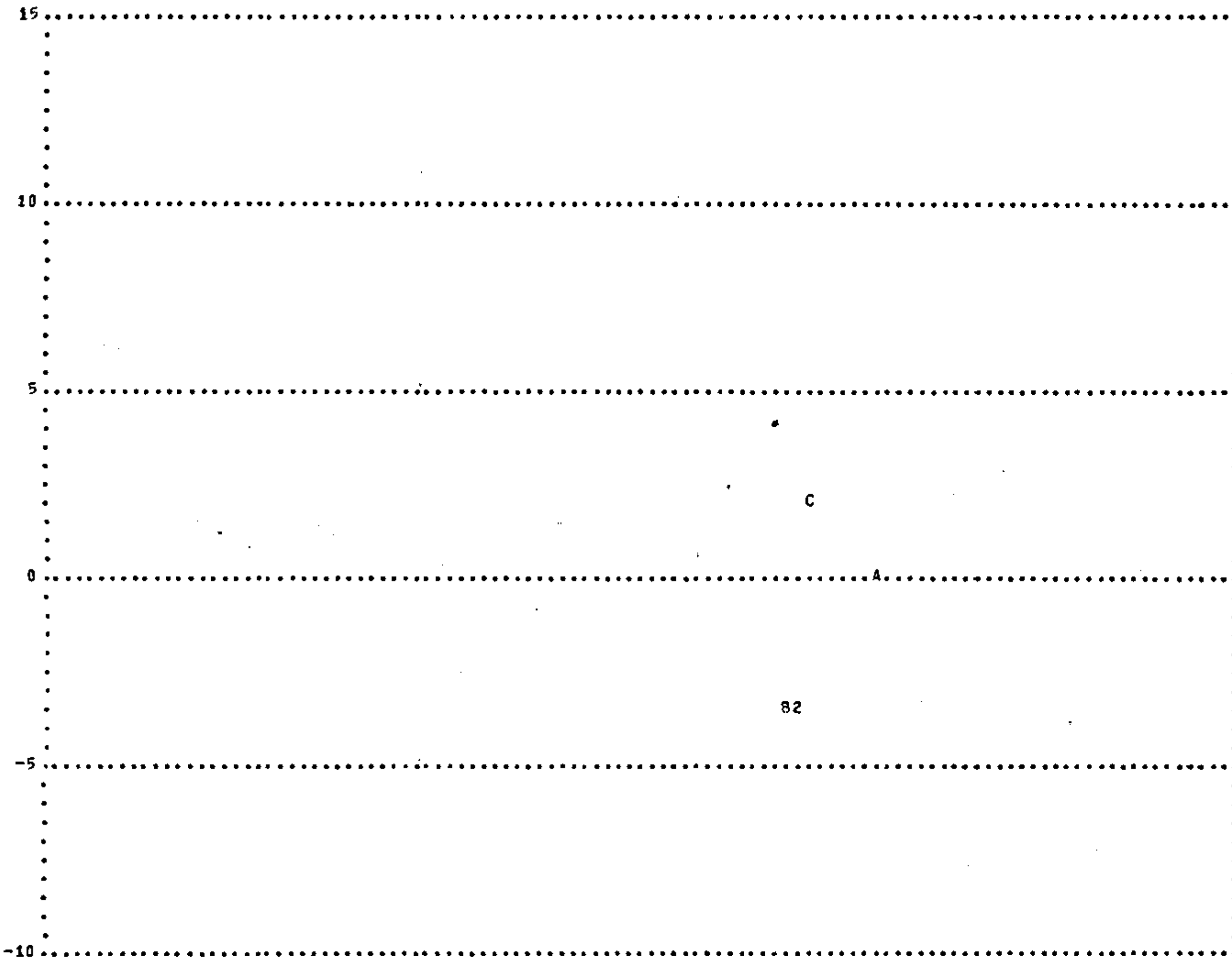


EPOCH = 2.25 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	10.83	.06	-.41	.80	1	-14.15	9.42	A
	2	7.36	-3.89	-2.28	-1.82	1	-16.99	-.67	B
	*	6.39	2.77	-.98	5.76	3	.00	17.00	
DEFENSE									
	A	10.71	.07	-.38	.54	1	-14.42	9.01	
	B	7.23	-3.90	-2.26	-1.54	1	-2.13	16.87	
	C	7.60	2.43	-.63	4.49	3	0.00	-17.00	

Figure 3-15j. 3-on-3: Example 2

EPOCH = 2.50 SECONDS



EPOCH = 2.50 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	10.68	.25	-.80	.58	1	-15.68	6.57	A
	2	7.04	-3.85	-1.33	-.17	1	16.98	.79	3
	*	6.20	4.28	-.53	6.30	0	.00	17.00	
DEFENSE									
	A	10.55	.22	-.78	.66	1	-12.42	11.61	
	B	6.89	-3.99	-1.26	-.03	1	-1.41	16.94	
	C	7.60	2.43	0.00	0.00	2	-10.26	13.55	

Figure 3-15k. 3-on-3: Example 2

CHAPTER IV

VALIDATING THE MODEL

Chapter III describes the model used in the remainder of the thesis. This chapter describes the validation of the model's results. It would have been preferable to collect data on each of the component parts of the model to verify the validity of that portion. The only data available, however, were game films of complete plays. Plays are sequences of interrelated decisions and it is difficult to isolate the interactions and study them apart from the rest of the play. Such is the state of affairs, in fact, that even professional scouts occasionally have difficulty ascertaining abilities from the films because it is so difficult to determine the players' objective. Thus validation by parts is infeasible, and instead the results of the entire model is required to validate any part of it.

Section 4.1 describes the process of obtaining data from the game films of the T-Night football scrimmage held on May 7, 1976. This data is listed in Tables 4-5 through 4-8.

In Section 4.2, the estimation of the parameters C_1, C_2, C_3 and σ_C necessary for the utilization of the model are made. The parameter C_1 (and the corresponding parameters C^{E_i} and C^{P_i}) are estimated using a non-linear estimation procedure with numerical differentiation. Parameters C_2 and C_3 are estimated using a grid search, minimizing an average sum-of-squares error. The parameter σ_C is estimated using a simulation of 40-yard dash times and actual measurement of the variances of the players' 40-yard dash times.

The method of the verification of the model is given in Section 4.3. The results of the simulation, the data collected, and the comparison of the two are listed in Tables 4-6 through 4-8.

4.1 Source of Data

Data for the validation of the model was taken from the game film of the Georgia Tech T-Night football scrimmage held on May 7, 1976. A scrimmage was chosen over a "real" game since the required data (weight, strength and speed) was known for all the participants. This data is regularly compiled by the Georgia Tech coaches and was made available for this purpose. The data analyzed was the players' weight, number of pounds the player could bench-press, and 40-yard dash times. The times were converted into C^P's and C^E's using the procedure outlined in Section 4.2.1.

The game film consisted of 16 millimeter film shot at twenty-four frames per second. This film was then taken to a microfilm reader and photocopied. On each play analyzed, every sixth frame (corresponding to epochs of 0.25 seconds) was recorded starting on the frame prior to the initial frame showing any motion of the players.

From each frame, 44 values (22 players x 2 coordinates) of positional data had to be obtained. The film represents a functional transformation from three-space to two-space, which then had to be transformed back to three-space and the x-y coordinates saved as data. In the transformation, relative distances on the field are not preserved on film. Straight lines are recorded as straight lines, however. Thus it was possible to draw a grid for the y-coordinate as the hash marks were clearly visible on the film. Likewise, the x-coordinate of both sets of hash marks are known ($\pm$ 8.89 yards). The data was then obtained by the following steps:

1. An estimate of the point on the football field directly under the front numerals of the player's jersey was made.

2. An estimate of the y-coordinate value of this point from the (nearly-parallel) y-coordinate grid marks was made.

3. Similar triangles on the plane of the playing field* between the hash marks (or the marks on the sidelines) and the point on the field were set up. Using the trigonometric laws of similar triangles and the y-coordinate value from step 2, the x-coordinate value was computed.

The estimation of step 1 is most critical--a deviation of 1/16 of an inch on the photocopy results in an x-coordinate change of up to 1/2 yard.

One play was chosen to collect data on (a Wishbone Counter-Dive vs. a 50 Defense)**. The values of the x- and y-coordinates are listed for each player in Tables 4-5 through 4-8.

4.2 Estimation of Parameters

The estimating of the parameters C_1, C_2, and C_3 is in fact the

*The playing field is really not a plane, as it slopes down from the center-line to the sidelines. From the camera angle in question, however, this was not a serious problem.

**This offense and defense are presented in Chapter V.

"tuning" of the model. Whereas finding a proper value for C_1 is perhaps of more general interest, finding C_2 and C_3 merely is an exercise of a common-sense grid search of suitable values.

4.2.1 Estimation of C_1

The estimation of C_1 by necessity hinges on the use of (3.8) with $x(0 = \dot{x}(0) = 0$, i.e.

$$x(t) = \frac{C_x}{C_1}t - \frac{C_x}{C_1^2}(1-e^{-e_1 t}) \tag{4.1}$$

or

$$x(t) = C_x \left[\frac{t}{C_1} - \frac{(1-e^{-C_1 t})}{C_1^2} \right] \tag{4.2}$$

which is linear in C_x and non-linear in C_1. It is clear from (4.2) that the estimation of C_1 is dependent on the simultaneous estimation of C_x. Also, if C_1 is given, a 40-yard dash time (or any distance for that matter) fixes the value for C_x, which could then be used for C^{E_i} or C^{P_i}.

Suppose N data points are available for a runner starting from a standstill and running in a straight line in the x-coordinate direction (a data point is defined as a x_i, t_i pair). Assume an additive error to the system equation (4.2),

$$x_i = C_x \left[\frac{t_i}{C_1} - \frac{(1-e^{-C_1 t_i})}{C_1^2}\right] + a_i \tag{4.3}$$

Holding C_1 constant (for the time being) and solving a least-squares problem with respect to C_x gives

$$\sum_{i=1}^{N} a_i \frac{\partial a_i}{\partial C_x} = 0 \tag{4.4}$$

or

$$C_x = \frac{\sum_{i=1}^{N} x_i Q_i}{\sum_{i=1}^{N} Q_i^2} \tag{4.5}$$

where $Q_i = t_i/C_1 - (1 - \exp(-C_1 t_i))/C_1^2$. Likewise, holding C_x constant and solving for C_1 gives the normal equation

$$\sum_{i=1}^{N} a_i \frac{\partial a_i}{\partial C_1} = 0 \tag{4.6}$$

or

$$\sum_{i=1}^{N} \left[C_x Q_i - x_i\right] \left[\frac{(1-e^{-C_1 t_i})e^{-C_1 t_i}}{C_1} - \frac{t_i}{C_1^2} + \frac{2(1-e^{-C_1 t_i})}{C_1^3}\right] = 0 \tag{4.7}$$

This is a cubic equation in C_1 with embedded exponentials in $C_1 t_i$--in short too difficult to solve analytically. Thus two trial values of

C_1 (C_1' and $C_1' + 0.01$) can be input into the left hand side of (4.7) and the next iteration's C_1 is an interpolation between the two. An input value of C_1 is needed for the first iteration, and there is convergence for any "close" value; in fact the surface defined by the normal equations is sufficiently well-behaved that convergence takes place for any reasonable input C_1.

Whereas fitting the constants to the data is rather simple and straight-forward, finding the input data to fit is another matter. Most authors are content to use world records to fit running curves and thus offer no data for less than 50 yards.

Keller [14] presents such a model, and attempts to fit an equation through world records from 50 yards through 10,000 meters. The Keller model is identical to equation (4.1) for distances less than 291 meters. The value for C_1 calculated by a non-linear estimation procedure is 1.121 seconds^{-1}.

Hill offers data for two rather poor (by today's standards) runners:

Table 4-1. Hill's Running Data

Time: Runner 1	Time: Runner 2	Distance
0.65	0.80 seconds	1 yards
1.07	1.28	3
1.56	1.82	6
2.10	2.42	10
2.72	3.07	15
3.28	3.73	20
4.36	4.91	30
5.42	6.06	40
6.51	7.22	50
7.56	8.37	60

which yields $C_x = 7.531$ and $C_1 = 0.791$ for runner 1 and $C_x = 5.522$ and $C_1 = 0.624$ for runner 2. It should be noted that neither fit is very good, resulting in standard deviations of the order of 0.25 yards.

Taking data off game films presents two problems:

1. Generally speaking, football players tend to avoid running in straight lines.

2. It is difficult to tell if a player is putting full effort into his running.

Never-the-less an attempt was made, giving the following data:

Table 4-2. Game Film Running Data

Time	Distance
0.25 seconds	0.50 yards
0.50	1.40
0.75	2.80
1.00	4.30
1.25	5.91

This yields $C_x = 16.81$ and $C_1 = 2.450$. In this estimation, the fit is quite good, resulting in a standard deviation of 0.04 yards. This player normally runs the 40-yard dash (in track clothes) in 4.55 seconds. For the value of C_1 this corresponds to a C_x of 23.65 yards/second^2. The ratio of the two values is 0.7109--a factor which represents the added burden of the football padding and clothes over those used in the clocking of the 40-yard dash. This factor is used to convert the players' time in the dash (t_i) to their speed on the playing field:

$$C^{E_i} \text{ (or } C^{P_i}) = \frac{(40)(0.7109)}{\left[2.45t_i - 1 + e^{-2.45t_i}\right]} \tag{4.9}$$

4.2.2 Estimation of C_2 and C_3

The value of C_2 and C_3 determine the movement of the center of motion and the rotation around the center of motion of the participants of a block. Using the data gathered from Section 4.1, it is possible to evaluate the effect of differing values of C_2 and C_3 by a grid search. Letting Δ_i denote the distance from the observed position to the position determined by the model for the i'th participant of a block, the criterion used to estimate C_2 and C_3 is to minimize the sum-of-squares error, $\sum_{i=1}^{2K} \Delta_i^2$, where K is the total number of blocks evaluated. The model was run for the same play for which data was gathered in Section 4.1 with parameters $C_1 = 2.45$ and $\sigma_C = 0$, and for the various values of C_2 and C_3. The average error, $\sqrt{\sum_i \Delta_i^2/2K}$, for those values of C_2 and C_3 are listed in Table 4-3 below.

Table 4-3. Average Error for Values of C_2 and C_3

C_3	C_2				
	1.0	1.5	2.0	3.0	5.0
0.1	0.3830	0.3843	0.3906	0.4242	0.4590
0.2	0.3822	0.3713	0.3897	0.4111	0.4294
0.3	0.3994	0.3857	0.3771	0.3882	0.4576

It is illustrative to show the physical significance of the various values of C_2. For this example, assume a straight-ahead block between two opposing players (P and E) with equal and opposite initial velocity, with $W^P = W^E = 200$ pounds, and $S^P = 300$ pounds and

S^E = 400 pounds. The movement of the center of the blockers for differing values of C_2 is listed in Table 4-4 below.

Table 4-4. Blocking Results for Values of C_2

			C_2		
Time	1.0	1.5	2.0	3.0	5.0
0.00	0.00	0.00	0.00	0.00	0.00
0.25	0.01	0.01	0.02	0.02	0.04
0.50	0.03	0.05	0.06	0.10	0.16
0.75	0.07	0.11	0.14	0.21	0.35
1.00	0.13	0.19	0.23	0.38	0.63

The choice for C_2 is 1.5, as it yields the minimum sum-of-squares and also provides reasonable results. The corresponding choice for C_3 is 0.2.

4.2.3 Estimation of σ_C

The estimation of σ_C is not an easy task. The only data available for this purpose on the participants of the play analyzed is three sets of 40-yard dash times. This is the one-dimensional case:

$$x(t) = x(0) + \left[(C_x+a_i) \frac{t}{C_1} + \frac{(C_x+a_i) - C_1\dot{x}(0)}{C_1^2} (1-e^{-C_1 t}) \right] \tag{4.9}$$

$$\dot{x}(t) = \frac{(C_x + a_i)}{C_1} (1 - e^{-C_1 t}) + \dot{x}(0)\, e^{-C_1 t} \tag{4.10}$$

where a_i is normal and independently distributed with variance σ_c^2, to meet the requirements in Section 3.8. Instead of attempting to determine the relationship between σ_C and 40-yard dash times analytically, a simulation approach is used. A 40-yard dash is simulated two hundred times, using (4.9) and (4.10) and epochs of 0.25 seconds. The variance of the dash times thus obtained depends both on the input σ_C and C_x*. The relationship between these values are presented in Figure 4-1. Appendix A lists the program used in generating Figure 4-1.

The estimate of σ_C is then made by determining each player's σ_{C_i} (using the player's variance of his dash times and his average dash time, and the use of Figure 4-1), and then averaging the σ_{C_i}. The value thus obtained is 1.417 yards.

4.3 Model Verification

The validation of the model is based on a theorem due to Alt [2]. Alt shows** that

1. If $\vec{z}$ is distributed bi-variate normal with unknown mean and covariance, and

2. if N observations $\vec{z}_i$ are drawn from the distribution,

and

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*The dependence on C_x is related to the fact that a slower runner will cover 40 yards in a greater number of epochs and thus more a_i's will be generated in the simulation.

**Theorem 5.7, page 112.

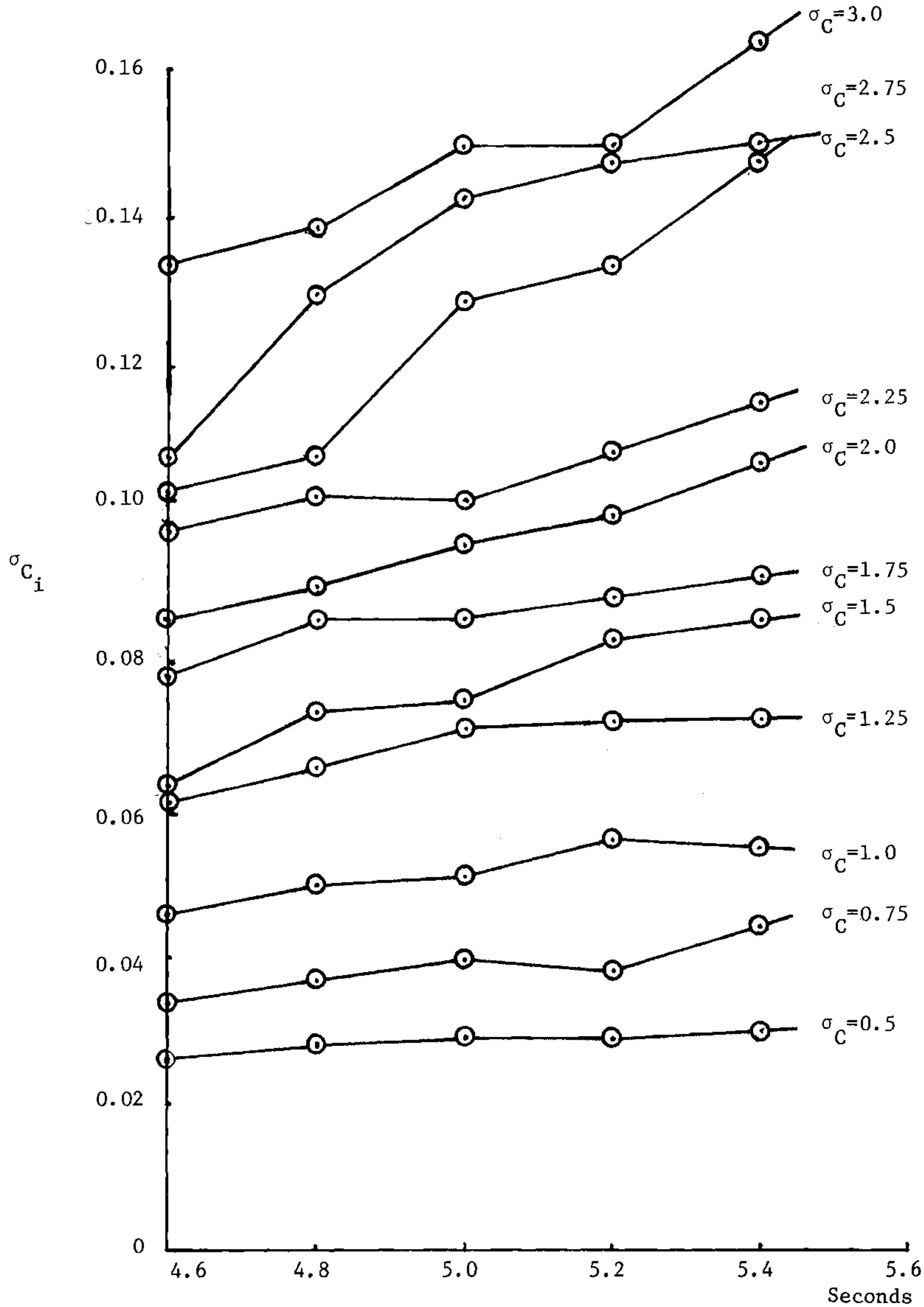

Figure 4-1. Relationship of σ_C to σ_{C_i}

3. $S = 1/(N-1) \sum_i (\vec{z}_i - \vec{z}_0)^t (\vec{z}_i - \vec{z}_0)$, where $\vec{z}_0 = (1/N) \sum_i \vec{z}_i$, and

4. if $\vec{z}^*$ is drawn from the same distribution, then

$$F = \frac{1}{2}\left(\frac{N^2-2N}{N^2-1}\right) (\vec{z}^* - \vec{z}_0)\ S^{-1}\ (\vec{z}^* - \vec{z}_0)^t \tag{4.11}$$

is $F(2,N-2)$-distributed.

The play was simulated twenty-five times on the play analyzed with the following parameters: 0.25 epochs, $C_1 = 2.45$, $C_2 = 1.5$, $C_3 = 0.2$, and $\sigma_C = 1.417$.

Each player's position at an epoch is assumed normally distributed. The positional data is used as the $\vec{z}_i$ ($N = 25$ here), and invoking the above theorem, the critical region for the statistic F as defined by (4.11) becomes $F > F(2,23,\alpha)$. These tests were performed for the 0.5 second, 1.0 second, and 1.5 second epochs of the play. The data is listed in Tables 4-5 through 4-8.

Table 4-5. Player Initial Positions and Attributes

	Player	Position	(x , y)	Speed	Weight	Strength
Offense	1	RE	6.60,-1.00	14.85	203	265
	2	RT	4.00,-1.00	13.77	240	245
	3	RG	2.20,-0.60	13.94	206	315
	4	C	0.00,-0.40	15.34	243	405
	5	LG	-2.20,-0.60	14.54	235	335
	6	LT	-4.00,-0.80	14.74	264	265
	7	WR	-15.00,-0.80	15.92	172	220
	8	QB	0.10,-1.30	16.36	200	235
	9	FB	0.20,-3.95	15.75	216	295
	10	HB	1.90,-5.30	16.82	200	270
	11	HB	-1.90,-5.30	16.42	181	265
Defense	1	CB	11.25, 3.00	15.86	210	350
	2	E	6.60, 1.15	15.86	198	320
	3	T	4.05, 1.30	14.24	251	280
	4	LB	3.30, 3.85	14.85	226	290
	5	MG	0.60, 1.00	14.69	231	285
	6	SS	3.10,10.00	15.40	196	265
	7	LB	-2.20, 3.60	15.75	212	315
	8	T	-2.85, 2.10	14.34	253	285
	9	E	-6.00, 1.60	15.86	206	315
	10	CB	-11.75, 3.70	16.11	220	315
	11	WS	-14.90, 7.60	15.51	190	185

Table 4-6. Player Positions at 0.5 Seconds

	Player	(xo , yo)	(x* , y*)	S^{-1}	F
Offense	1	6.31, 0.17	6.10, 0.20	98.44,-26.52 -26.52,206.48	2.282
	2	3.39,-0.53	3.50,-0.50	132.54,-55.64 -55.64,136.64	0.687
	3	2.90, 0.39	2.80, 0.50	110.21,-14.49 -14.49,123.38	1.336
	4	0.31,-0.18	0.35, 0.10	146.37,-100.8 -100.8, 128.81	3.679
	5	-2.39, 0.61	-2.50, 0.80	105.95,-36.39 -36.39,142.24	1.550
	6	-3.93, 0.48	-4.00, 0.40	130.75, 11.33 11.33, 91.72	0.660
	7	-15.05, 0.53	-15.10, 0.60	126.92,-34.16 -34.16,134.72	0.529
	8	0.46,-1.47	0.40,-1.40	157.98, -6.56 -6.56, 98.55	0.523
	9	-0.23,-2.68	-0.25,-2.80	167.48, 53.31 53.31,171.62	1.470
	10	1.94,-3.96	1.80,-3.90	100.72, 3.65 3.65,112.53	1.257
	11	-3.31,-5.30	-3.40,-5.30	94.77,-17.51 -17.51, 92.74	0.325
Defense	1	10.40, 3.03	10.40, 2.90	61.26,-42.67 -42.67,109.72	0.800
	2	6.33, 1.03	6.15, 1.05	72.68, 20.90 20.90,176.19	1.138
	3	3.87, 0.99	3.60, 1.00	110.46, -9.23 -9.23,110.55	3.644
	4	3.25, 3.58	3.10, 3.40	48.12, -1.17 -1.17, 36.22	1.034
	5	0.49, 0.60	0.40, 0.80	254.26,-140.9 -140.9, 99.15	5.045
	6	3.08, 9.96	3.10,10.00	65.03,-15.55 -15.55,107.35	0.072
	7	-2.01, 2.65	-2.10, 2.80	105.79, 40.32 40.32,187.06	1.840
	8	-2.91, 1.73	-2.90, 1.70	148.08,-54.44 -54.44,152.66	0.066
	9	-6.02, 1.56	-6.00, 1.70	112.38, 62.78 62.78,196.05	2.120
	10	-11.64, 2.40	-11.60, 2.40	103.20, 5.61 5.61,108.80	0.062
	11	-14.93, 7.61	-14.85, 7.70	76.15, -5.49 -5.49, 90.56	0.536

Table 4-7. Player Positions at 1.0 Second

	Player	(xo , yo)	(x* , y*)	S^{-1}	F
Offense	1	5.61, 0.30	5.10, 0.80	7.56, 1.65 1.65, 13.45	2.228
	2	2.52, 1.35	2.50, 1.40	45.36,-10.87 -10.87, 30.36	0.047
	3	3.03, 0.75	3.15, 1.10	52.64,-15.72 -15.72, 19.49	0.821
	4	0.54,-0.14	0.70, 0.50	63.14,-54.89 -54.89, 56.09	5.935
	5	-2.12, 1.98	-1.70, 2.30	48.62, -5.37 -5.37, 6.21	3.599
	6	-3.68, 1.79	-3.90, 1.60	39.32, -5.78 -5.78, 3.82	0.737
	7	-14.75, 2.87	-15.00, 3.00	33.42,-22.49 -22.49, 41.78	1.973
	8	0.65,-1.39	0.65,-1.40	23.56, 1.73 1.73, 28.57	0.001
	9	-0.94,-0.17	-1.05,-0.50	25.25, 12.08 12.08, 40.56	2.507
	10	2.05,-1.70	2.30,-1.40	30.76, -7.10 -7.10, 40.25	2.189
	11	-6.10,-5.32	-6.10,-5.30	21.57, -5.00 -5.00, 30.75	0.006
Defense	1	9.71, 3.30	10.40, 3.30	10.55, -4.83 -4.83, 23.22	2.304
	2	5.52, 1.11	6.30, 1.40	5.56, -0.90 -0.90, 14.87	2.115
	3	3.80, 0.92	3.80, 1.20	41.36,-32.95 -32.95, 43.04	1.611
	4	2.64, 2.51	3.20, 3.00	17.50, -6.90 -6.90, 23.94	3.709
	5	0.74, 0.64	0.40, 0.80	51.30,-39.97 -39.97, 36.54	8.298
	6	2.26, 9.67	2.10, 9.70	11.31, -7.31 -7.31, 33.87	0.193
	7	-1.34, 1.83	-1.70, 2.4	57.95, 3.55 3.55, 3.14	3.309
	8	-2.52, 0.59	-3.00, 1.70	20.43, 6.34 6.34, 5.18	1.999
	9	-6.01, 1.53	-6.10, 1.60	18.65, 9.57 9.57, 42.79	0.099
	10	-11.00, 0.26	-10.90, 0.40	24.77, 1.08 1.08, 7.06	0.179
	11	-14.01, 8.23	-14.30, 8.50	20.69, 2.17 2.17, 39.63	1.984

Table 4-8. Player Positions at 1.5 Seconds

	Player	(xo , yo)	(x* , y*)	S^{-1}	F
Offense	1	4.82,-0.00	5.10, 0.50	2.44, -2.95 -2.95, 10.26	0.903
	2	2.39, 1.90	2.40, 2.20	7.75, -1.33 -1.33, 5.43	0.218
	3	3.02, 0.83	3.50, 1.10	15.94,-12.33 -12.33, 15.90	0.764
	4	0.86, 0.27	0.90, 0.80	19.63,-19.58 -19.58, 22.82	2.572
	5	-1.68, 1.86	-1.30, 2.10	7.50, -5.12 -5.12, 10.22	0.341
	6	-2.97, 1.31	-3.65, 1.50	2.54, -3.64 -3.64, 28.98	1.488
	7	-13.23, 4.97	-13.60, 5.30	9.95, -3.05 -3.05, 7.23	1.397
	8	0.72,-1.17	1.70,-1.40	5.10, -0.10 -0.10, 3.45	2.338
	9	-1.18, 2.80	-1.70, 2.40	11.88, 5.35 5.35, 19.61	4.008
	10	2.11, 1.29	1.90, 1.50	22.37, 0.69 0.69, 27.48	0.982
	11	-9.33,-5.34	-8.95,-5.30	13.61, -0.12 -0.12, 15.41	0.924
Defense	1	7.64, 4.44	8.40, 4.40	5.17, -4.02 -4.02, 20.13	1.474
	2	4.56, 0.71	5.25, 1.30	2.88, -4.30 -4.30, 13.24	1.126
	3	3.74, 1.05	4.20, 0.80	7.09, -3.89 -3.89, 5.05	1.242
	4	2.42, 2.66	2.80, 3.20	9.31, -4.19 -4.19, 8.28	0.946
	5	1.12, 1.03	0.40, 1.10	10.98,-10.57 -10.57, 12.66	3.166
	6	0.22, 9.29	1.00, 9.40	4.94, -2.08 -2.08, 16.69	1.250
	7	-1.50, 1.29	-1.50, 2.40	4.72, -2.01 -2.01, 1.39	0.770
	8	-1.82,-0.21	-3.10, 1.90	3.57, 1.04 1.04, 4.89	1.922
	9	-5.83, 1.74	-6.10, 2.20	5.80, -0.42 -0.42, 11.27	1.335
	10	-9.84,-1.85	-10.00,-1.60	19.88, -4.93 -4.93, 13.94	0.813
	11	-13.32,10.21	-13.80, 9.80	11.31, 3.35 3.35, 17.46	3.103

The values of $F(2,23,\alpha)$ for $\alpha = 0.10$, 0.05, and 0.01 are 2.55, 3.42, and 5.66, respectively. Assigning H_0 as $\vec{z}^*$ being drawn from the same distribution of $\vec{z}_i$, Table 4-9 lists the frequency of accepting H_0 for the data.

Table 4-9. Fraction of Time H_0 is Accepted

α	0.5 Seconds	1.0 Seconds	1.5 Seconds	Total
0.10	19/22	17/22	18/22	54/66
0.05	19/22	19/22	21/22	59/66
0.01	22/22	20/22	22/22	64/66

The model is accepted at the 0.05 level.

CHAPTER V

DESCRIPTION OF PLAYS TO BE MODELED

In this chapter, two offensive plays and two defensive plays are modeled, and the model is run for each combination of the offensive-defensive pairs. The offensive plays (Section 5.1) are chosen from the Wishbone T, as this is the formation for which the data used in the verification procedure described in Chapter IV is available. Two defenses common against the Wishbone are chosen. These alignments are diagrammed in Section 5.2. Sections 5.3 and 5.4 present the play diagrams and results of the four offense-defense pairs.

5.1 Offense

The Wishbone offense is run from an unvarying initial formation (disregarding a mirror-image), regardless of the play called. This formation (with arbitrarily assigned player numbers) is diagrammed in Figure 5-1.

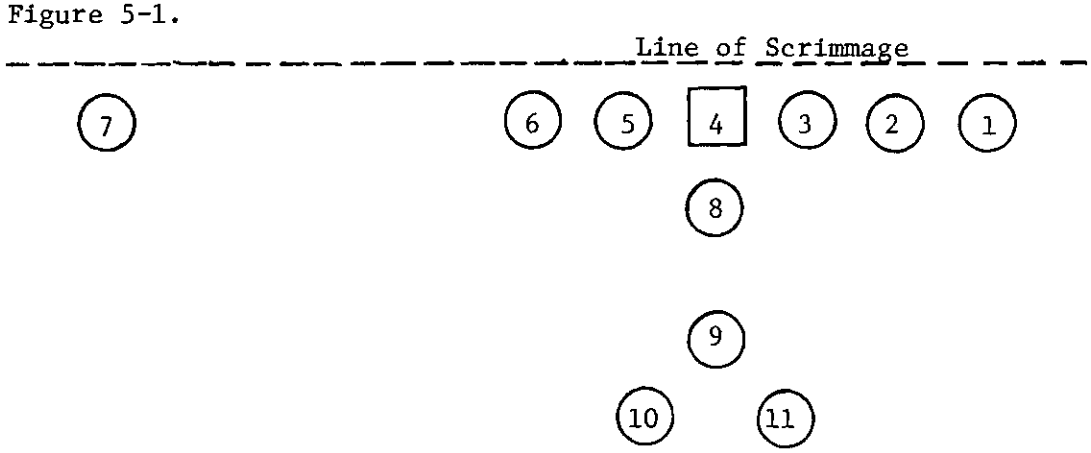

Figure 5-1. Wishbone Offensive Formation

Player 4 is the center, players 5 and 3 the guards, players 6 and 2 the tackles, player 1 the tight end, and player 7 the spread end. Player 8 is the quarterback, player 9 the fullback, and players 10 and 11 the halfbacks. In addition, the adjectives "onside" and "offside" are applied to players on the side of the direction of the play and away from the direction of the play, respectively. Hence if the play is toward the left, player 5 is the onside guard and player 10 is the offside halfback.

The two offensive plays chosen from the Wishbone offense are the Counter Dive and the Predetermined Fullback Play. In the Counter Dive, the ball is faked to the fullback and handed off to the offside halfback. In the Predetermined Fullback Play*, the halfbacks run parallel to the line of scrimmage and in the same direction. The ball is then given to the fullback running (possibly) behind the guard's block. For a more complete description of these plays, and the entire Wishbone offense, see Rodgers and Smith [19]. Due to the difficulty of presenting these plays without considering the opposing defense, the diagrams of the plays are postponed until Sections 5.3 and 5.4.

5.2 Defense

The defensive formations commonly used against the Wishbone are the Oklahoma-50 and the Split-Six. These are diagrammed below in Figures 5-2 and 5-3.

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*"Predetermined" indicates that the fullback has been designated to receive the quarterback's handoff regardless of the defense's reaction. This differentiates the play from an option, where the hand-off is dependent on the defense.

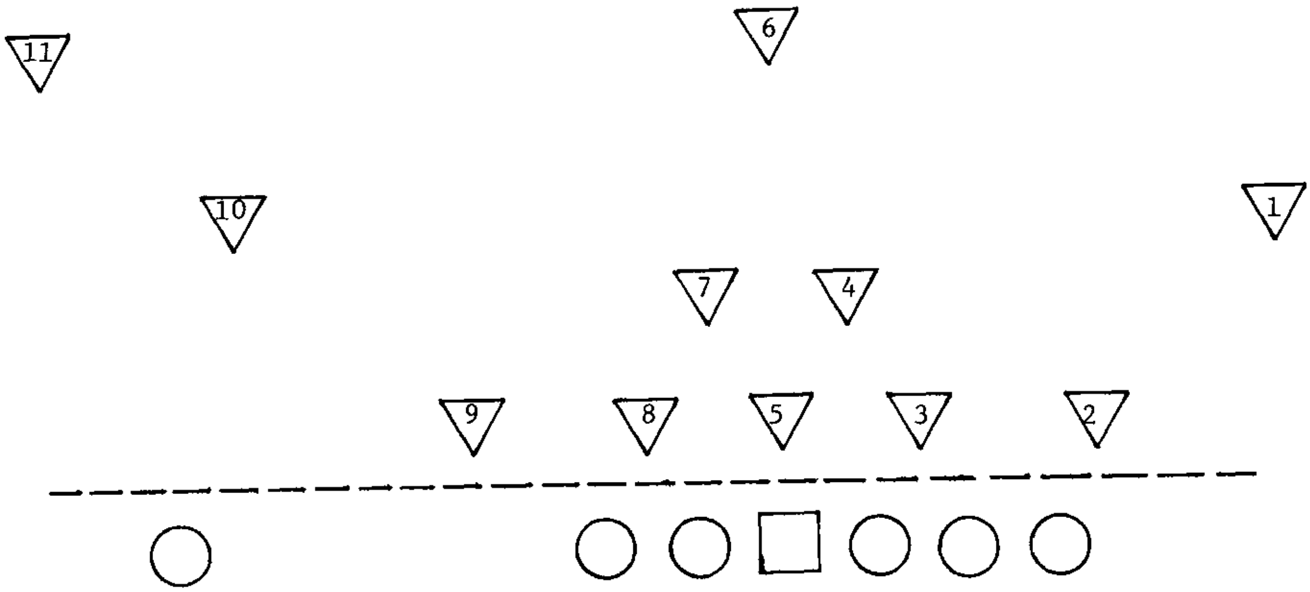


Figure 5-2. Wishbone vs. Oklahoma-50

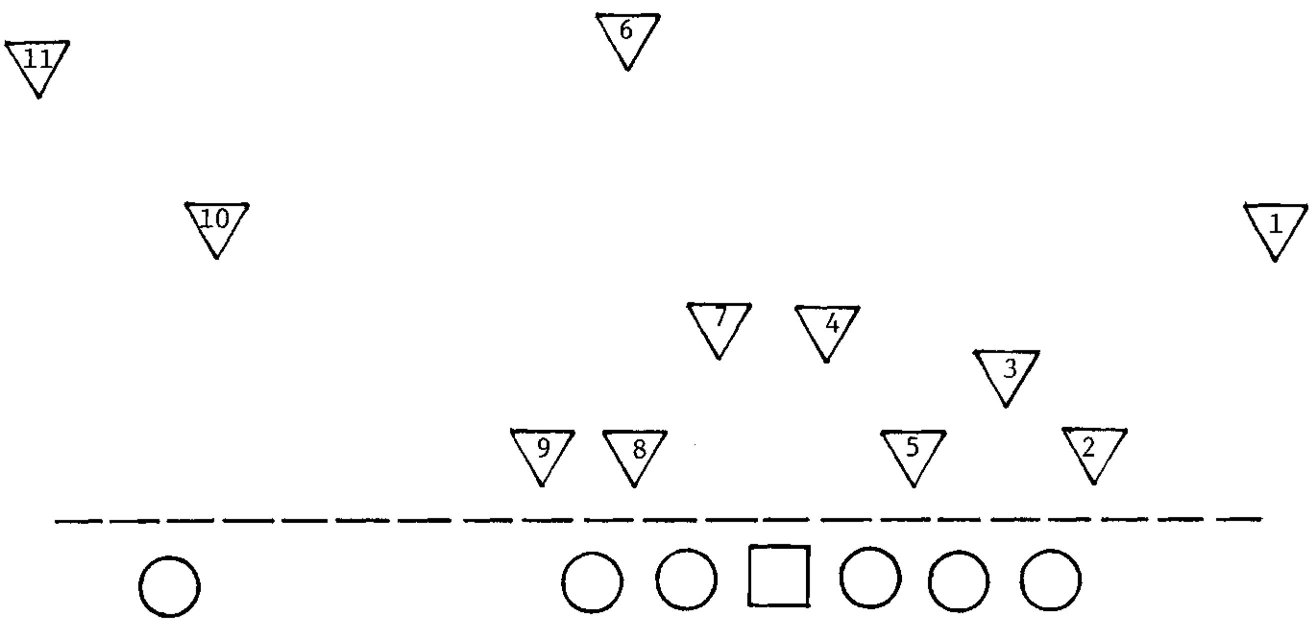


Figure 5-3. Wishbone vs. Split-Six

Players 1 and 10 are cornerbacks, player 6 is the strong safety, player 11 is the quick safety, players 4 and 7 are the linebackers. Players 2 and 8 are the defensive ends, and players 8 and 3 are the defensive tackles. Player 5 is the middle guard.

5.3 Counter Dive

The Counter Dive is diagrammed in Figure 5-4 against the Oklahoma-50 defense, and in Figure 5-5 against the Split-Six defense.

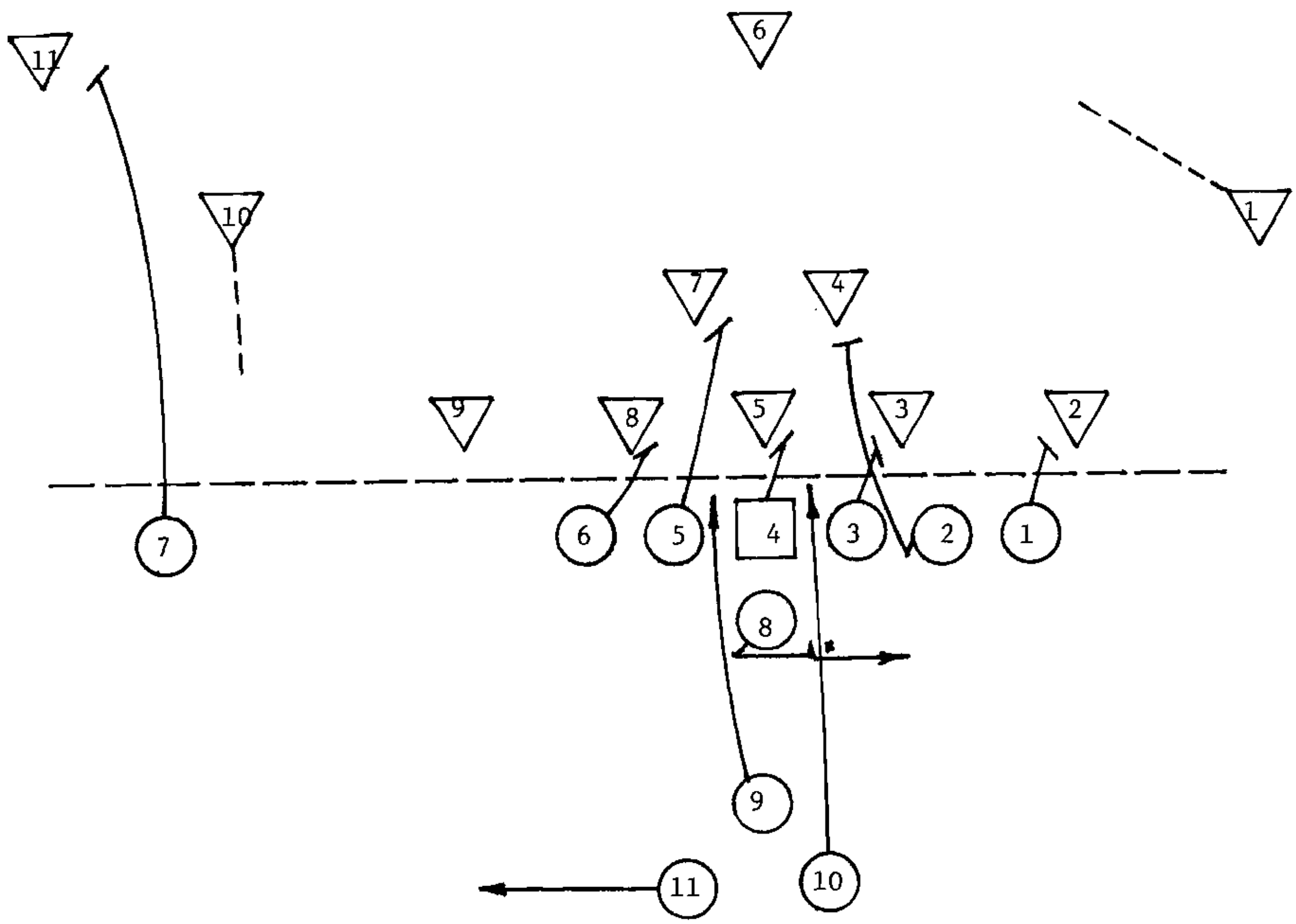


Figure 5-4. Counter Dive vs. Oklahoma-50

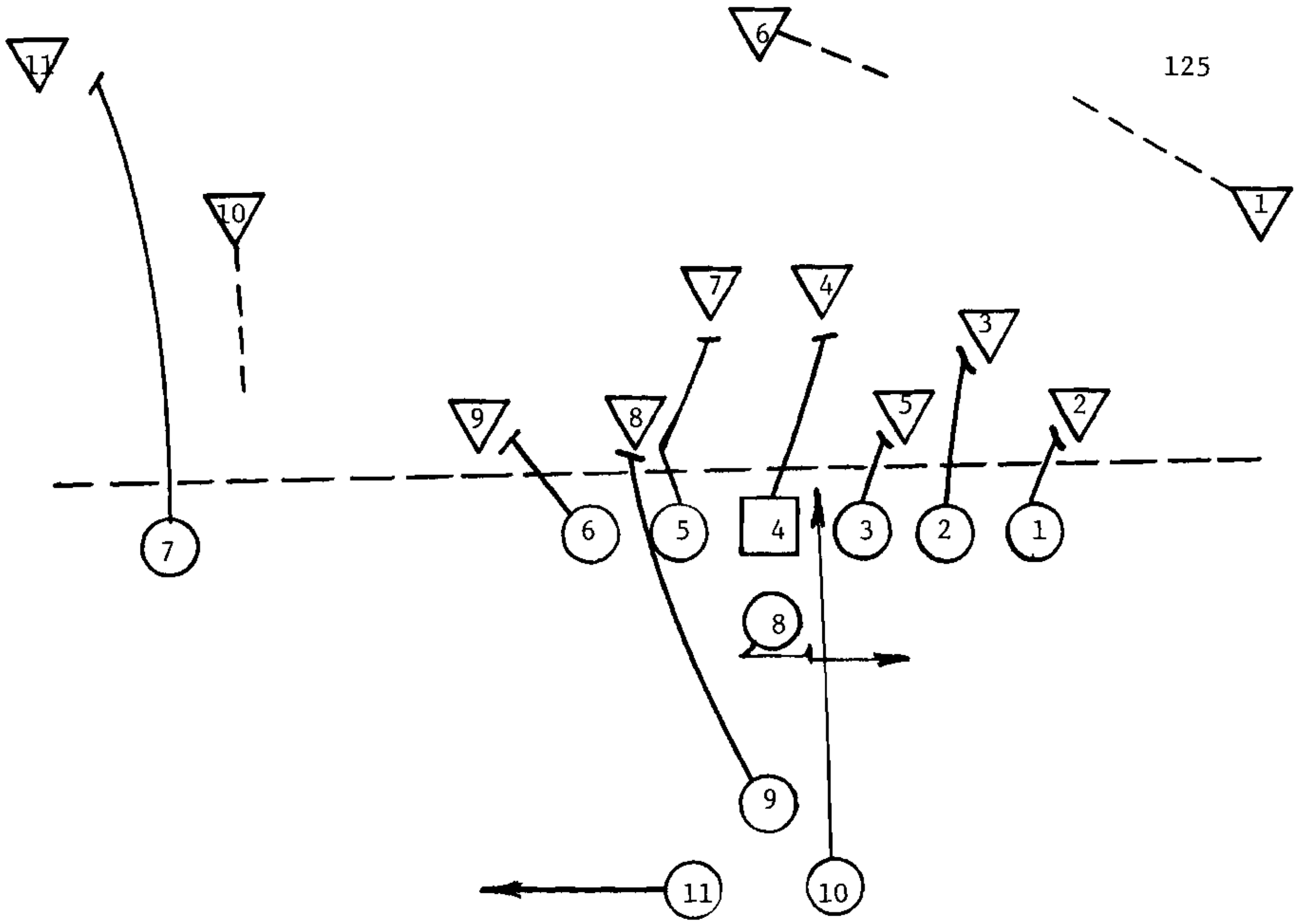


Figure 5-5. Counter Dive vs. Split-Six

Against the 50, the offensive players 2 and 3 switch blocking assignments due to the poor blocking angle 2 would have on the defensive tackle. The step-around "fold-block" accomplishes this purpose. As the analysis shows, the center's block on the middle is quite important to the development of the play.

Figure 5-5 shows the Counter Dive versus the Split-Six. Offensive player 5 (the onside guard) momentarily blocks defensive player 8 (the defensive tackle) and then releases him to the fullback after the handoff fake. The guard then blocks the linebacker (player 7) who is required to maintain his position by the fullback's threat.

5.3.1 Results of Counter Dive vs. 50 Defense

The play was simulated twenty-five times, starting on the 50 yard line. The player's attributes listed in Table 4-5 were used. The gains in Table 5-1 resulted.

Table 5-1. Results of Counter Dive vs. Oklahoma-50

Run	Gain	Tackled By
1	7.69	1
2	3.40	4,5
3	5.51	4,6
4	5.78	1
5	4.20	4,5,6
6	2.78	4
7	3.50	4,6
8	2.28	4,5,7
9	5.05	4
10	7.06	6
11	2.77	5
12	4.73	4
13	2.62	5
14	50.00	-
15	6.21	1
16	7.08	1,2,5,7
17	3.01	4,5
18	7.14	6
19	3.61	4
20	1.51	5
21	9.90	1,6
22	1.43	5
23	5.86	6
24	5.20	7
25	3.90	4

The average gain, not including the touchdown, is 4.68 yards; including the touchdown as 50 yards, the average gain is 6.49 yards. Considering only the nine plays where defensive player 5 (the middle guard) was a tackler, the average gain is 3.14. For the other 15 plays (disregarding the touchdown), the average is 5.59 yards. Thus a good block by the center adds an average of over two yards to the gain of the play.

A sample of the output for this offense-defense pair is given in Figure 5-6a through 5-6k.

5.3.2 Results of Counter Dive vs. Split-Six

Again the play was simulated twenty-five times with the player attributes of Table 4-5. The results are listed in Table 5-2.

Table 5-2. Results of Counter Dive vs. Split-Six

Run	Gain	Tackled By
1	50.00	-
2	1.57	3,5
3	4.52	1,4
4	1.67	5
5	1.78	3
6	3.29	3,5
7	5.00	1
8	2.18	3,5
9	2.70	3,5
10	12.26	11
11	1.91	3,5
12	2.77	3
13	2.47	3
14	3.29	4
15	2.47	3,4
16	4.21	1,4
17	9.32	6
18	4.52	1
19	4.29	1,4
20	50.00	-
21	1.95	3
22	2.90	3,7
23	1.99	3,5
24	4.43	1
25	2.35	3

The average gain, not including the two touchdowns, is 3.65

yards. With the touchdowns, the average run is 7.35 yards. The blocks on defensive players 3 and 5 seem most critical.

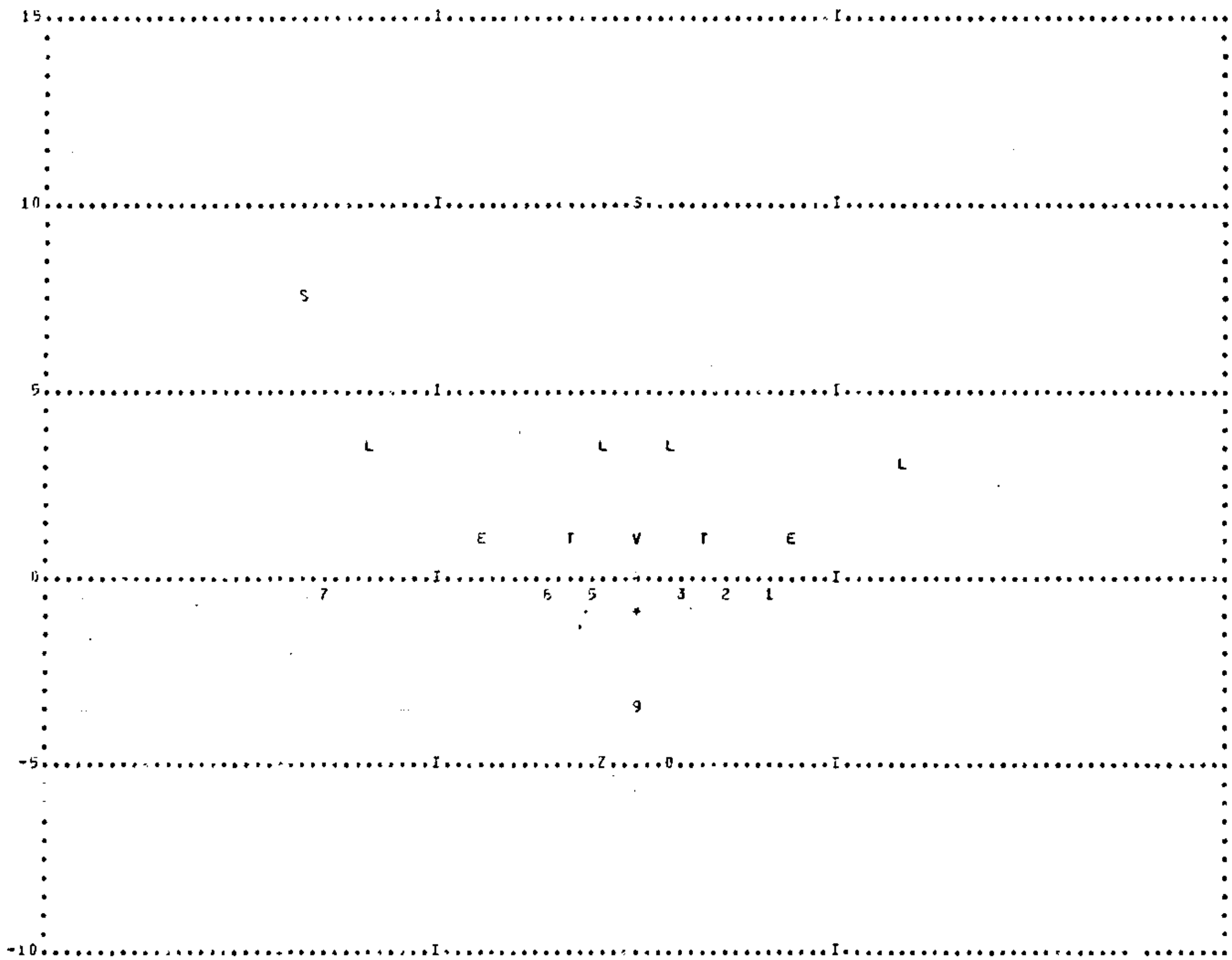
For plays not going for touchdowns, the relatively smaller gain against the Split-Six is reasonable. Against the 50, the fake on defensive player 5 draws him out of position for a subsequent tackle on the halfback. Against the Split-Six, the same fake draws player 5 into the running corridor. Likewise, the blocking angle on 3 is relatively poor against the Split-Six, whereas care was taken to insure a good block against the 50.

A sample output for Counter Dive versus the Split-Six defense is given in Figure 5-7a through 5-7j.

5.4 Predetermined Fullback Play

The Predetermined Fullback Play is diagrammed below against the Oklahoma-50 and Split-Six Defenses.

EPOCH = 0.00 SECONDS

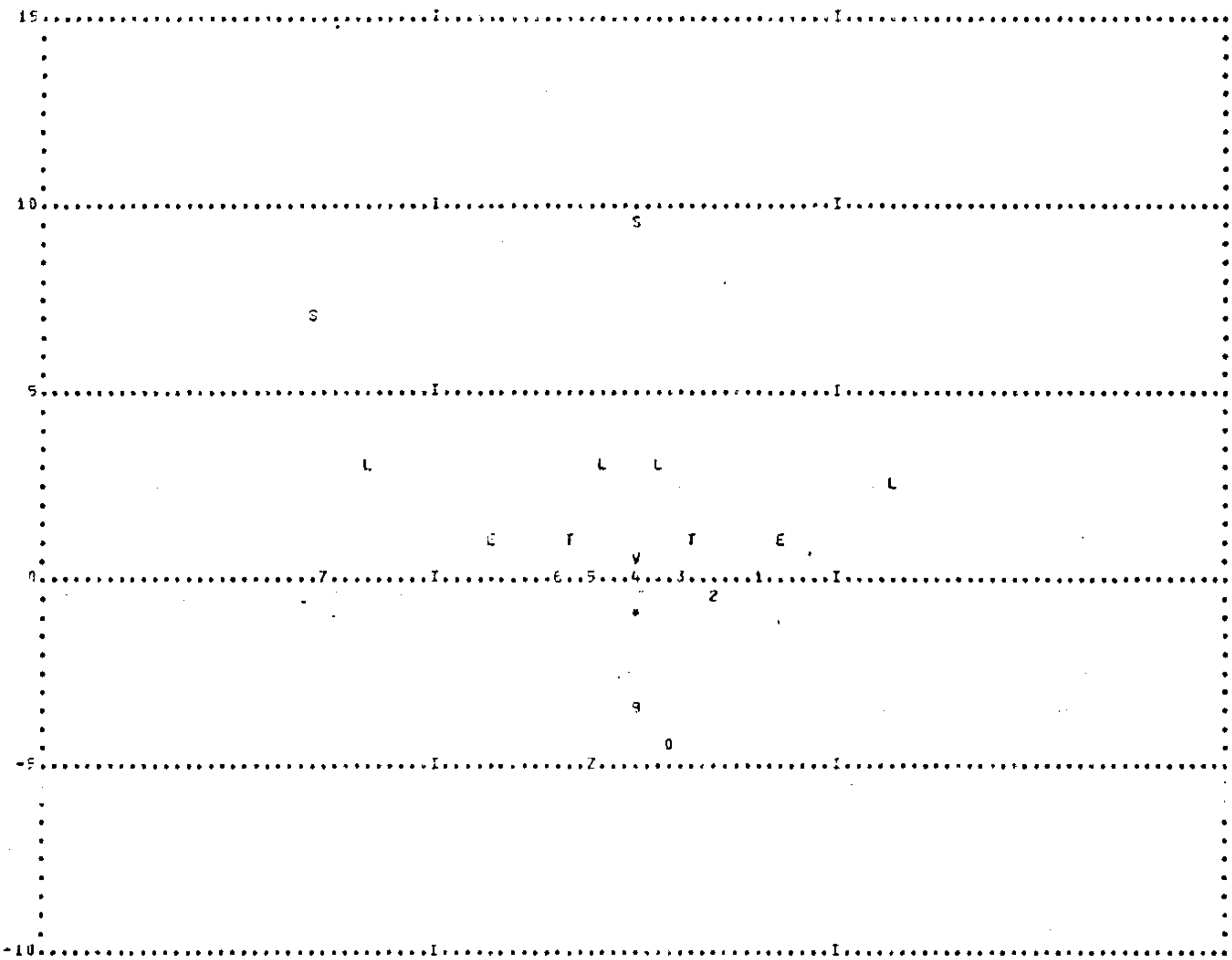


EPOCH = 0.00 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	6.00	-.80	0.00	0.00	0	-.29	15.71	E
	2	4.00	-.80	0.00	0.00	0	-14.82	.93	L
	3	2.00	-.80	0.00	0.00	0	3.52	13.99	T
	4	0.00	-.40	0.00	0.00	0	4.57	15.93	V
	5	-2.00	-.80	0.00	0.00	0	-7.16	12.33	L
	6	-4.00	-.80	0.00	0.00	0	2.27	15.64	T
	7	-14.00	-.30	0.00	0.00	0	-2.95	16.39	S
	*	0.00	-1.30	0.00	0.00	0	4.42	-2.33	
	9	0.00	-3.90	0.00	0.00	0	-5.55	14.20	L
	0	1.80	-5.30	0.00	0.00	0	2.32	18.06	
	Z	-1.80	-5.30	0.00	0.00	0	-19.56	1.78	
DEFENSE									
	L	12.00	3.50	0.00	0.00	0	-12.35	-1.62	
	E	7.00	1.40	0.00	0.00	0	-1.12	-.48	
	T	3.00	1.40	0.00	0.00	0	-.33	-.76	
	L	1.50	3.50	0.00	0.00	0	-.59	-.74	
	V	0.00	1.00	0.00	0.00	0	-.51	-15.31	
	S	0.00	10.00	0.00	0.00	0	1.32	-.82	
	L	-1.50	3.50	0.00	0.00	0	-.58	-17.32	
	T	-3.00	1.40	0.00	0.00	0	-.26	.59	
	E	-7.00	1.40	0.00	0.00	0	.57	.31	
	L	-12.00	3.50	0.00	0.00	0	-.65	-17.27	
	S	-15.00	7.50	0.00	0.00	0	.27	-.31	

Figure 5-6a. Counter-Dive vs. Oklahoma 50 (Simulation Results)

EPOCH = .25 SECONDS

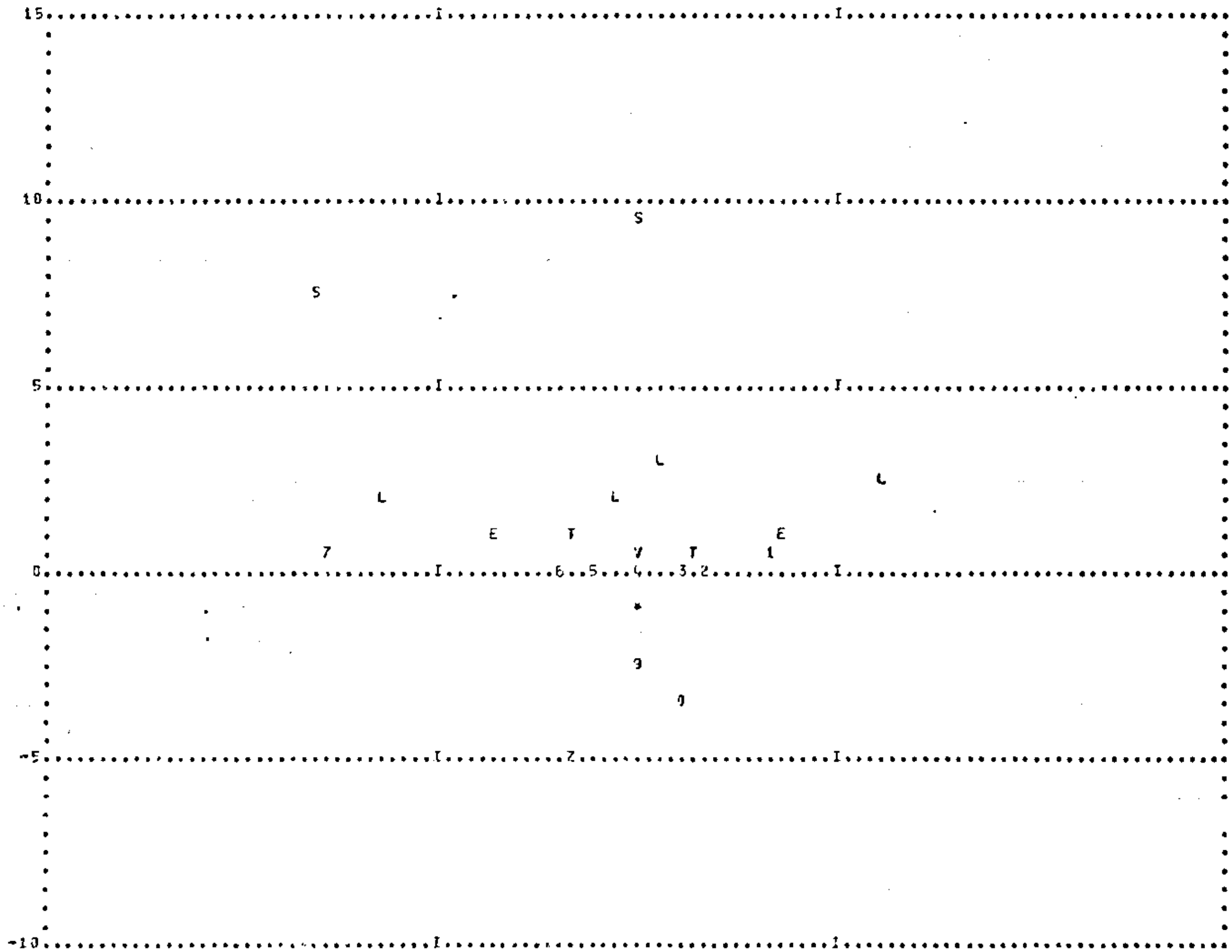


EPOCH = .25 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	5.99	-.40	-.05	2.94	0	4.67	16.50	E
	2	3.62	-.78	-2.77	.17	0	-1.76	13.46	L
	3	2.05	-.44	.86	2.62	0	4.48	13.99	T
	4	.10	-.10	.18	.24	1	7.30	13.26	V
	5	-2.18	-.48	-1.34	2.32	0	4.54	15.40	L
	6	-3.94	-.40	.42	2.92	0	-1.72	12.57	T
	7	-14.09	-.38	-.59	3.06	0	.50	15.72	S
	*	.11	-1.36	.83	-.44	0	-1.29	2.30	
	9	-.14	-3.53	-1.04	2.55	0	-5.17	13.78	L
	0	1.85	-4.65	.43	3.36	0	2.84	14.74	
	Z	-2.30	-5.25	-3.66	.33	0	-14.81	-.08	
DEFENSE									
	L	11.68	2.05	-2.31	-.30	0	-.71	-.59	
	E	6.97	1.39	-.21	-.09	0	-14.71	-3.07	
	T	2.59	1.38	-.06	-.14	0	-.09	-16.35	
	L	1.46	3.46	-.19	-.20	0	-1.02	-5.65	
	V	.00	.75	.51	.34	1	4.65	-14.41	
	S	.03	9.48	.25	-.15	0	-1.12	-1.94	
	L	-1.51	3.05	-.11	-3.24	0	4.27	.20	
	T	-3.01	1.42	-.05	.11	0	-1.14	-15.40	
	E	-6.99	1.41	.11	.06	0	.10	-1.24	
	L	-12.02	3.06	-.12	-3.23	0	6.61	-13.43	
	S	-14.99	7.49	.05	-.06	0	2.18	2.57	

Figure 5-6b. Counter-Dive vs. Oklahoma 50 (Simulation Results)

EPOCH = .50 SECONDS

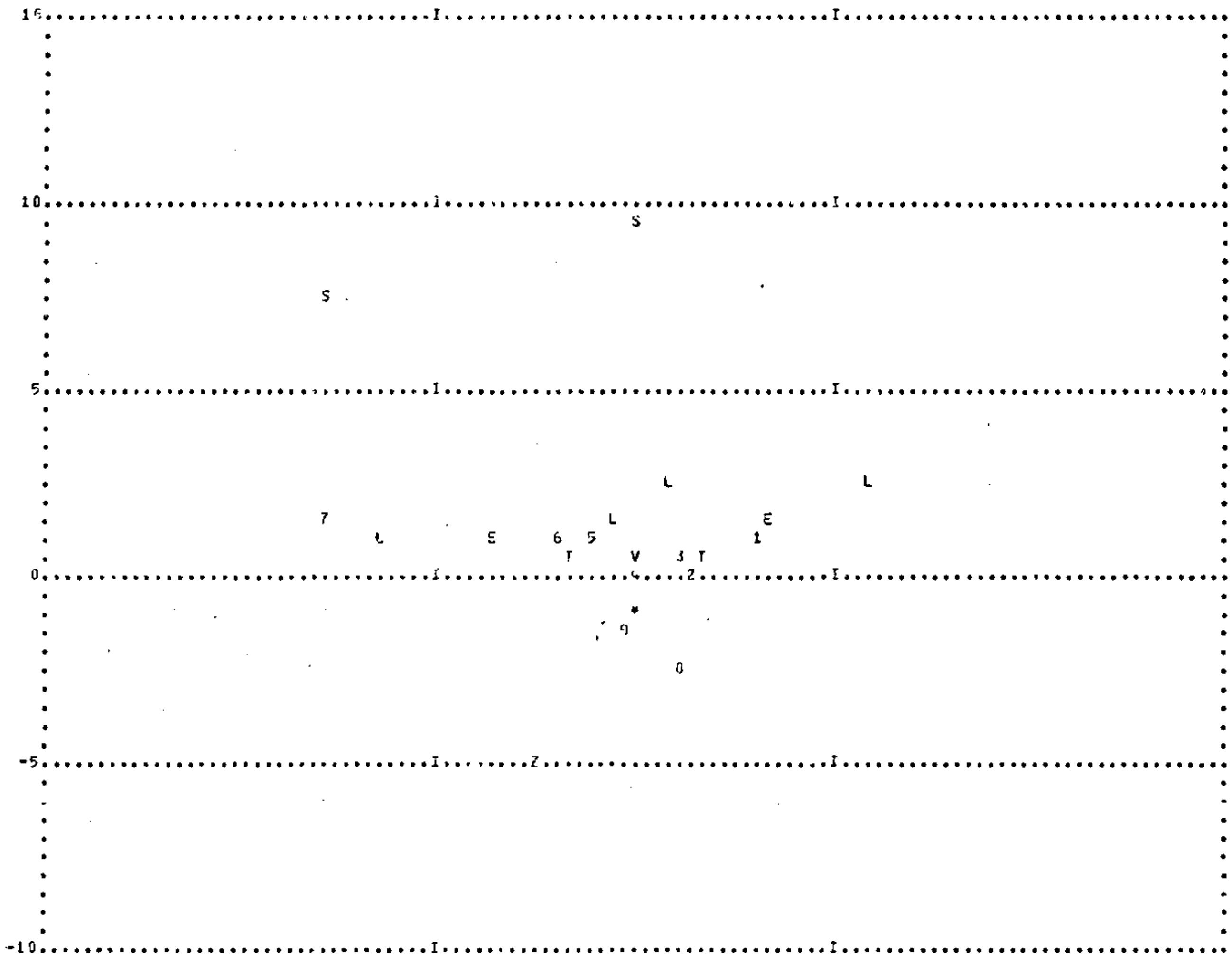


EPOCH = .50 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	6.10	.58	.64	4.68	1	-2.99	34.75	E
	2	3.06	-.40	-1.85	2.61	0	.17	15.46	L
	3	2.35	.41	1.19	4.03	1	3.65	11.24	T
	4	.23	-.02	.46	.36	1	9.54	13.71	V
	5	-2.31	.35	.19	4.13	0	4.82	12.60	L
	6	-3.91	.47	-.18	5.93	0	-3.85	14.96	T
	7	-14.17	.60	-.20	4.60	0	.31	15.27	S
	*	.24	-1.38	.21	.19	0	1.04	2.00	
	9	-.47	-2.65	-1.53	4.02	0	-5.66	12.75	L
	0	2.01	-3.62	.77	4.39	0	.75	11.55	
	Z	-3.37	-5.19	-4.75	.16	0	-16.27	.01	
DEFENSE									
	L	11.23	2.69	-1.38	-.27	0	-1.82	-.37	
	E	6.55	1.29	-2.55	-.62	1	-15.56	-1.70	
	T	2.98	.92	-.05	-3.23	1	-5.02	-13.06	
	L	1.40	3.28	-.29	-1.17	0	6.30	-13.56	
	V	.11	.82	.68	.33	1	3.49	-13.91	
	S	.05	9.90	-.37	-.45	1	4.74	-4.31	
	L	-1.43	2.45	.74	-1.72	0	.09	-15.53	
	T	-3.05	1.64	-.24	-2.62	0	13.17	-6.42	
	E	-6.96	1.39	.08	-.20	0	-.68	-.37	
	L	-11.87	2.11	1.17	-4.25	0	4.26	-11.54	
	S	-14.93	7.55	.43	.49	0	15.54	3.35	

Figure 5-6c. Counter-Dive vs. Oklahoma 50 (Simulation Results)

EPOCH = .75 SECONDS

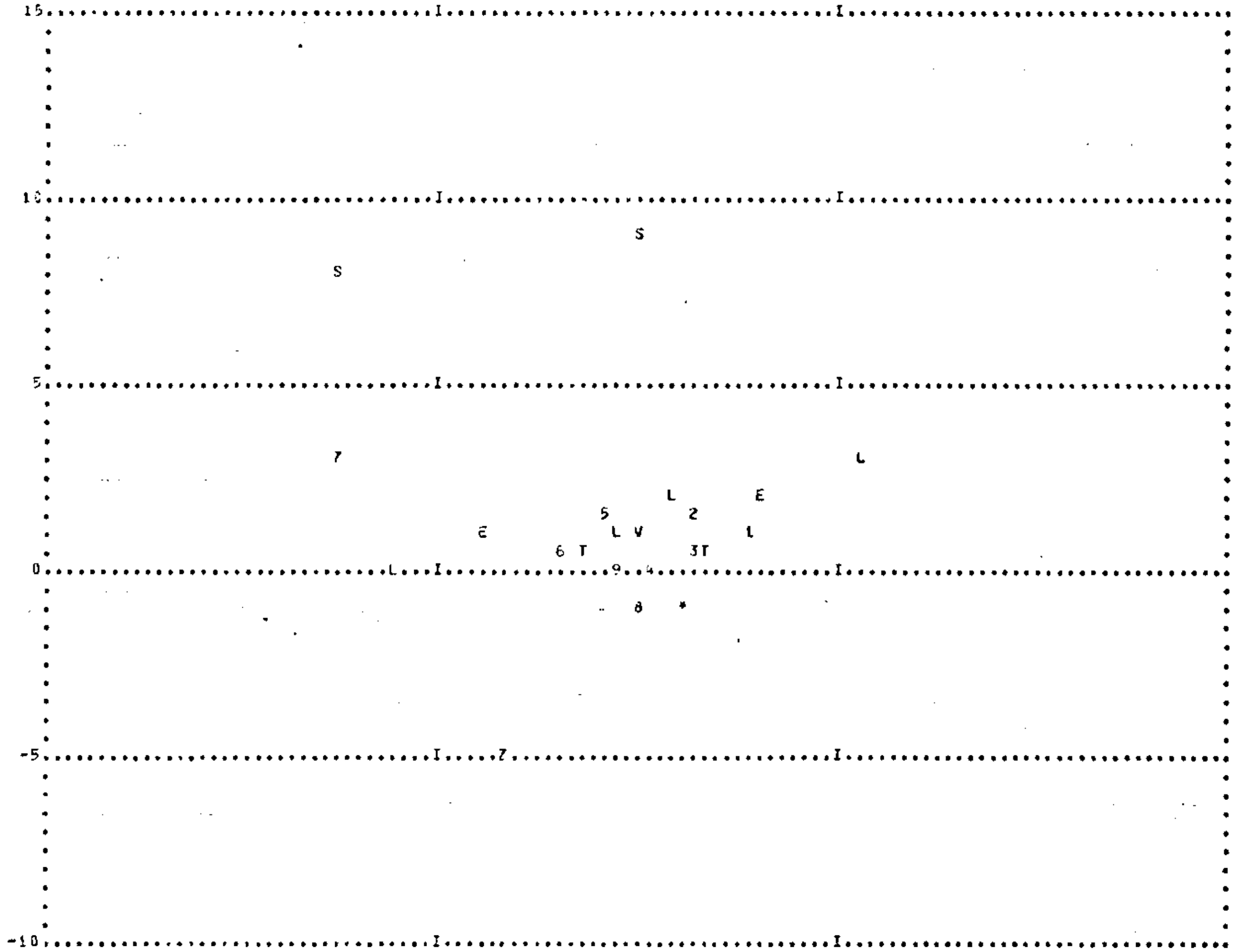


EPOCH = .75 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	5.81	1.17	-1.25	2.66	1	7.75	-12.53	E
	2	2.72	.49	-.96	4.31	0	5.14	12.75	L
	3	2.39	.55	.71	.74	1	14.12	-4.70	T
	4	.42	.09	.63	.48	1	7.67	13.59	V
	5	-2.15	1.45	1.00	4.60	1	11.94	-11.35	L
	6	-3.90	1.00	.06	1.31	1	13.43	-2.58	T
	7	-14.20	1.95	-.05	5.35	0	15.40	9.24	S
	*	.39	-1.29	.31	.48	0	.16	4.28	
	9	-.36	-1.60	-1.89	4.56	0	2.07	14.67	L
	0	2.16	-2.67	.56	4.04	0	1.30	16.89	
	Z	-4.68	-5.16	-5.62	.09	0	-17.10	.29	
DEFENSE									
	L	10.93	2.53	-1.09	-.22	0	-11.74	12.46	
	E	6.27	1.87	-1.41	-.49	1	-14.89	.68	
	T	3.17	.80	.73	-.19	1	.16	11.52	
	L	1.51	2.71	1.02	-3.17	0	.66	.23	
	V	.26	.92	1.01	.50	1	-.65	13.41	
	S	.20	9.69	.97	-1.13	0	-.49	.27	
	L	-1.28	1.73	.42	-3.83	1	-11.64	-9.03	
	T	-3.01	.99	.29	-.81	1	13.99	-.94	
	E	-6.97	1.36	-.09	-.18	0	-1.79	-1.31	
	L	-11.54	1.01	1.43	-4.47	0	6.73	-10.89	
	S	-14.45	7.73	3.14	.98	0	8.32	10.62	

Figure 5-6d. Counter-Dive vs. Oklahoma 50 (Simulation Results)

EPOCH = 1.00 SECONDS

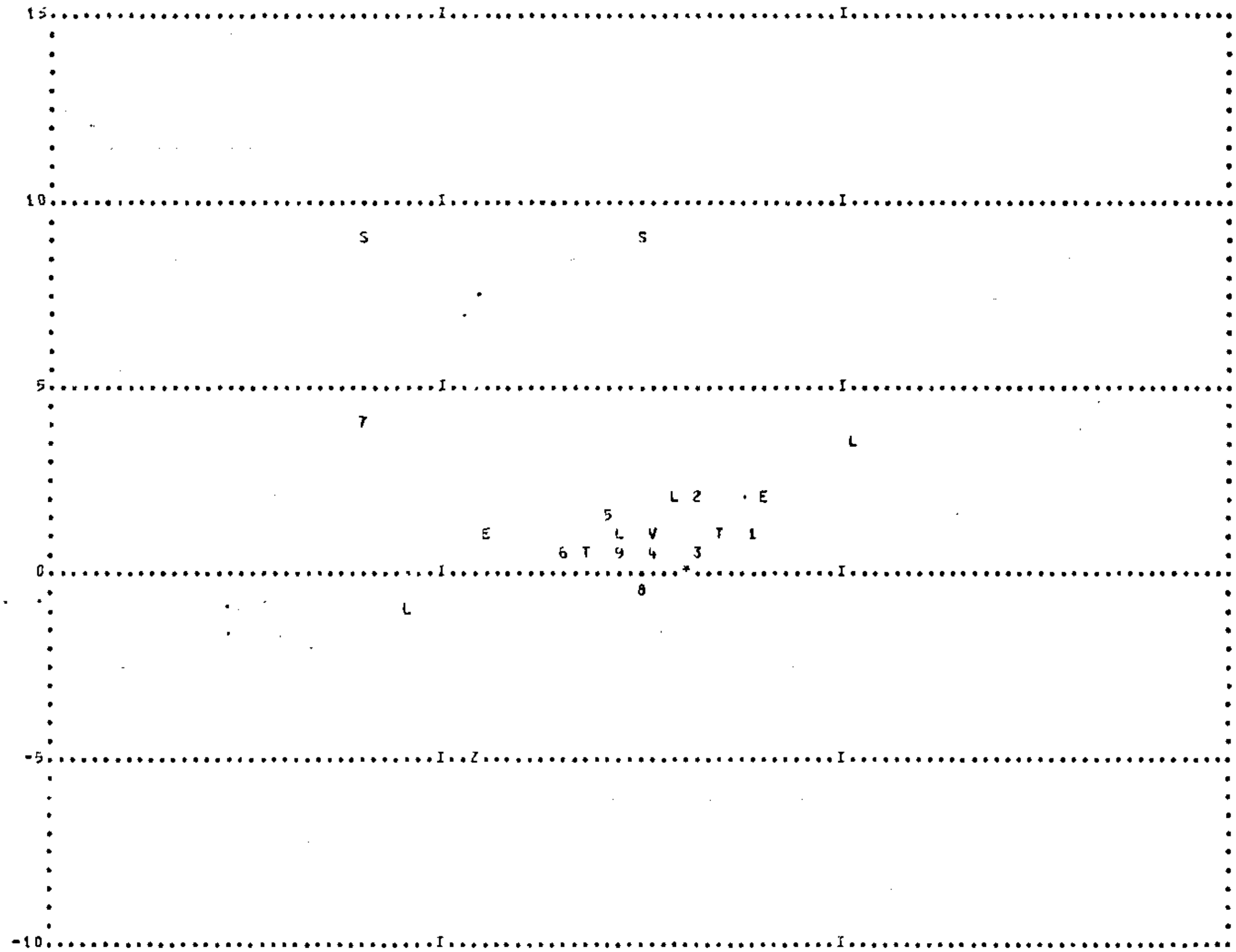


EPOCH = 1.00 SECONDS

PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE								
1	5.47	1.41	-1.74	.75	1	9.38	-10.78	E
2	2.67	1.62	.44	4.72	0	14.55	.48	L
3	2.54	.60	.65	.08	1	10.64	7.84	T
4	.65	.25	.82	1.06	1	-2.58	16.76	V
5	-1.99	1.88	-.27	1.00	1	12.67	-6.33	L
6	-3.78	.93	.74	-.37	1	15.20	-1.09	T
7	-13.21	3.09	2.85	4.62	0	12.91	9.36	S
8	.36	-1.09	.20	1.06	0	.22	4.77	
9	-1.20	-.38	-.64	5.18	0	1.45	14.62	L
*	2.31	-1.37	.54	5.67	0	1.64	17.31	
Z	-6.17	-5.14	-6.24	.10	0	-15.05	-.75	
DEFENSE								
L	10.42	3.11	-2.79	2.21	0	-10.92	10.70	
E	5.91	2.13	-1.22	-1.37	1	-14.51	2.34	
T	3.32	.89	.83	.92	1	-6.38	-10.19	
L	1.72	2.12	.68	-1.67	0	.62	.02	
V	.47	1.12	1.11	1.27	1	.46	15.19	
S	.35	9.50	.36	-.49	0	-2.00	-3.51	
L	-1.28	1.43	.72	-.54	1	-17.77	1.45	
T	-2.86	.77	.76	.37	1	-12.87	-1.35	
E	-7.93	1.26	-.58	-.44	0	1.48	.23	
L	-11.15	-.10	2.04	-4.46	0	5.58	-12.52	
S	-13.65	8.19	3.26	2.52	0	5.51	15.14	

Figure 5-6e. Counter-Dive vs. Oklahoma 50 (Simulation Results)

EPOCH = 1.25 SECONDS

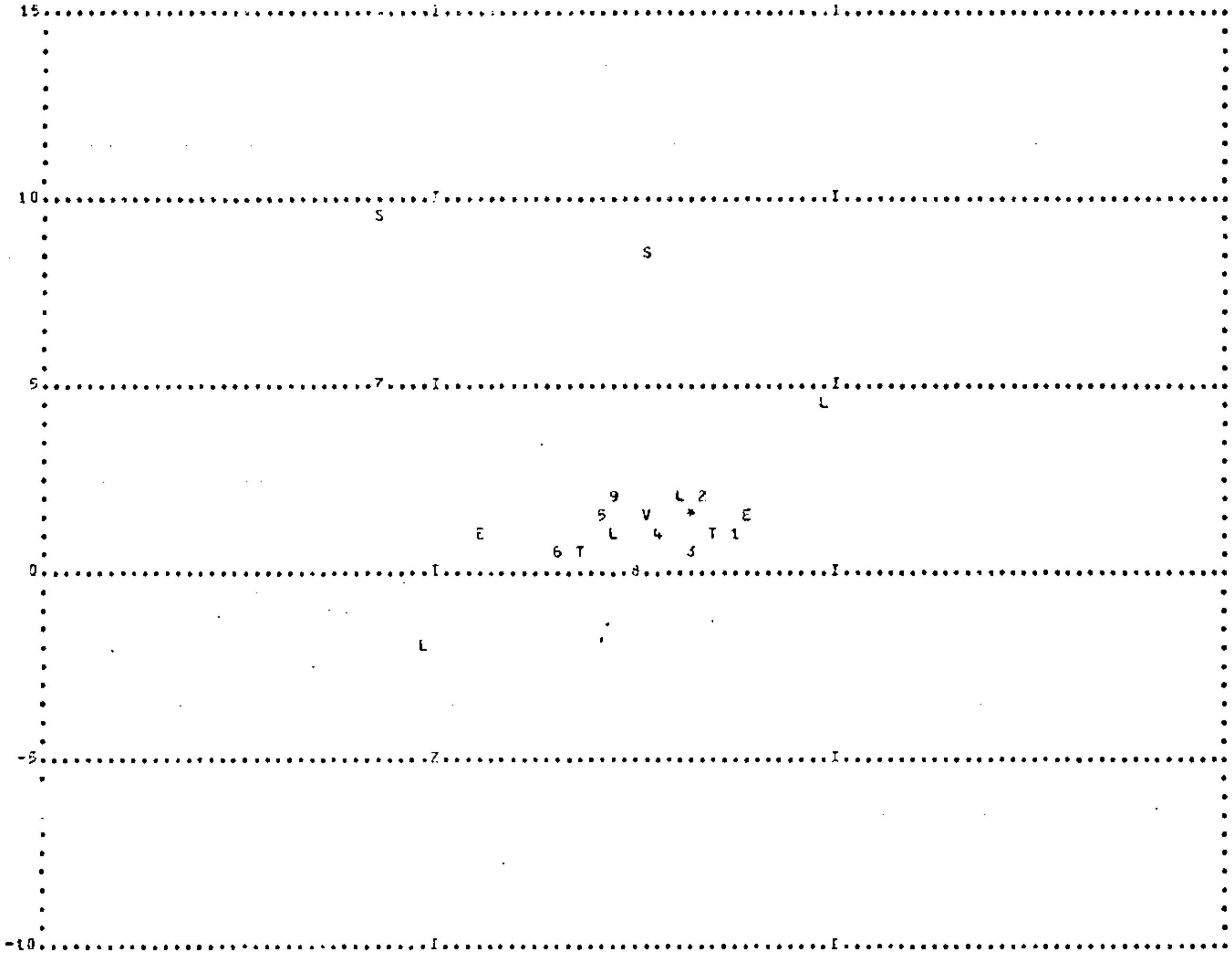


EPOCH = 1.25 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	5.10	1.31	-1.89	-.50	1	5.94	-3.80	E
	2	2.91	2.25	1.05	2.06	1	-12.92	-1.97	L
	3	2.74	.73	.40	.58	1	9.04	7.03	T
	4	.58	.65	.97	1.74	1	-1.62	16.84	V
	5	-2.09	1.85	-.24	-.13	1	12.95	-9.41	L
	6	-3.69	.90	.22	-.07	1	16.05	.08	T
	7	-12.95	4.19	3.94	4.26	0	11.26	12.84	S
	8	.40	-.77	.13	1.47	0	.58	5.96	
	9	-1.28	.96	-.07	5.54	0	-4.03	15.08	L
	*	2.46	.14	.60	5.31	0	1.77	15.02	
	Z	-7.72	-5.14	-6.20	-.08	0	-17.63	1.60	
DEFENSE									
	L	3.62	3.79	-3.55	3.20	0	-16.59	5.83	
	E	5.50	2.05	-1.31	-1.46	1	-5.80	.56	
	T	3.52	1.60	.80	.62	1	15.54	1.08	
	L	1.93	2.22	1.01	.25	1	9.45	-8.64	
	V	.71	1.48	.86	1.56	1	15.01	2.50	
	S	.34	9.29	-.35	-.38	0	13.93	-8.95	
	L	-1.30	1.35	-.03	-.32	1	-3.95	-.68	
	T	-2.78	.78	.22	.36	1	-1.98	1.11	
	C	-7.06	1.19	.07	-.19	0	-.16	-.23	
	L	-10.57	-1.26	2.15	-4.76	0	13.54	3.12	
	S	-12.89	9.05	2.79	4.20	0	16.01	4.25	

Figure 5-6f. Counter-Dive vs. Oklahoma 50 (Simulation Results)

EPOCH = 1.50 SECONDS

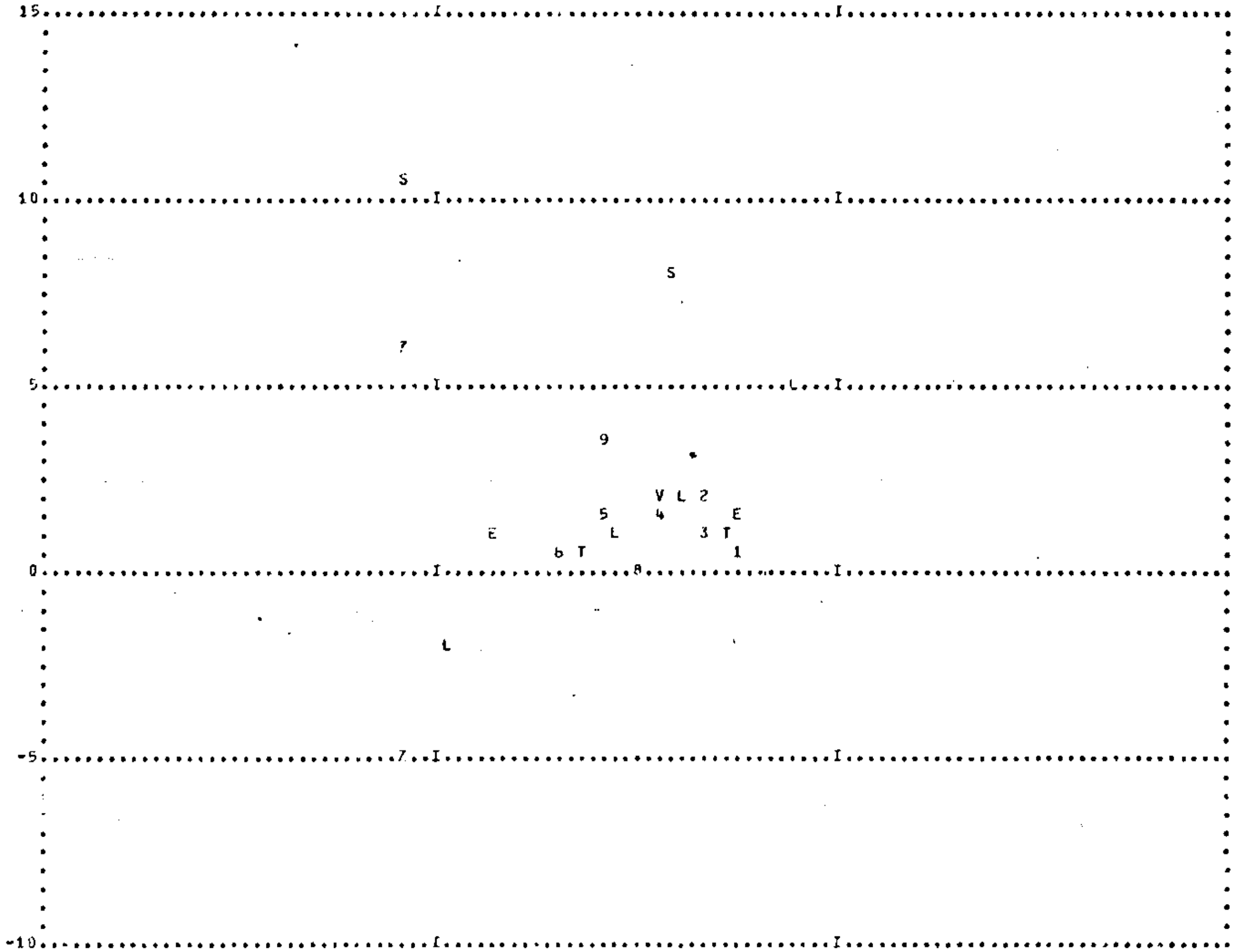


EPOCH = 1.50 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	4.71	1.53	-1.76	-1.24	1	1.57	14.60	E
	2	3.00	2.24	.63	-.09	1	12.34	-6.34	L
	3	3.00	.91	1.29	.87	1	11.74	10.09	T
	4	1.12	1.10	1.26	2.01	1	-.57	15.22	V
	5	-1.99	1.81	.80	-.24	1	10.68	-10.41	L
	6	-3.67	.93	.20	.17	1	15.50	-2.38	T
	7	-11.92	5.32	4.24	4.71	0	14.66	9.23	S
	8	.45	-.40	.21	1.53	0	-.37	-.04	
	9	-1.40	2.39	-.79	5.82	0	-5.93	12.46	L
	*	2.62	1.71	.66	6.23	0	.05	16.84	
	Z	-9.33	-5.11	-6.65	.25	0	1.55	-.95	
DEFENSE									
	L	8.53	4.54	-5.05	2.33	0	-12.65	7.83	
	E	5.03	1.78	-1.47	-1.68	1	-5.98	.02	
	T	3.79	1.16	1.22	.90	1	-12.69	5.61	
	L	2.14	2.19	.62	.46	1	-15.42	-1.90	
	V	.98	1.95	.94	1.33	1	2.95	-1.03	
	S	.75	0.82	2.65	-2.37	0	15.24	-5.88	
	L	-1.32	1.26	.10	-.31	1	14.51	1.22	
	T	-2.75	.84	.13	.09	1	16.35	3.90	
	E	-7.05	1.14	.01	-.15	0	16.93	.31	
	L	-9.82	-2.07	3.71	-1.39	0	13.97	4.53	
	S	-11.96	9.95	4.51	3.97	0	17.04	5.43	

Figure 5-6g. Counter-Dive vs. Oklahoma 50 (Simulation Results)

EPOCH = 1.75 SECONDS

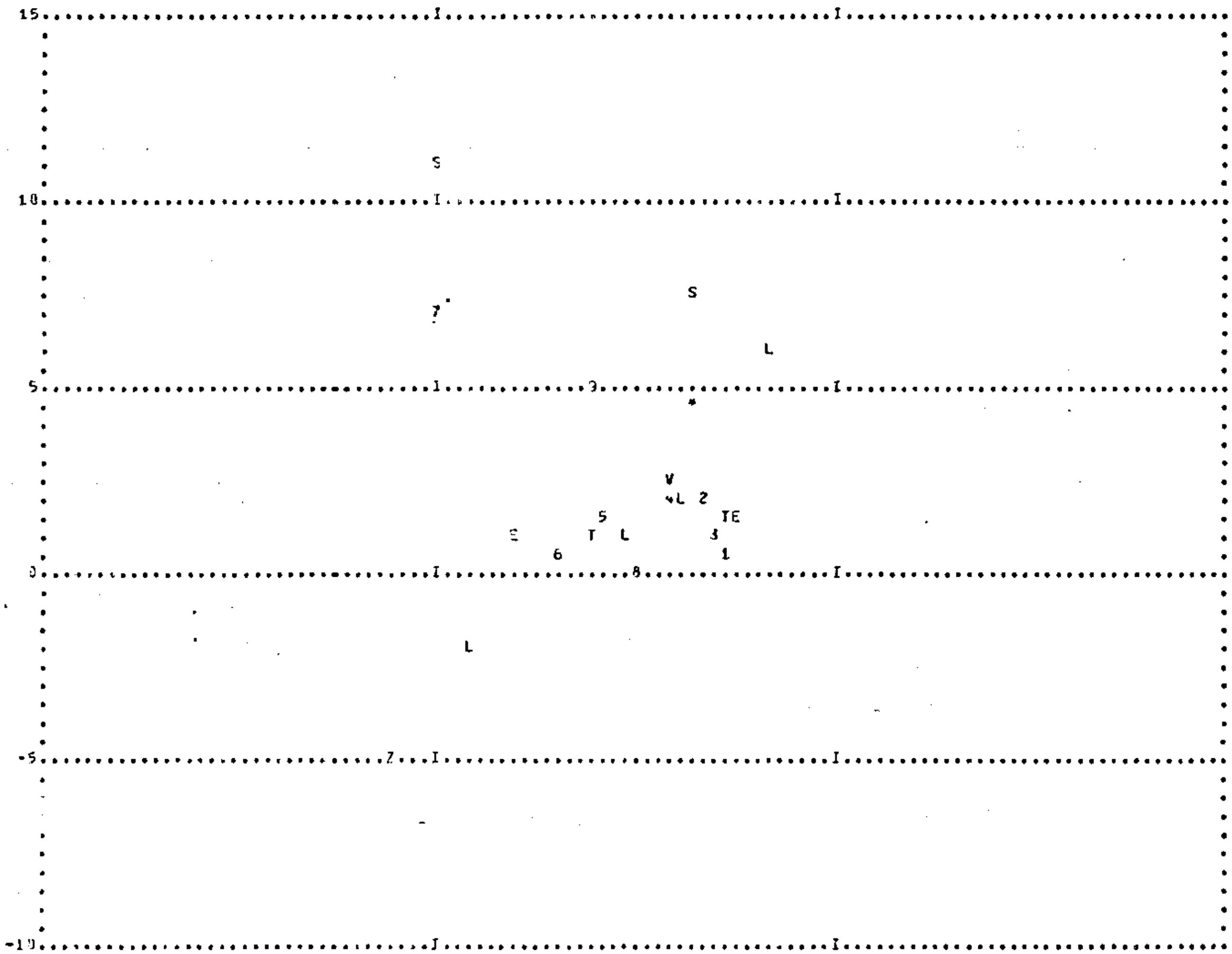


EPOCH = 1.75 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	4.56	.90	-.95	-.57	1	2.03	14.22	E
	2	3.15	2.24	.56	-.10	1	-14.62	2.63	L
	3	3.32	1.16	1.25	1.12	1	13.91	4.91	T
	4	1.41	1.56	1.10	1.09	1	.42	16.46	V
	5	-1.98	1.76	.84	-.40	1	11.29	-4.18	L
	6	-3.56	.95	.82	.04	1	14.54	-1.47	T
	7	-10.75	6.44	5.04	4.28	0	14.69	-.27	S
	8	.48	-.11	.04	.32	0	.03	-.19	
	9	-1.70	3.30	-1.55	5.48	0	-9.08	13.06	L
	*	2.74	3.30	.37	6.52	0	-11.14	14.37	
	Z	-10.54	-5.09	-3.32	-.04	0	2.48	-1.13	
DEFENSE									
	L	7.26	5.28	-5.11	3.02	0	-10.63	10.64	
	E	4.84	1.58	-.76	-.55	1	-12.01	13.58	
	T	4.10	1.43	1.29	1.61	1	-6.98	10.37	
	L	2.29	2.24	.56	.65	1	7.10	13.30	
	V	1.25	2.40	1.20	1.41	1	12.24	8.90	
	S	1.71	8.23	4.51	-2.39	0	8.72	-10.34	
	L	-1.17	1.24	.62	.38	1	12.49	11.01	
	T	-2.59	.90	.81	.59	1	11.18	5.29	
	E	-6.62	1.12	3.17	-.02	0	14.41	2.70	
	L	-5.77	-2.32	4.62	-.23	0	13.97	4.81	
	S	-10.68	10.69	5.63	2.56	0	13.18	2.46	

Figure 5-6h. Counter-Dive vs. Oklahoma 50 (Simulation Results)

EPOCH = 2.00 SECONDS

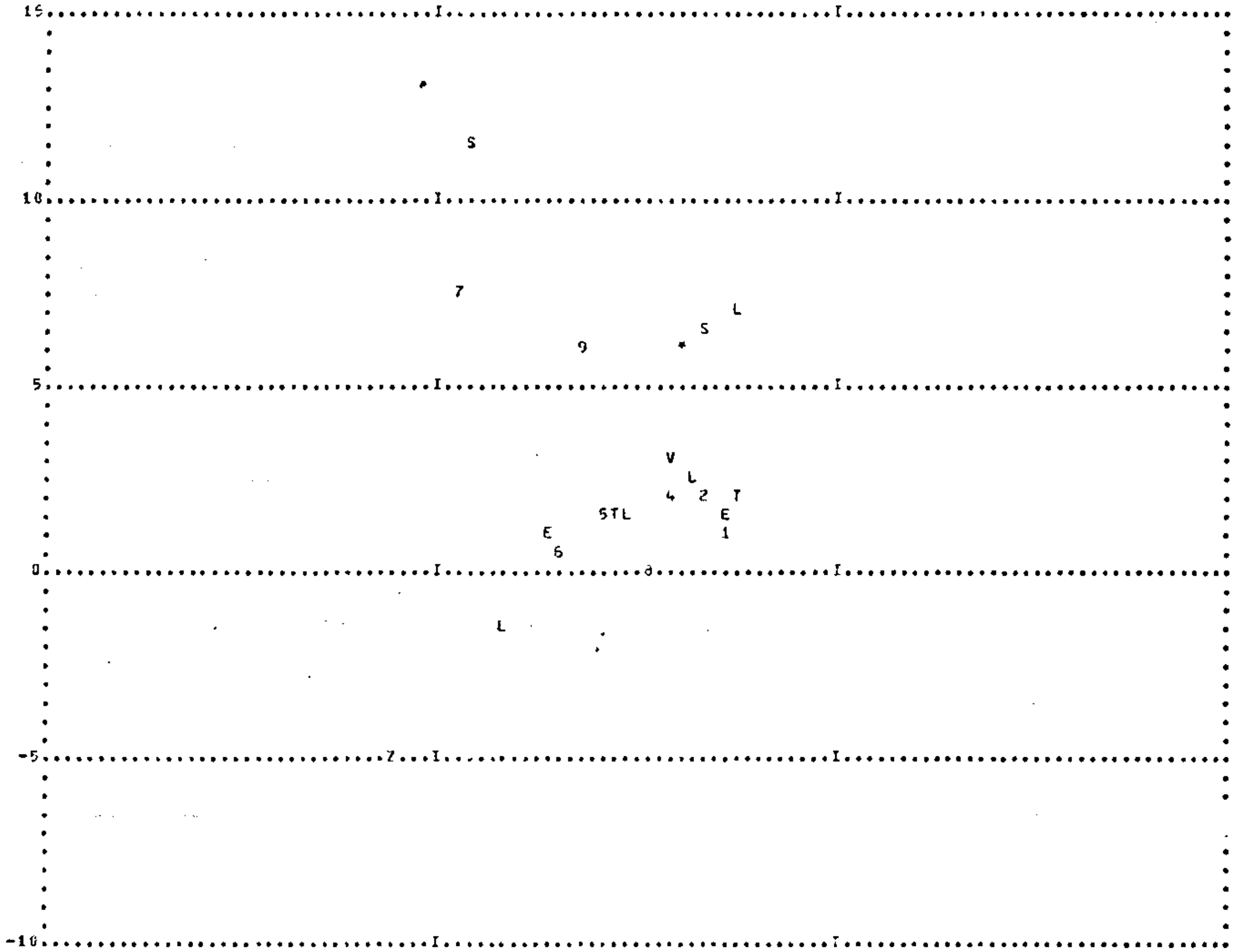


EPOCH = 2.00 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	4.47	.94	-1.08	.41	1	3.41	14.76	E
	2	3.26	2.30	.49	.26	1	-12.88	3.22	L
	3	3.67	1.50	1.31	1.40	1	14.21	9.62	T
	4	1.71	2.01	1.67	1.60	1	3.22	13.62	V
	5	-1.78	1.31	1.02	.22	1	17.23	-4.54	L
	6	-3.54	.95	0.30	0.00	2	16.13	3.42	T
	7	-9.44	7.23	5.44	2.27	0	15.55	1.35	S
	8	.49	.64	.04	.41	0	1.15	.41	
	9	-2.20	5.16	-2.35	5.41	0	-11.71	11.22	L
	*	2.52	4.89	-1.86	6.22	0	.48	16.24	
	Z	-11.09	-5.13	-1.33	-.73	0	-1.20	.72	
DEFENSE									
	L	6.04	6.15	-4.71	3.71	0	-9.73	11.80	
	E	4.61	1.64	-.45	-.15	1	-6.51	13.60	
	T	4.42	1.84	1.55	1.76	1	-6.89	11.87	
	L	2.42	2.38	.44	1.05	1	1.56	17.24	
	V	1.63	2.84	1.22	1.43	1	3.76	13.79	
	S	2.78	7.52	4.07	-3.23	0	-15.17	-1.84	
	L	-.97	1.40	.74	.33	1	9.44	13.40	
	T	-2.15	1.14	2.53	1.50	0	10.51	9.49	
	E	-5.65	1.19	4.41	.49	0	14.08	6.63	
	L	-7.54	-2.24	5.12	.77	0	14.41	12.03	
	S	-3.29	11.28	5.52	2.01	0	14.11	7.69	

Figure 5-6i. Counter-Dive vs. Oklahoma 50 (Simulation Results)

EPOCH = 2.25 SECONDS

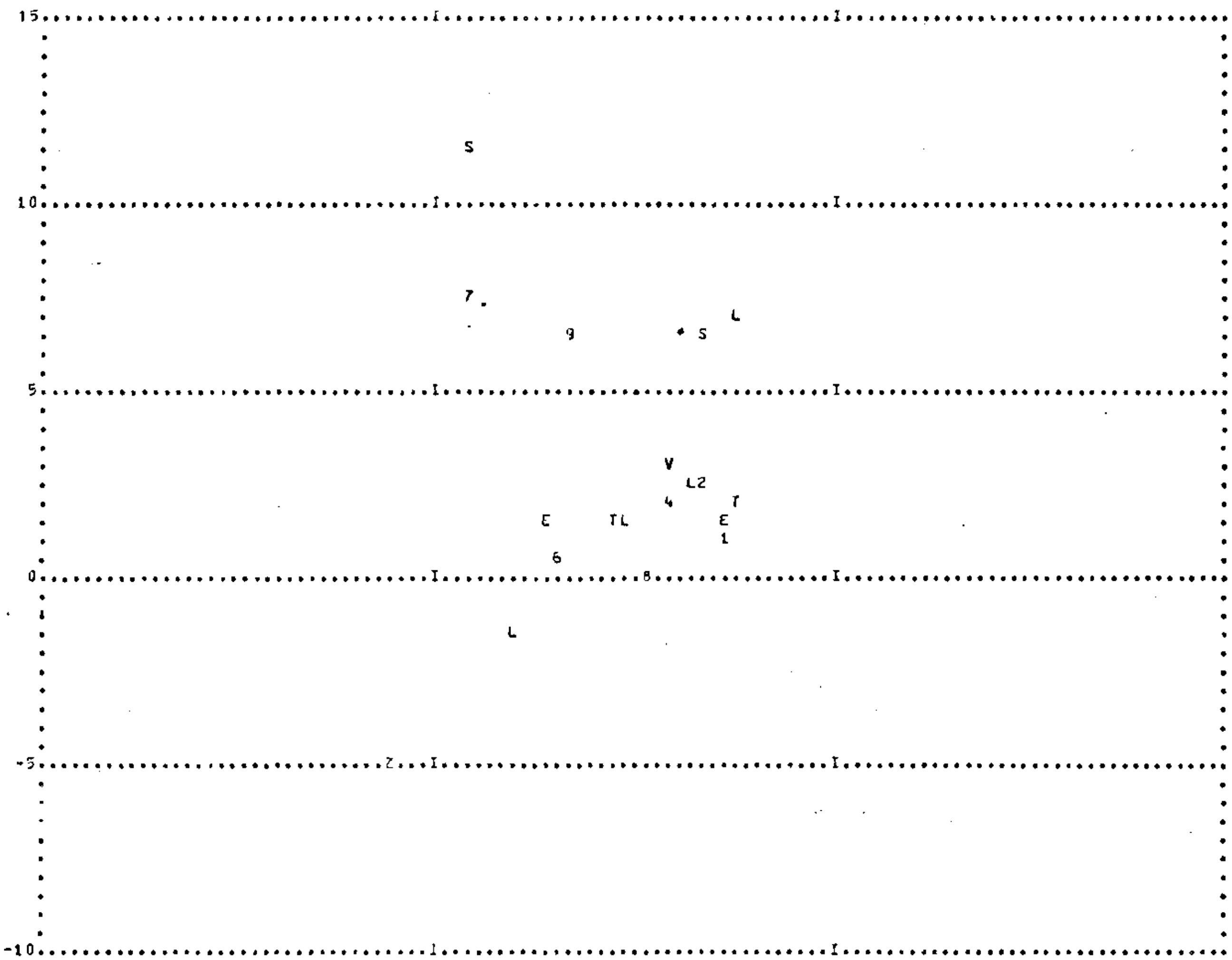


EPOCH = 2.25 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	4.25	1.06	-1.05	.61	1	-.86	16.44	E
	2	3.38	2.48	.57	.34	1	-12.45	6.49	L
	3	-4.06	1.91	1.43	1.74	1	10.63	8.95	T
	4	1.86	2.27	.64	1.27	1	.39	15.53	V
	5	-1.53	1.96	1.23	.65	1	14.29	3.91	L
	6	-3.54	.95	0.00	0.00	2	12.86	4.22	T
	7	-5.02	7.59	5.86	1.48	0	11.07	8.15	S
	8	.52	.13	.24	.30	0	1.33	-2.11	
	9	-2.94	6.46	-3.46	3.03	0	-12.75	8.25	L
	*	2.18	6.47	-.93	0.41	0	-2.66	14.47	
	Z	-11.37	-5.15	-.95	.01	0	2.09	-2.22	
DEFENSE									
	L	4.93	7.17	-4.32	-4.29	0	-9.32	11.07	
	E	4.39	1.76	-.61	-.60	1	-7.25	13.67	
	T	4.78	2.32	1.66	1.99	1	-5.63	9.05	
	L	2.55	2.71	.37	1.01	1	-.33	14.43	
	V	1.97	3.21	.49	.75	1	1.03	13.80	
	S	3.15	6.67	-.63	-2.09	0	1.89	-15.18	
	L	-.69	1.67	1.02	1.48	1	6.93	14.97	
	T	-1.41	1.53	3.33	2.48	0	7.70	10.15	
	E	-4.46	1.45	5.02	1.51	0	12.08	3.94	
	L	-6.22	-1.79	5.47	2.67	0	10.57	11.44	
	S	-7.89	11.66	5.63	2.53	0	15.14	7.13	

Figure 5-6j. Counter-Dive vs. Oklahoma 50 (Simulation Results)

EPOCH = 2.30 SECONDS

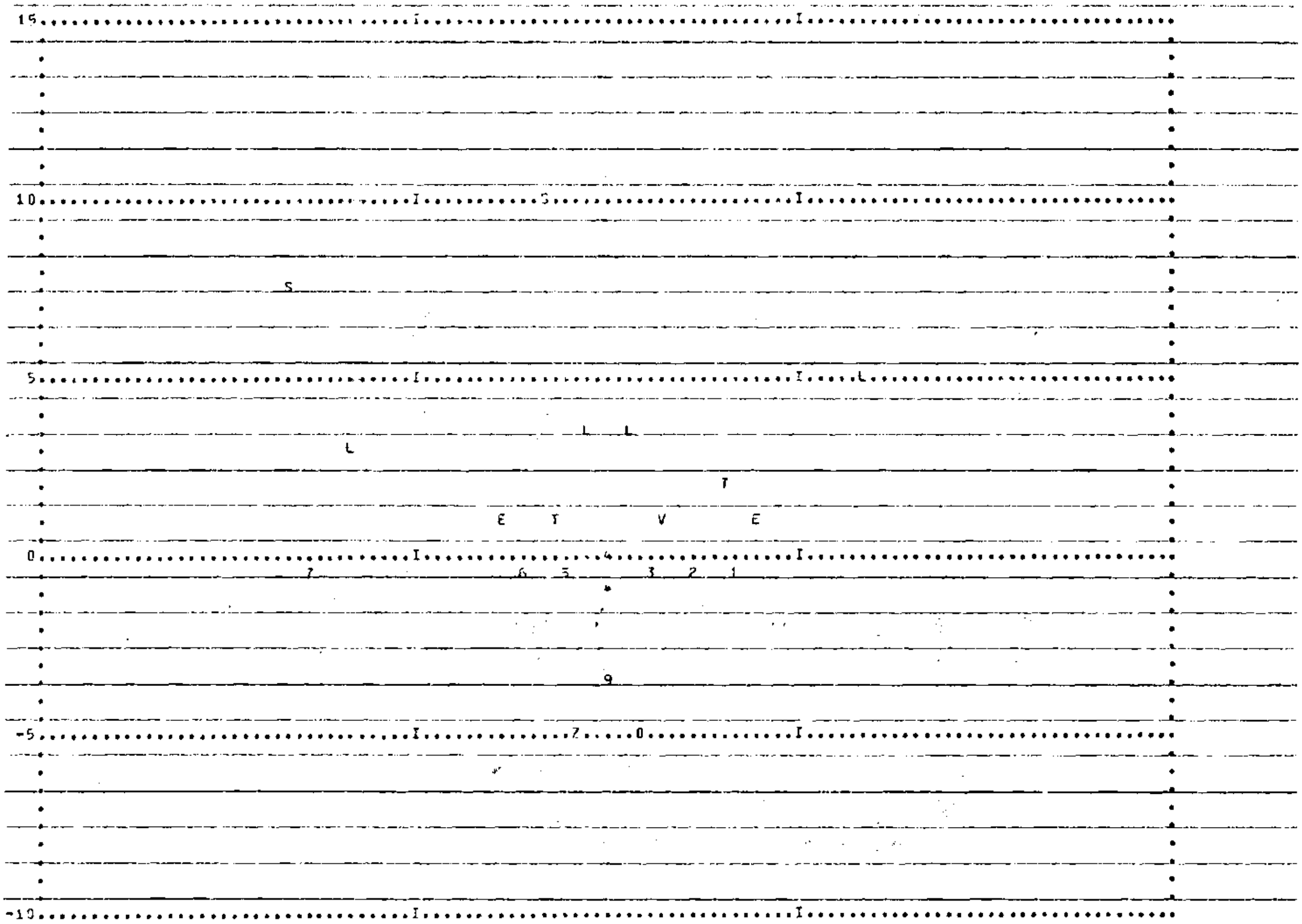


EPOCH = 2.30 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	4.20	1.09	-.93	.54	0	0.00	0.00	E
	2	3.41	2.51	.50	.74	0	0.00	0.00	L
	3	4.06	1.91	0.00	0.00	2	10.63	8.95	T
	4	1.86	2.27	0.00	0.00	2	.39	15.53	V
	5	-1.47	1.99	1.09	.57	0	0.00	0.00	L
	6	-3.54	.95	0.00	0.00	2	12.86	4.22	T
	7	-7.73	7.77	5.70	1.09	0	11.97	3.15	S
	8	.54	.14	.27	.17	0	1.33	-2.11	
	9	-3.12	6.71	-3.66	4.24	0	-12.75	8.25	L
	*	2.13	6.79	-.95	6.35	3	-2.66	14.47	
	Z	-11.41	-5.15	-.74	-.10	0	2.09	-2.22	
DEFENSE									
	L	4.71	7.39	-4.26	4.31	0	-9.32	11.07	
	E	4.34	1.75	-1.06	.11	0	-7.29	13.67	
	T	4.95	2.42	1.21	2.18	0	-5.63	9.05	
	L	2.56	2.78	.31	1.56	0	-.33	14.43	
	V	1.97	3.26	.48	1.32	0	1.03	13.80	
	S	3.12	6.75	-.47	-2.57	3	1.69	-15.18	
	L	-.64	1.76	1.23	1.07	0	6.93	14.07	
	T	-1.24	1.76	3.31	2.07	0	7.70	10.15	
	E	-4.21	1.53	5.01	1.75	0	12.08	8.94	
	L	-5.95	-1.65	5.34	2.40	0	10.57	11.44	
	S	-7.01	11.94	5.69	2.07	0	15.14	7.13	

Figure 5-6k. Counter-Dive vs. Oklahoma 50 (Simulation Results)

EPOCH = 0.00 SECONDS

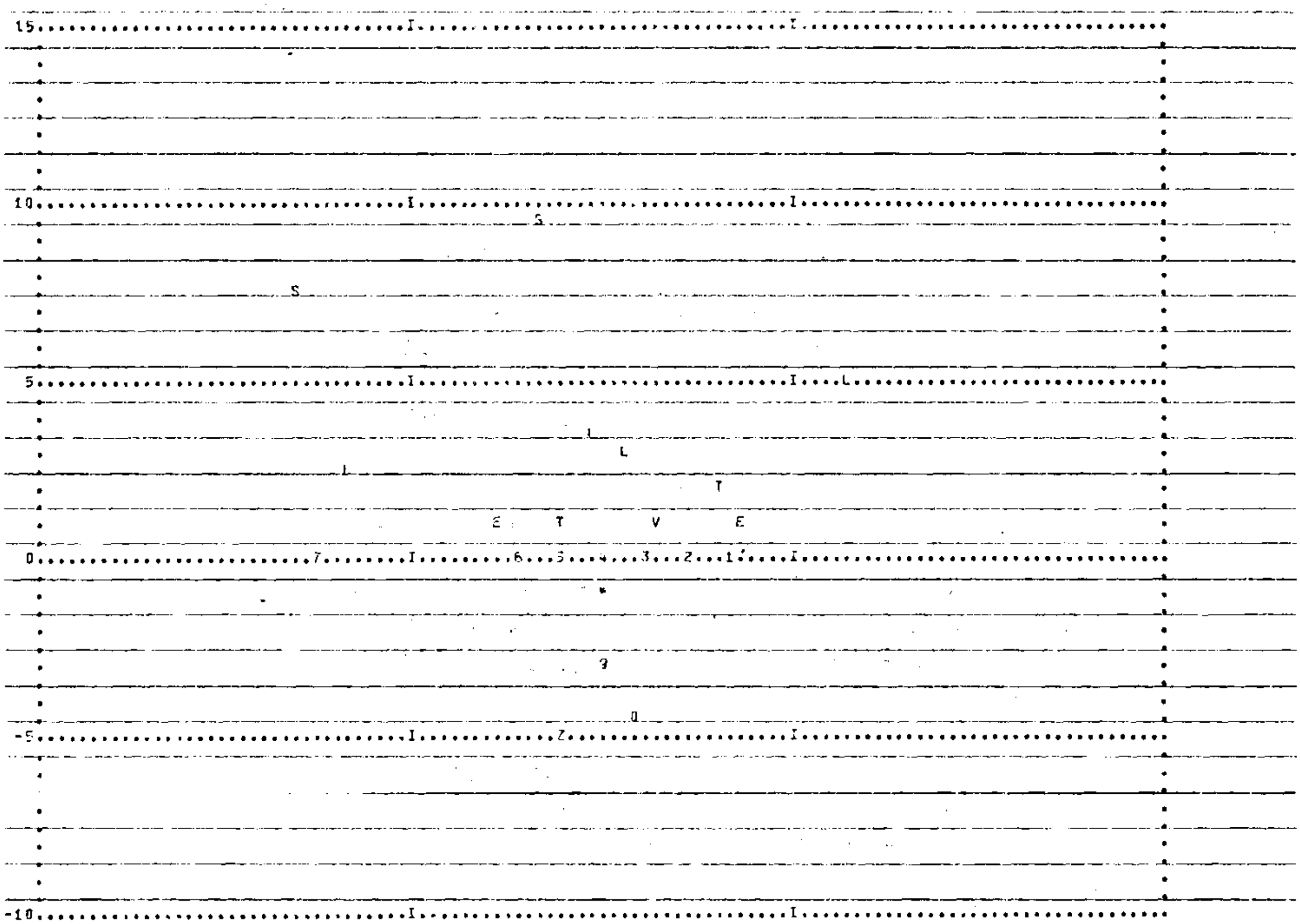


EPOCH = 0.00 SECONDS

PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE								
1	6.00	-.80	0.00	0.00	0	1.99	14.21	E
2	4.00	-.80	0.00	0.00	0	5.50	12.17	T
3	2.00	-.80	0.00	0.00	0	3.42	14.28	V
4	0.00	-.40	0.00	0.00	0	6.84	14.14	L
5	-2.00	-.80	0.00	0.00	0	-5.83	15.69	T
6	-4.00	-.80	0.00	0.00	0	-6.26	13.49	E
7	-14.00	-.80	0.00	0.00	0	2.21	14.60	S
*	0.00	-1.30	0.00	0.00	0	6.45	-2.80	
9	0.00	-3.50	0.00	0.00	0	-3.54	16.22	T
0	1.80	-5.30	0.00	0.00	0	1.47	18.32	
Z	-1.80	-5.30	0.00	0.00	0	-12.13	-.88	
DEFENSE								
L	12.00	5.00	0.00	0.00	0	-14.72	.90	
E	7.00	1.40	0.00	0.00	0	-1.52	-2.11	
T	5.50	2.20	0.00	0.00	0	.84	-2.20	
L	1.00	3.50	0.00	0.00	0	.53	-.34	
V	2.00	1.40	0.00	0.00	0	.32	1.34	
S	-3.75	10.00	0.00	0.00	0	-2.54	-1.00	
L	-1.00	3.50	0.00	0.00	0	.70	1.54	
T	-2.50	1.40	0.00	0.00	0	.38	-.72	
E	-5.00	1.40	0.00	0.00	0	-.74	.19	
L	-12.00	3.00	0.00	0.00	0	-2.34	-18.17	
S	-15.00	7.50	0.00	0.00	0	.03	.07	

Figure 5-7a. Counter-Dive vs. Split-Six Defense (Simulation Results)

EPOCH = .25 SECONDS

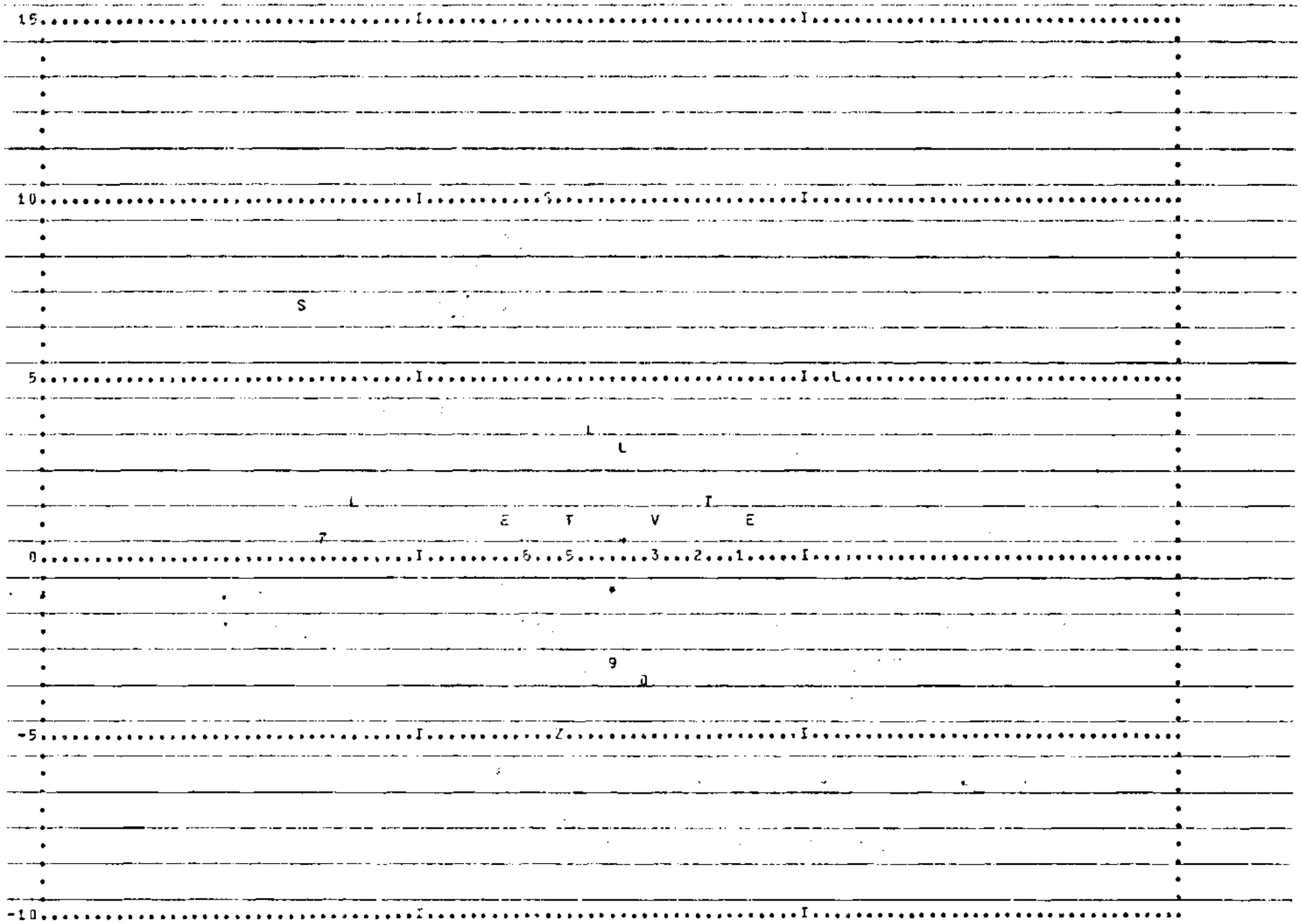


EPOCH = .25 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	6.05	-.43	.37	2.66	0	3.22	13.48	E
	2	4.14	-.49	1.03	2.28	0	4.49	14.09	T
	3	2.10	-.43	.71	2.67	0	2.83	14.43	V
	4	.18	-.84	1.28	2.34	0	6.59	14.40	L
	5	-2.15	-.40	-1.09	2.33	0	-2.84	14.79	T
	6	-4.16	-.46	-1.17	2.31	0	-2.92	14.29	E
	7	-13.44	-.42	.41	2.74	0	-.38	16.63	S
	*	.17	-1.37	1.21	-.52	0	.02	-.91	
	9	-.89	-3.48	-.66	5.33	0	-7.55	14.00	T
	0	1.84	-4.63	.27	5.43	0	-1.83	13.37	
	Z	-2.11	-5.30	-2.27	-.01	0	-16.93	1.57	
DEFENSE									
	L	11.62	2.02	-2.75	.17	0	-13.18	-.61	
	E	6.26	1.35	-.28	-.39	0	-15.35	2.69	
	T	5.52	2.14	.16	-.41	0	-12.93	-2.07	
	L	1.91	3.49	.10	-.05	0	-2.93	-4.31	
	V	2.51	1.43	.06	.25	0	-2.33	-17.66	
	S	-3.67	9.97	-.47	-.13	0	.63	2.96	
	L	-.98	3.53	.13	.25	0	-2.57	.93	
	T	-2.45	1.38	.16	-.14	0	11.93	-3.19	
	E	-5.02	3.40	-.14	.04	0	3.50	1.63	
	L	-12.06	2.53	-.44	-3.40	0	3.79	-12.16	
	S	-15.00	7.20	.11	.01	0	3.42	-1.54	

Figure 5-7b. Counter-Dive vs. Split-Six Defense (Simulation Results)

EPOCH = .50 SECONDS

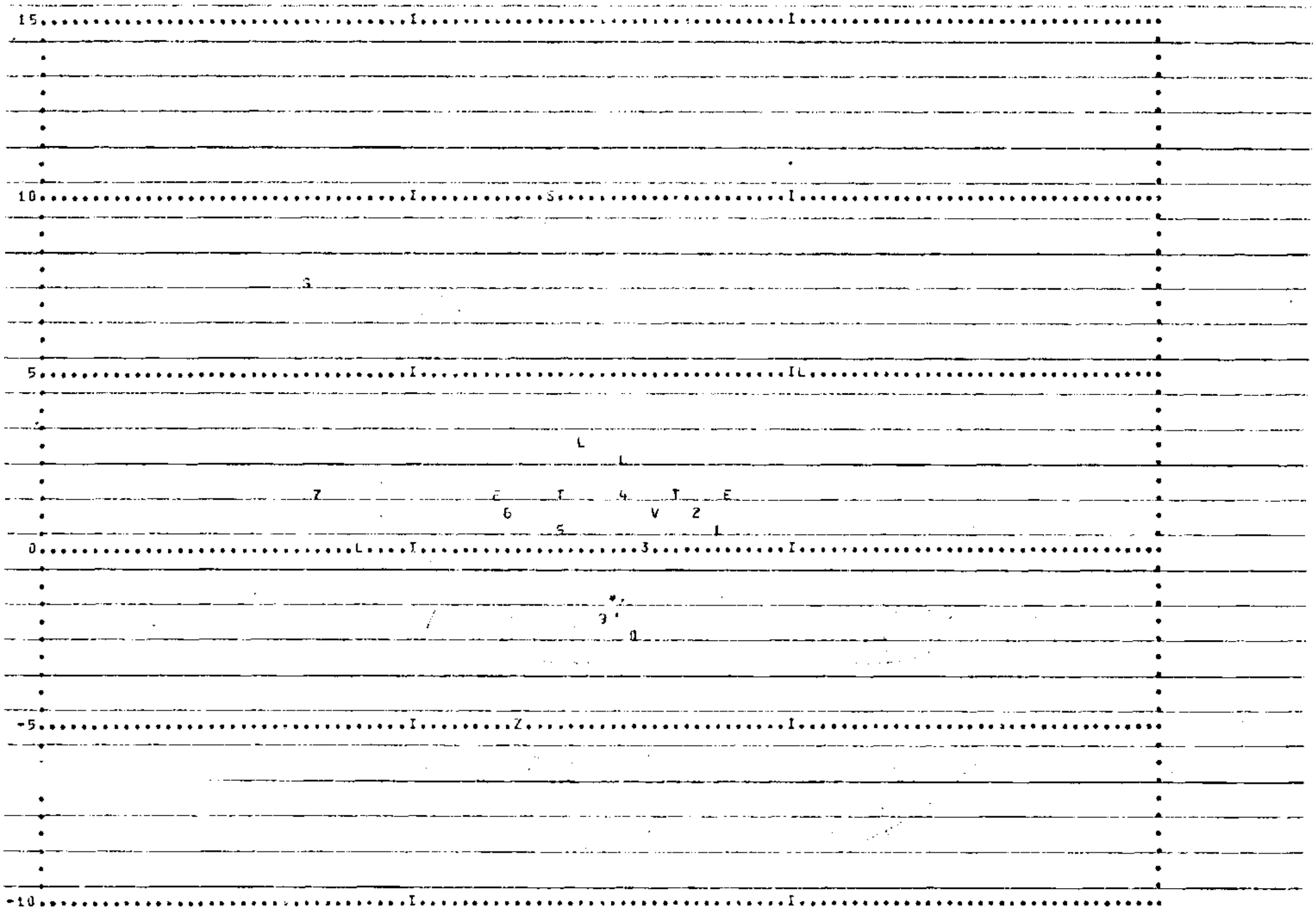


EPOCH = .50 SECONDS

PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE								
1	6.20	.41	.80	3.96	1	-2.21	12.64	E
2	4.45	.30	1.40	3.37	0	-.31	15.76	T
3	2.27	.26	.23	.46	1	.54	13.35	V
4	.58	.63	1.92	-.12	0	6.38	13.78	L
5	-2.37	.39	.95	1.38	1	.25	12.18	I
6	-4.46	.39	-1.19	4.03	0	-1.33	15.25	E
7	-13.98	.52	.15	4.63	0	-1.00	13.83	S
4	.39	-1.49	.66	-.45	0	1.31	.69	
9	-.13	-3.32	-.94	3.34	0	-3.77	15.67	T
0	1.24	-3.64	-.19	4.36	0	2.48	12.53	
Z	-2.97	-5.26	-4.41	.23	0	-17.41	-.27	
DEFENSE								
L	10.77	5.04	-3.95	-.02	0	-12.67	-.09	
E	6.51	1.34	-3.02	.27	1	-14.92	-2.29	
I	4.82	1.96	-3.47	-.70	0	-13.36	-.02	
L	.98	3.37	-.33	-.34	0	7.47	-14.79	
V	2.49	1.19	.21	.23	1	-3.48	-14.69	
S	-3.14	10.07	-.14	.45	0	10.38	.50	
L	-1.03	3.61	-.97	.51	0	-1.12	-7.94	
T	-2.20	1.28	.94	.44	1	11.16	-9.37	
E	-5.01	1.45	.17	.32	0	.35	-1.54	
L	-12.04	1.53	.47	-4.11	0	5.17	-13.92	
S	-14.24	7.48	.42	-.29	0	15.16	.87	

Figure 5-7c. Counter-Dive vs. Split-Six Defense (Simulation Results)

EPOCH = .75 SECONDS

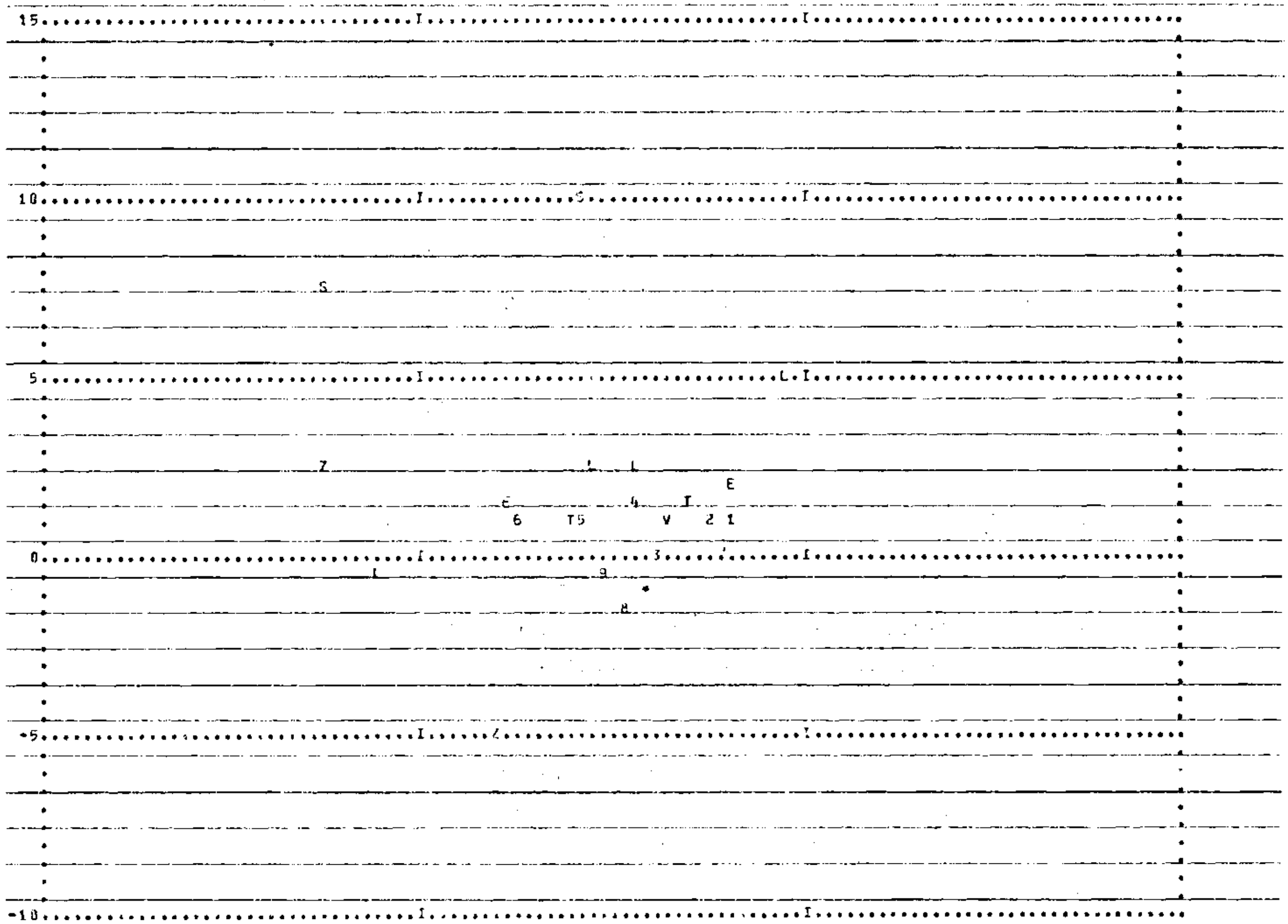


EPOCH = .75 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	5.97	.95	-1.79	2.24	1	7.75	-14.05	E
	2	4.60	1.32	-2.35	2.77	1	-14.98	.18	T
	3	2.32	.35	.23	.36	1	10.44	8.94	V
	4	1.11	1.95	0.00	0.00	2	-2.98	15.01	L
	5	-2.23	.62	.84	.99	0	7.33	11.99	L
	6	-4.63	1.22	-.59	2.82	1	-5.61	-14.90	E
	7	-13.87	1.74	-.10	5.10	0	14.70	5.59	S
	*	.56	-1.56	.69	-.12	2	3.95	.77	
	9	-.40	-2.29	-1.21	4.74	0	-7.63	15.05	T
	0	1.87	-2.71	.46	4.79	0	-.69	17.99	
	Z	-4.25	-5.22	-5.64	.11	0	-17.64	.28	
DEFENSE									
	L	9.70	6.03	-4.51	-.03	0	-16.77	1.56	
	E	6.12	1.91	-1.01	-.85	1	-16.84	-.97	
	T	3.93	1.95	-1.27	-2.30	1	-.69	13.25	
	L	1.11	2.83	0.00	0.00	2	6.59	-8.92	
	V	2.55	1.28	.15	.25	1	-12.73	-10.46	
	S	-2.70	10.10	2.50	.31	0	6.59	-.75	
	L	-1.15	3.46	-.46	-1.33	0	-4.67	-12.51	
	T	-2.04	1.51	.52	.26	0	-2.47	-14.55	
	E	-2.09	1.86	-.37	-.52	1	2.49	.21	
	L	-11.92	.46	1.26	-.33	0	2.34	-10.58	
	S	-14.46	7.54	3.03	.72	0	3.00	11.75	

Figure 5-7d. Counter-Dive vs. Split-Six Defense (Simulation Results)

EPOCH = 1.00 SECONDS

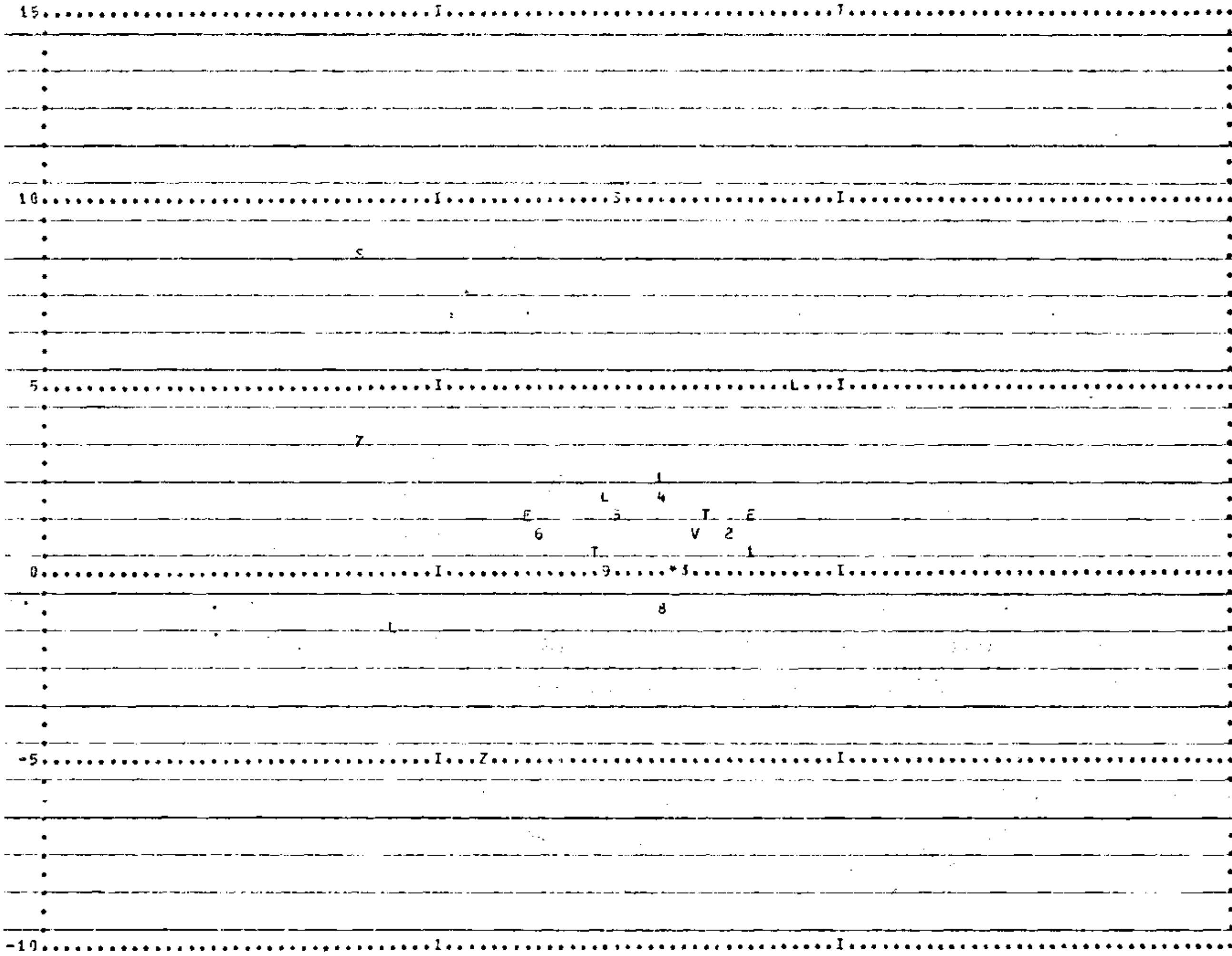


EPOCH = 1.00 SECONDS

PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE								
1	5.62	1.09	-1.94	.44	1	11.05	-11.59	E
2	4.51	1.23	-.73	-.58	1	-12.87	4.57	T
3	2.40	.42	-.14	.25	1	10.15	10.53	V
4	1.10	1.07	.22	.17	1	-2.47	15.09	L
5	-1.07	1.12	1.42	2.78	0	9.73	10.71	L
6	-4.66	1.16	-.01	-.45	1	-5.79	10.93	E
7	-13.51	2.64	2.69	3.81	0	12.39	8.30	S
8	.79	-1.56	1.12	.98	0	4.22	3.58	
9	-.83	-1.00	-2.08	5.57	0	-17.80	.58	T
*	1.92	-1.37	.07	5.91	0	-1.07	20.41	
Z	-5.75	-5.19	-6.35	.11	0	-15.74	.05	
DEFENSE								
L	8.43	5.07	-5.59	.29	0	-14.61	2.00	
E	5.72	2.07	-1.29	-1.01	1	-15.03	1.35	
T	3.70	1.15	-.71	-.43	1	-15.25	.58	
L	1.13	2.05	-.10	.15	1	8.13	-11.60	
V	2.52	1.36	.53	.24	1	-15.28	-8.06	
S	-1.93	10.13	3.04	-.31	0	11.05	-.01	
L	-1.35	2.99	-1.12	-3.76	0	-5.12	-13.27	
T	-2.92	1.74	-.76	-2.43	0	2.69	-13.51	
E	-5.12	1.78	-.24	-.17	1	-.30	-1.91	
L	-11.42	-.72	1.49	-4.58	0	4.97	-15.14	
S	-13.65	7.06	3.32	7.42	0	6.42	13.66	

Figure 5-7e. Counter-Dive vs. Split-Six Defense (Simulation Results)

EPOCH = 1.25 SECONDS

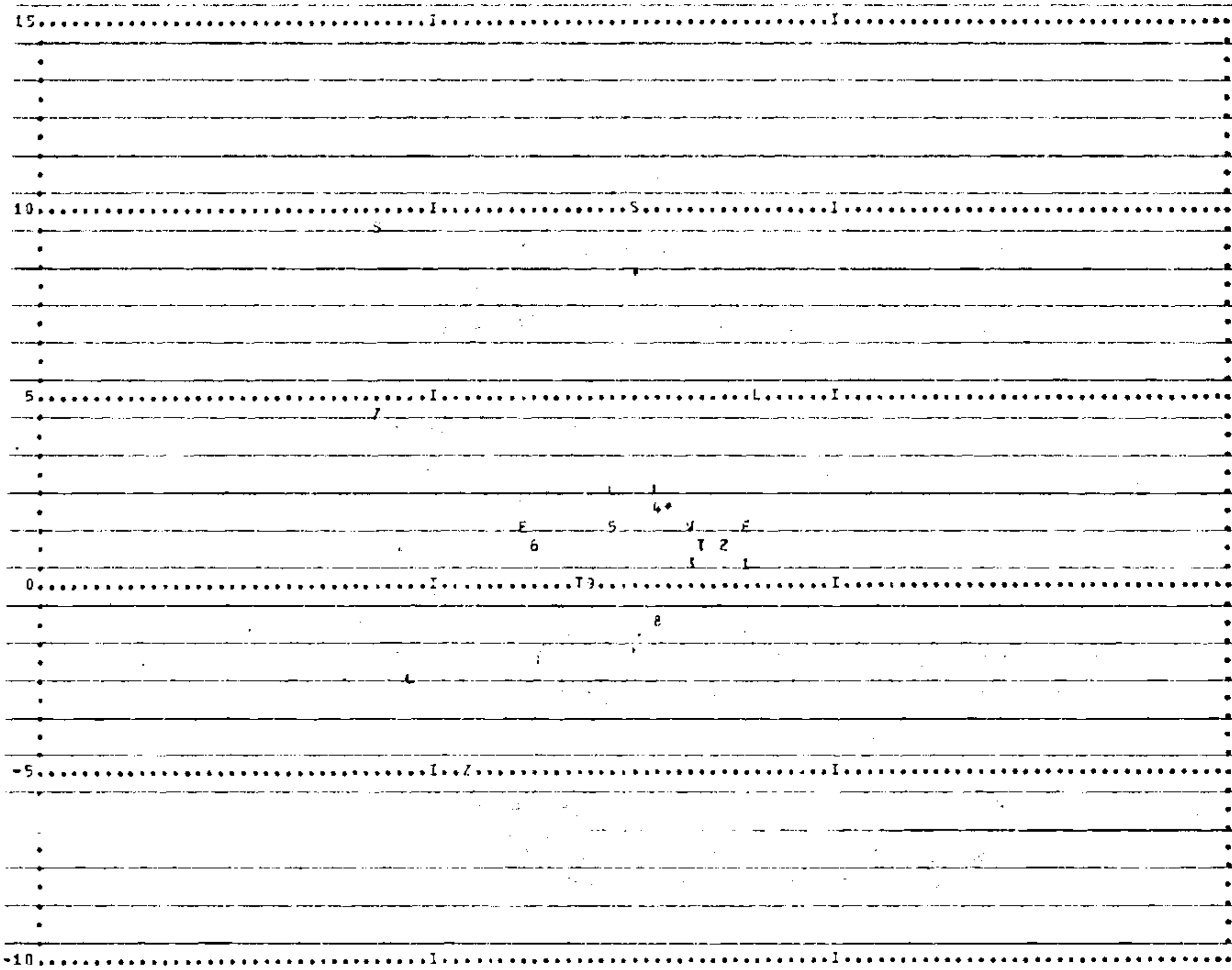


EPOCH = 1.25 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	5.25	.93	-2.03	-.75	1	8.12	-11.52	E
	2	4.26	1.13	-1.23	-.36	1	-16.37	1.15	T
	3	2.48	.48	-.19	.28	1	8.04	10.04	V
	4	1.11	2.03	.30	.31	1	1.36	15.27	L
	5	-1.42	1.57	.19	-.01	0	9.39	12.14	L
	6	-4.68	[illegible]	-.25	.04	1	-10.16	11.99	E
	7	-12.69	3.77	3.77	3.52	0	9.70	12.69	S
	8	1.11	-1.43	1.39	.77	[illegible]	2.85	5.52	
	9	-1.95	-.12	-1.85	-.33	1	-16.77	.50	T
	*	1.90	.27	-.16	7.02	0	-.40	17.32	
	Z	-7.35	-5.17	-6.39	.07	0	-14.40	-.45	
DEFENSE									
	L	7.01	5.17	-5.76	.34	0	-14.68	1.10	
	E	5.27	1.90	-1.31	-1.75	1	-4.41	-1.56	
	T	3.43	1.53	-1.14	-1.09	1	-3.56	-2.73	
	L	1.17	2.90	-.03	.13	1	11.17	-9.35	
	V	2.57	1.54	.46	.27	1	-5.25	-13.63	
	S	-3.10	10.13	3.82	-.81	0	15.59	-.13	
	L	-1.52	2.46	1.23	.39	1	15.19	1.86	
	T	-2.11	.60	-1.72	-1.64	0	-5.32	-1.75	
	E	-5.17	1.75	-.13	-.85	1	-.32	-.56	
	L	-11.09	-1.95	1.69	-5.31	0	16.90	3.90	
	S	-12.66	8.75	3.05	3.57	0	13.10	5.23	

Figure 5-7f. Counter-Dive vs. Split-Six Defense (Simulation Results)

EPOCH = 1.50 SECONDS

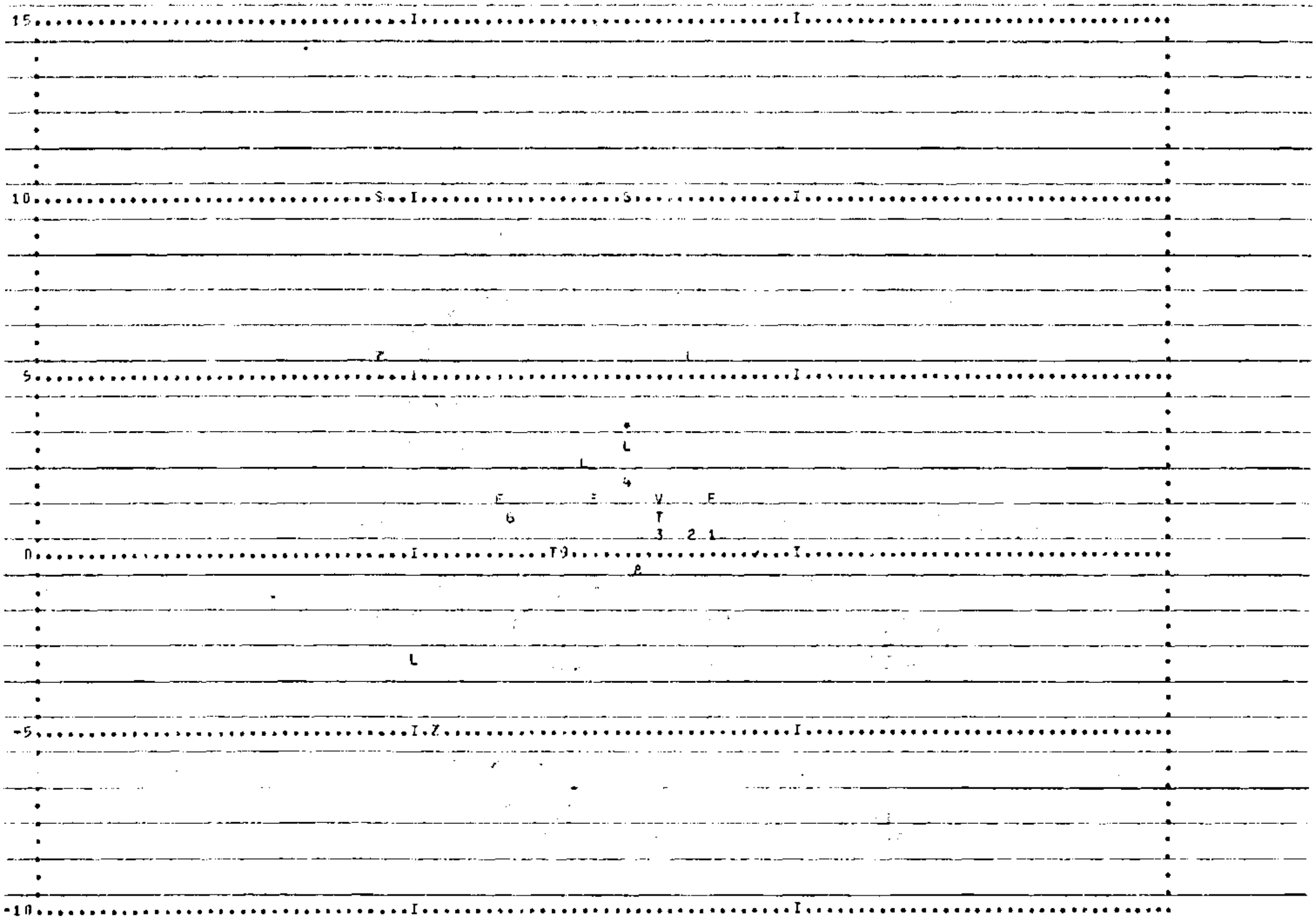


EPOCH = 1.50 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	3.24	.91	-.30	-.16	1	1.08	17.38	E
	2	4.17	1.06	-.80	-.43	1	-11.15	1.55	T
	3	2.53	.55	.04	.25	1	3.12	13.37	V
	4	1.15	2.11	.49	.44	1	3.19	16.93	L
	5	-1.24	1.72	.99	.81	1	1.09	15.62	L
	6	-4.75	1.17	-.39	.16	1	-10.00	10.75	E
	7	-11.74	4.77	3.66	4.33	0	13.93	9.89	S
	8	1.44	-1.16	1.29	1.45	0	.65	-1.40	
	9	-2.00	-.23	-1.97	-.55	1	-12.57	1.21	T
	*	1.80	2.02	-.16	7.05	0	-13.76	9.12	
	Z	-7.72	-5.16	-2.69	.04	0	-.03	.90	
DEFENSE									
	L	5.53	5.29	-5.93	.47	0	-13.33	3.57	
	E	5.22	1.69	-.05	-.28	1	-16.18	-.85	
	T	3.20	1.31	-.73	-.76	1	-13.29	9.47	
	L	1.25	2.98	.20	.30	1	-13.50	-.12	
	V	2.60	1.51	.35	.13	1	-11.79	7.74	
	S	.02	10.12	4.98	-.03	0	15.37	2.13	
	L	-1.33	2.53	.95	.33	1	4.10	.35	
	T	-2.57	.21	-1.33	-1.32	0	13.34	6.91	
	E	-5.24	1.77	-.35	-.03	1	15.73	3.06	
	L	-10.34	-2.85	4.09	-2.13	0	15.47	6.29	
	S	-11.93	9.64	4.25	3.26	0	15.97	4.49	

Figure 5-7g. Counter-Dive vs. Split-Six Defense (Simulation Results)

EPOCH = 1.75 SECONDS

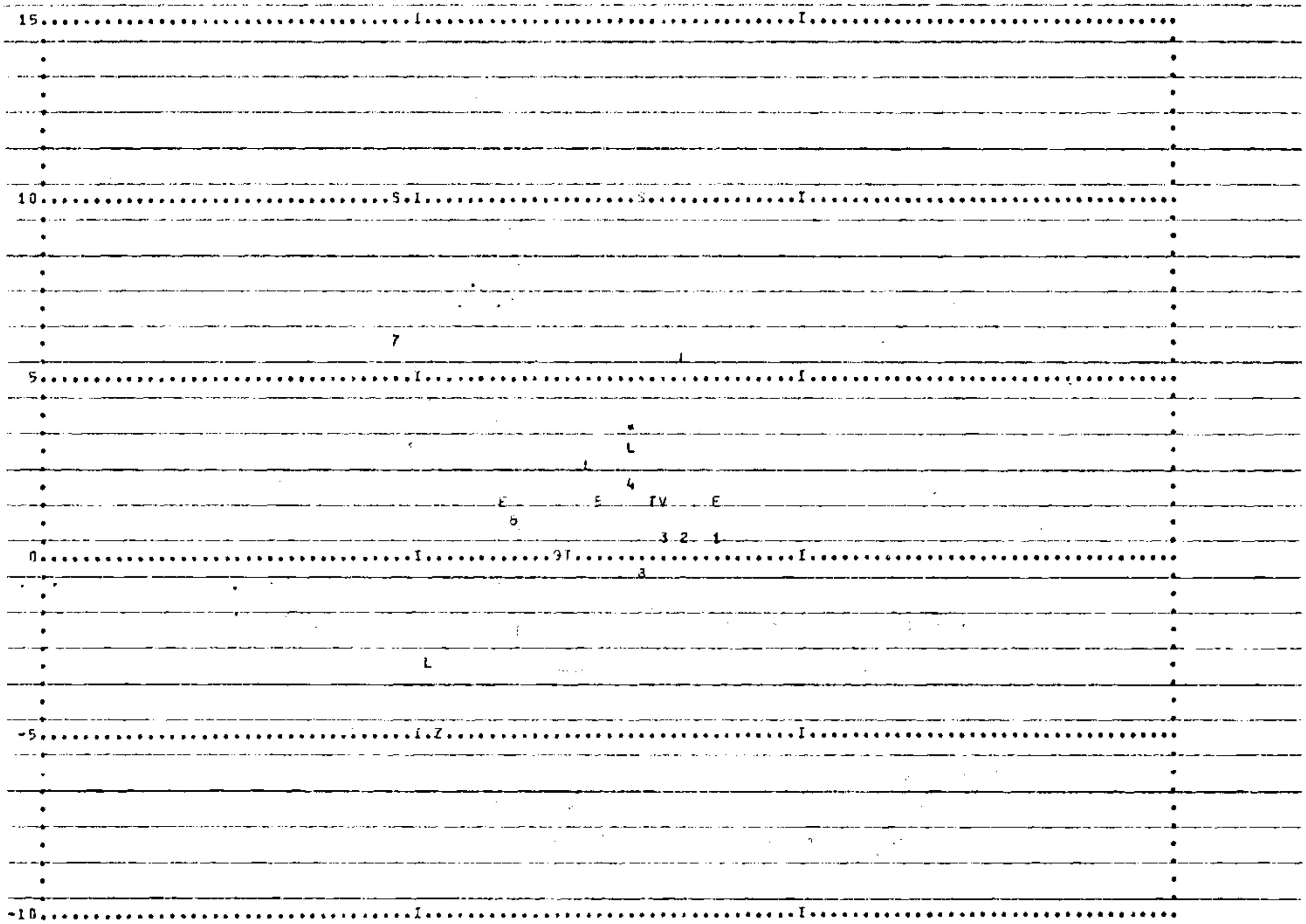


EPOCH = 1.75 SECONDS

PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE								
1	5.22	.91	-.37	-.00	1	-.28	14.01	E
2	4.02	.98	-.44	-.24	0	-13.13	6.47	T
3	2.52	.63	-.46	.57	1	.80	15.06	V
4	1.26	2.25	-.06	.72	1	2.66	15.92	L
5	-.99	1.97	1.06	1.13	1	2.70	15.85	L
6	-4.94	1.20	-.05	.15	1	-12.79	2.70	E
7	-10.66	5.63	4.70	4.20	0	9.45	1.10	S
8	1.70	-.93	.82	.52	0	-.19	.56	
9	-2.45	-.34	-1.97	-.55	1	6.09	10.67	T
*	1.48	3.98	-2.66	5.53	0	-9.38	14.24	
Z	-6.22	-5.13	-1.47	.19	0	-2.03	.76	
DEFENSE								
L	4.08	5.53	-5.75	1.09	0	-13.93	3.17	
E	5.18	1.67	-.00	-.02	1	-15.07	9.19	
T	2.62	1.43	-3.16	1.44	0	-9.24	12.72	
L	1.28	3.12	.49	.71	1	6.56	14.30	
V	2.54	1.61	-.01	.29	1	-8.31	14.78	
S	1.34	10.17	5.57	.36	0	13.09	-9.21	
L	-1.08	2.77	1.06	1.23	1	14.79	5.82	
T	-2.57	.14	1.54	.57	0	10.21	5.34	
E	-5.29	1.84	-.43	-.01	1	12.67	5.02	
L	-4.16	-3.09	5.11	.02	0	12.93	4.39	
S	-10.72	10.37	3.28	2.61	0	13.99	3.21	

Figure 5-7h. Counter-Dive vs. Split-Six Defense (Simulation Results)

EPOCH = 1.80 SECONDS



EPOCH = 1.80 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	5.22	.91	0.00	0.00	2	-.28	14.01	E
	2	3.96	.58	-1.00	.10	0	-13.13	6.47	T
	3	2.52	.63	0.00	0.00	2	.80	15.06	V
	4	1.26	2.25	0.00	0.00	2	2.66	15.92	L
	5	-.99	1.57	0.00	0.00	2	2.70	15.85	L
	5	-4.52	1.20	-.19	.30	1	-12.79	2.70	E
	7	-10.43	6.93	4.60	3.77	0	9.45	1.10	S
	8	1.74	-.90	.72	.49	0	-.15	.53	
	9	-2.53	-.37	-1.95	-.32	1	6.09	10.67	T
	*	1.34	3.65	-2.80	3.56	3	-9.38	14.24	
	Z	-8.29	-5.12	-1.39	.20	0	-2.03	.76	
DEFENSE									
	L	3.30	5.59	-5.74	1.11	0	-13.95	3.17	
	E	5.16	1.88	-.71	.42	0	-10.07	9.19	
	T	2.47	1.56	-3.18	1.37	0	-9.24	12.72	
	L	1.31	3.18	.72	1.30	3	6.36	14.30	
	V	2.53	1.63	-.60	.68	0	-6.31	10.70	
	S	1.62	10.18	5.59	-.09	0	13.69	-9.21	
	L	-1.21	2.24	1.63	1.40	0	14.75	5.92	
	T	-2.49	.18	1.65	.90	0	10.21	8.34	
	E	-5.30	1.64	-.24	-.18	1	12.17	5.02	
	L	-8.92	-3.08	5.13	.25	0	12.93	4.99	
	S	-10.45	10.20	5.52	2.46	0	15.94	3.21	

Figure 5-7i. Counter-Dive vs. Split-Six Defense (Simulation Results)

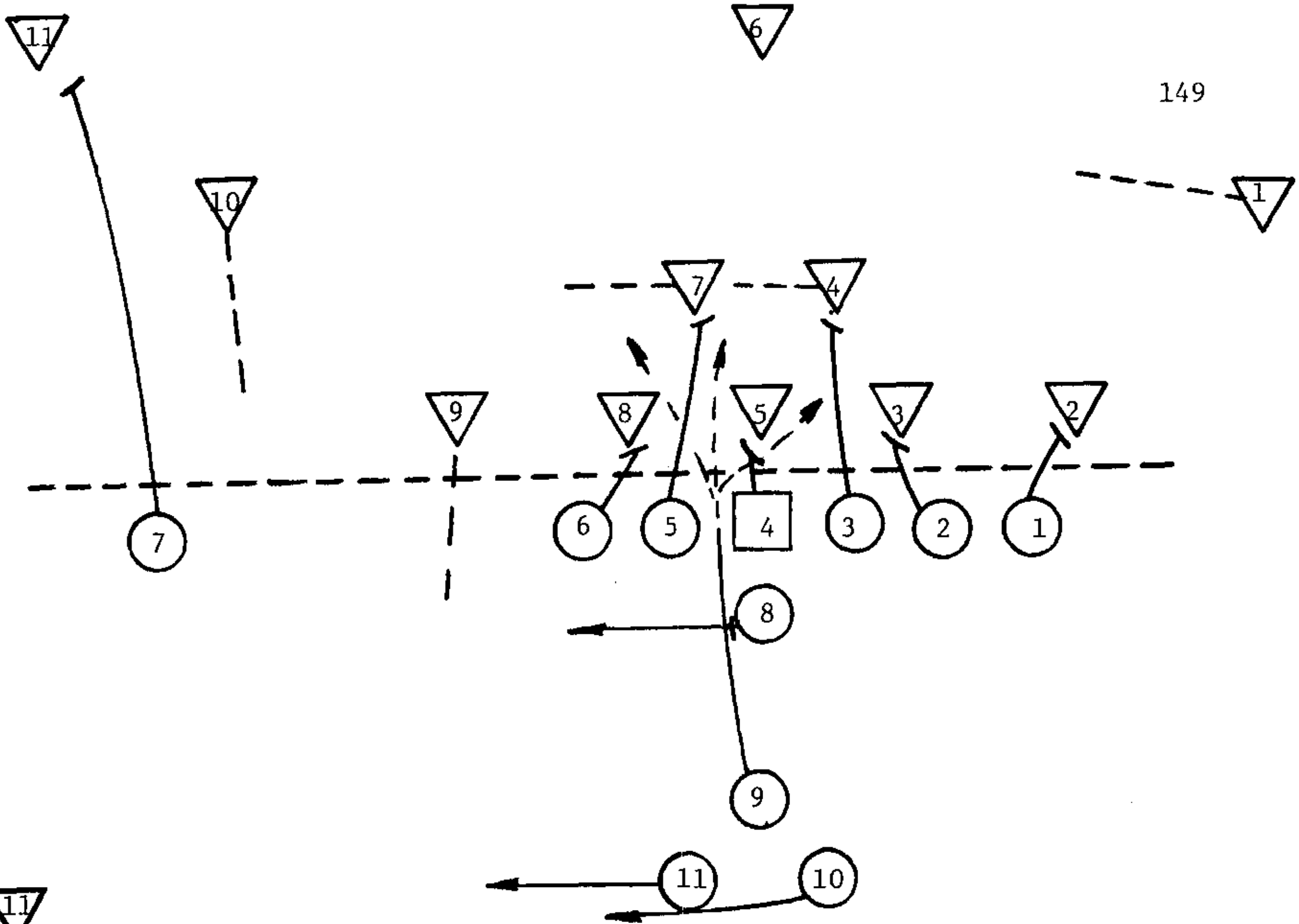


Figure 5-8. Predetermined Fullback vs. Oklahoma-50

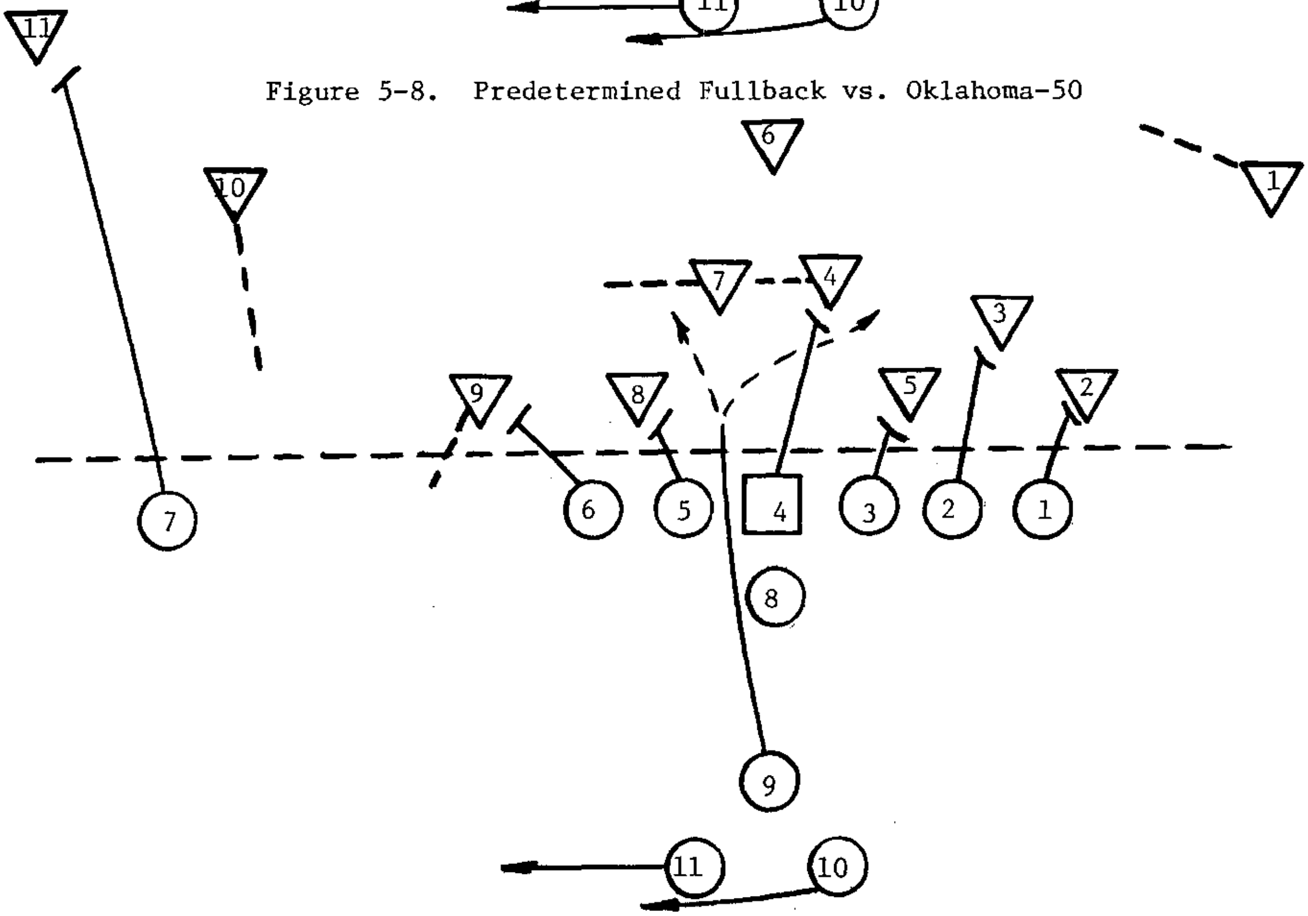


Figure 5-9. Predetermined Fullback vs. Split-Six

The play against the 50 is fairly straight-forward. After receiving the handoff, the fullback runs to daylight, either behind the center's block, or upfield on his own. (This corresponds to allowing the fullback to choose his own C^E according to the methods of Chapter Three, rather than constraining the motion.)

The problem comes in the Predetermined Fullback Play against the Split-Six Defense. There is no way to block both linebackers and players 5 and 8 as well. Players 5 and 8 are critical; without these blocks the play is stopped along the line of scrimmage. Thus only one linebacker can be blocked, and the choice is for the center to block 4. What saves the play is that the initial motion of the play looks like the triple option--a play for which player has outside responsibility. Thus the linebackers' first reaction is to move with the flow of the play. Thus he puts himself, for an instant, out of position, and it is up to the fullback to take advantage of this.

5.4.1 Results of Predetermined Fullback vs. 50 Defense

The results for the twenty-five runs of the play are listed in Table 5-3.

Table 5-3. Results of Predetermined Fullback vs. Oklahoma-50

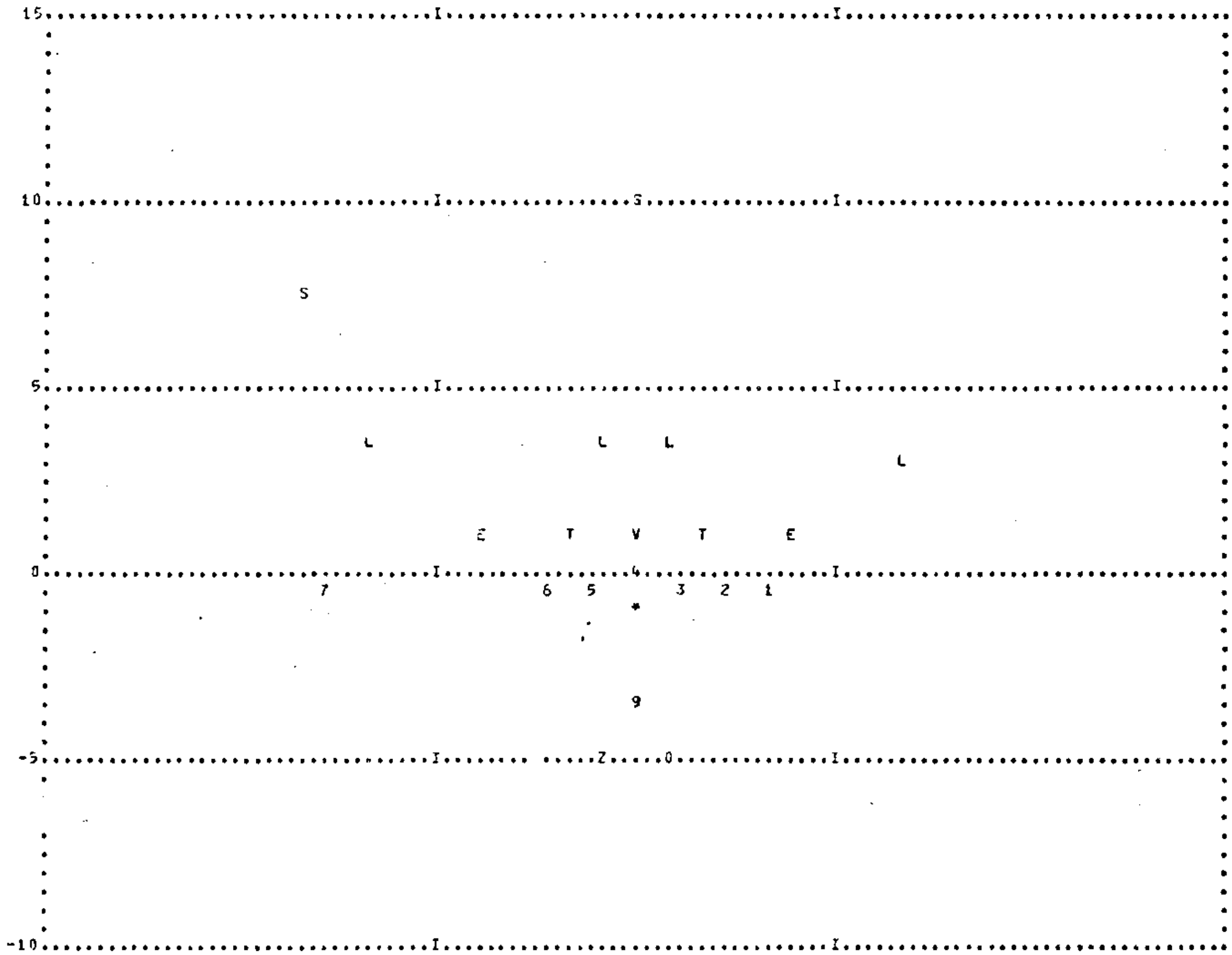
Run	Gain	Tackled By
1	2.63	5
2	1.95	8
3	2.65	5
4	1.28	8,9
5	50.00	-
6	5.57	8,9
7	1.74	5,8
8	3.32	5
9	3.39	5
10	5.28	5,9
11	2.30	5
12	4.23	7
13	50.00	-
14	1.28	8,9
15	5.28	5,9
16	2.26	8
17	6.86	1,5,7
18	3.53	8
19	4.14	7
20	4.87	8
21	2.00	8,9
22	2.74	8
23	0.77	3
24	13.73	1
25	3.36	4,5,9

The average gain, not including the two touchdowns, is 3.70. With the touchdowns, the average gain is 7.41 yards. A sample of the output is listed in Figure 5-10a through 5-10h.

5.4.2 Results of Predetermined Fullback vs. Split-Six

The results for the twenty-five runs of the play are listed in Table 5-4.

EPOCH = 0.00 SECONDS

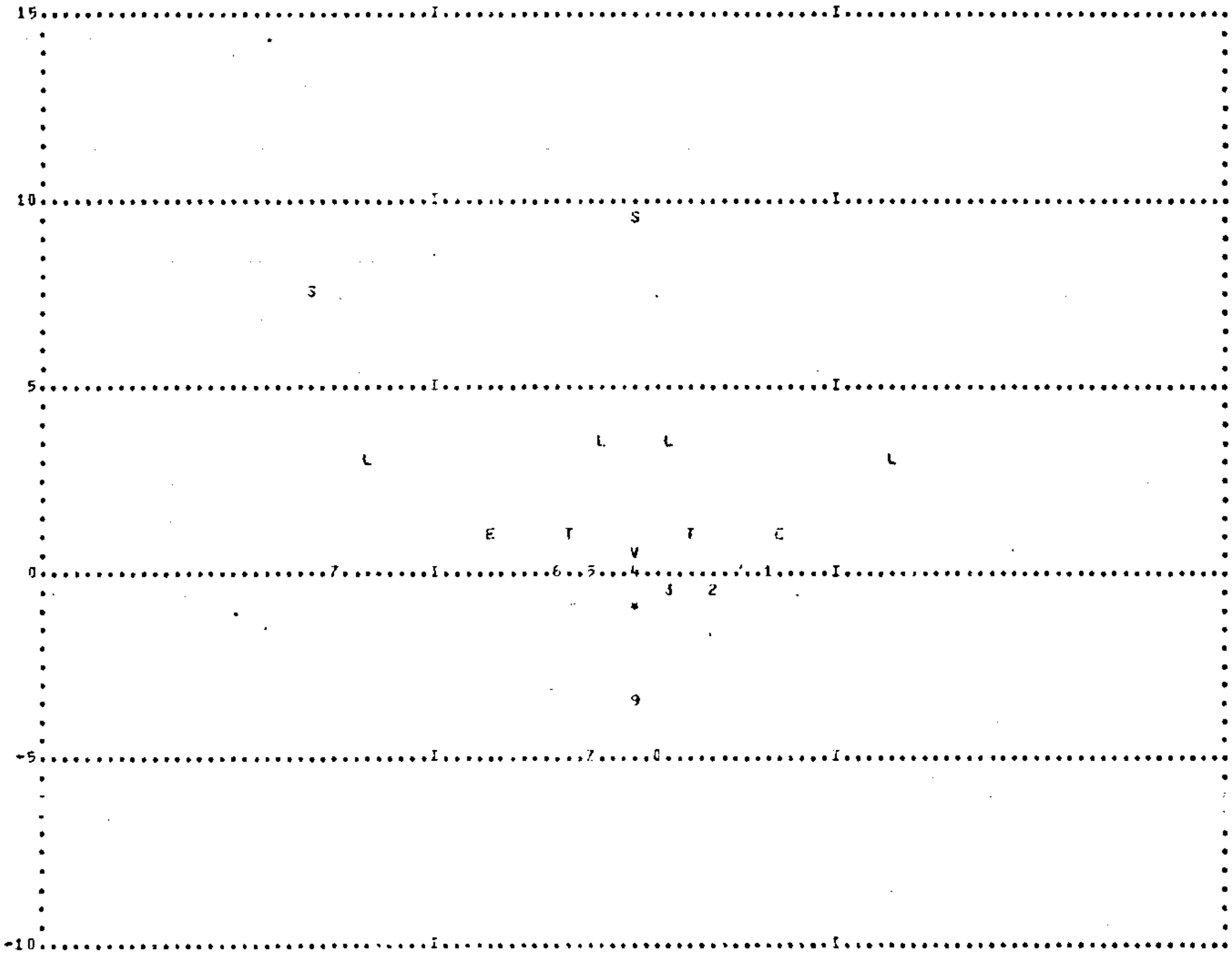


EPOCH = 0.00 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	6.00	-.80	0.00	0.00	0	2.47	15.75	E
	2	4.00	-.80	0.00	0.00	0	-13.45	1.84	T
	3	2.00	-.80	0.00	0.00	0	-9.10	10.62	L
	4	0.00	-.40	0.00	0.00	0	-1.31	16.22	V
	5	-2.00	-.80	0.00	0.00	0	-.20	14.23	L
	6	-4.00	-.80	0.00	0.00	0	7.96	13.19	T
	7	-14.00	-.50	0.00	0.00	0	.49	17.73	S
	*	0.00	-1.30	0.00	0.00	0	6.69	-5.49	
	9	0.00	-3.90	0.00	0.00	0	-4.75	13.36	
	0	1.80	-5.30	0.00	0.00	0	-22.20	-.29	
	Z	-1.80	-5.30	0.00	0.00	0	-17.59	-.47	
DEFENSE									
	L	12.00	3.00	0.00	0.00	0	-15.05	.75	
	E	7.00	1.40	0.00	0.00	0	-.55	-.30	
	T	3.00	1.40	0.00	0.00	0	-.01	1.07	
	L	1.50	3.50	0.00	0.00	0	1.57	2.46	
	V	0.00	1.00	0.00	0.00	0	.03	-.25	
	S	0.00	10.00	0.00	0.00	0	-2.19	-1.30	
	L	-1.50	3.50	0.00	0.00	0	-1.12	3.29	
	T	-3.00	1.40	0.00	0.00	0	-.07	.28	
	E	-7.00	1.40	0.00	0.00	0	1.02	-.66	
	L	-12.00	3.50	0.00	0.00	0	-.54	-17.44	
	S	-15.00	7.50	0.00	0.00	0	.32	.06	

Figure 5-10a. Predetermined Fullback vs. 50 Defense (Simulation Results)

EPOCH = .25 SECONDS

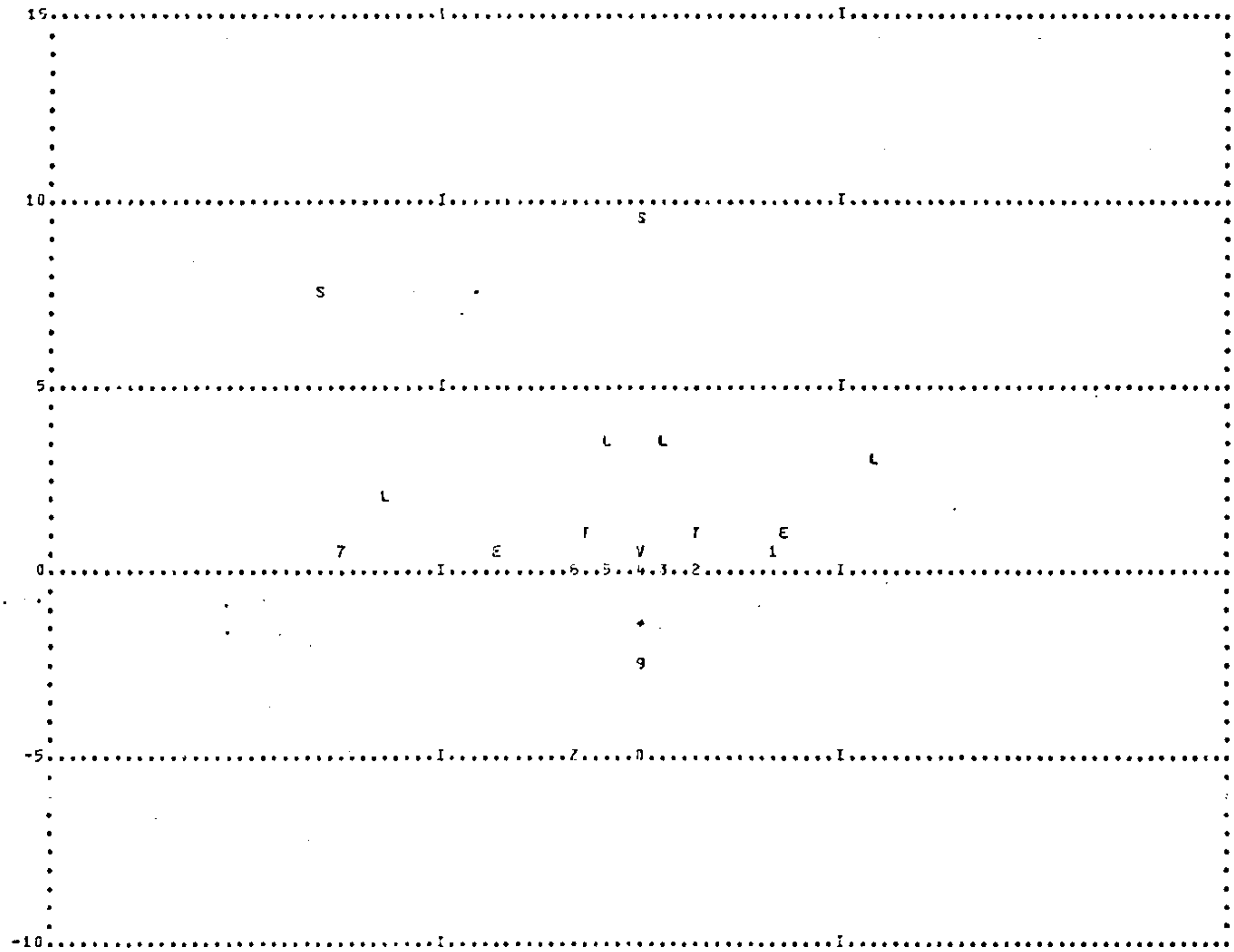


EPOCH = .25 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	6.06	-.39	.46	2.34	0	4.83	14.99	E
	2	3.65	-.75	-2.51	.34	0	-8.00	13.11	T
	3	1.77	-.53	-1.70	1.99	0	-8.49	11.13	L
	4	-.03	.02	0.00	0.00	2	.17	17.38	V
	5	-2.01	-.43	-.04	2.66	0	4.06	13.19	L
	6	-3.80	-.46	1.49	2.47	0	7.58	9.99	T
	7	-13.99	-.34	.09	3.32	0	1.03	15.27	S
	*	.17	-1.44	1.25	-1.03	0	.34	2.33	
	9	-.12	-3.56	-.89	2.50	0	-4.06	15.58	
	0	1.23	-5.31	-4.15	-.05	0	-16.82	-2.24	
	Z	-2.25	-5.31	-3.29	-.09	0	-17.88	.55	
DEFENSE									
	L	11.61	3.02	-2.01	.14	0	-13.49	6.09	
	E	6.99	1.39	-.10	-.06	0	-17.92	-.57	
	T	3.00	1.43	-.98	.20	0	-2.58	-11.53	
	L	1.94	3.58	.29	.45	0	-10.42	.35	
	V	.00	.99	.01	-.05	0	11.25	-3.16	
	S	-.06	9.97	-.41	-.24	0	-.04	-1.56	
	L	-1.53	3.58	-.21	.62	0	-3.71	1.54	
	T	-3.02	1.41	-.13	.05	0	8.15	-14.31	
	E	-6.97	1.38	.19	-.12	0	5.56	-14.44	
	L	-12.01	3.05	-.10	-3.26	0	5.31	-11.03	
	S	-14.98	7.50	.17	.31	0	1.71	.69	

Figure 5-10b. Predetermined Fullback vs. 50 Defense (Simulation Results)

EPOCH = .50 SECONDS

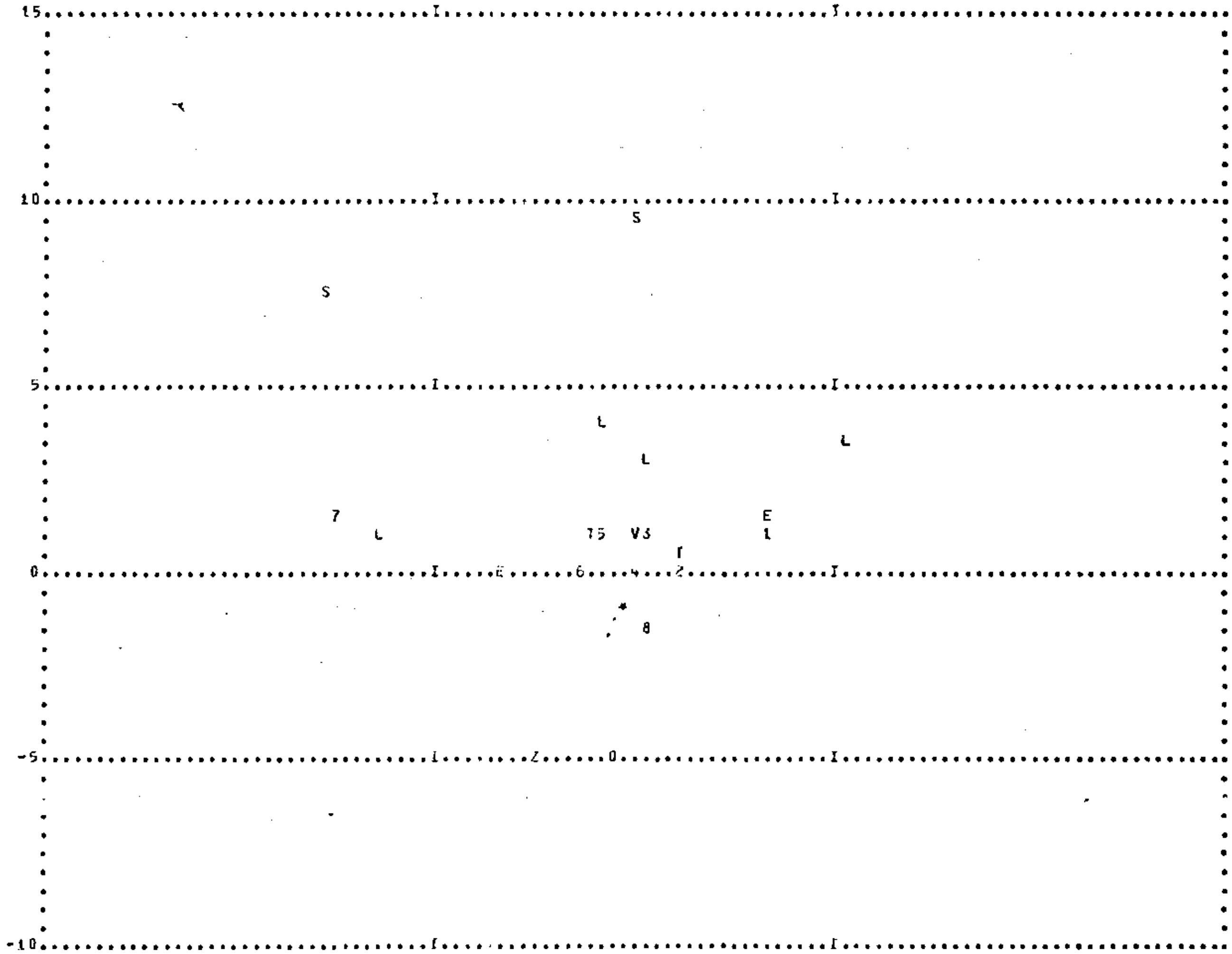


EPOCH = .50 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	6.27	.54	1.15	4.40	1	-4.81	15.12	E
	2	2.98	-.35	-2.96	2.64	0	-7.35	12.21	T
	3	1.23	.13	-2.51	3.16	0	-9.69	12.62	L
	4	.01	.03	.60	.18	1	.34	16.62	V
	5	-1.91	.40	.74	3.91	0	2.21	13.90	L
	6	-3.32	.26	2.22	3.20	1	12.65	9.07	T
	7	-13.94	.67	.24	4.55	0	-.52	16.97	S
	*	.41	-1.57	.74	-.12	0	-.01	.09	
	9	-.39	-2.69	-1.24	4.27	0	-5.22	16.27	
	0	.02	-5.38	-5.39	-.45	0	-16.11	1.73	
	Z	-3.33	-5.31	-5.13	.95	0	-18.29	-.58	
DEFENSE									
	L	10.74	3.20	-4.05	1.21	0	-12.42	8.66	
	E	6.51	1.37	-3.41	-.14	1	-15.70	.65	
	T	2.93	1.17	-.49	-2.05	0	-15.20	-.99	
	L	1.23	3.05	-2.07	.29	0	-.30	-9.45	
	V	.11	.99	.20	.40	1	-1.29	-12.21	
	S	-.13	9.88	-.23	-.42	0	8.03	-.96	
	L	-1.66	3.74	-.81	.62	0	-3.87	6.11	
	T	-2.83	1.05	1.46	-2.65	1	-9.47	-10.55	
	E	-6.79	.99	1.15	-2.77	0	9.28	-12.35	
	L	-11.90	2.16	.94	-3.43	0	3.40	-11.02	
	S	-14.90	7.22	.41	.13	0	14.20	2.62	

Figure 5-10c. Predetermined Fullback vs. 50 Defense (Simulation Results)

EPOCH = .75 SECONDS

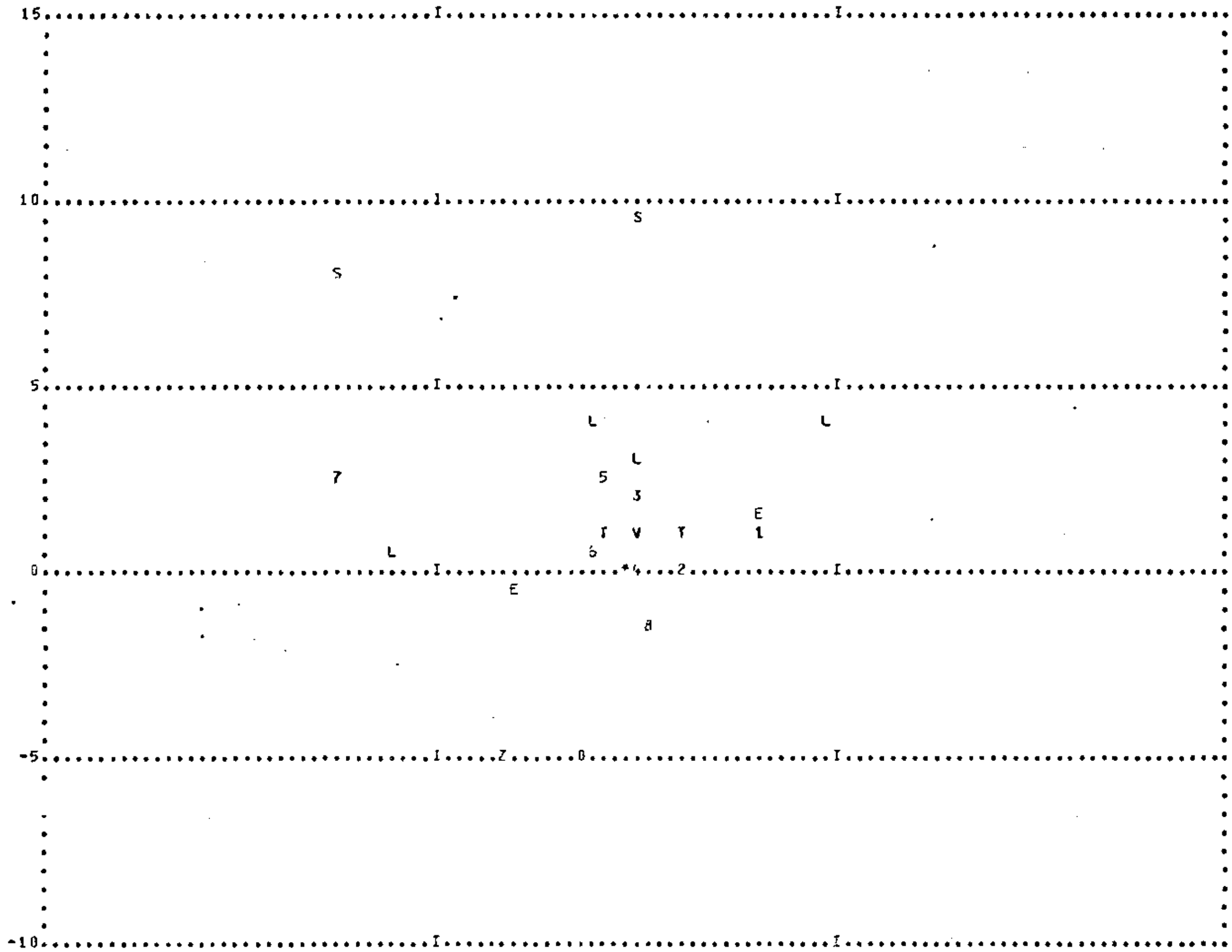


EPOCH = .75 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	6.05	1.10	-1.93	2.40	1	7.99	-11.09	E
	2	2.26	.19	-2.60	.95	1	11.13	1.99	T
	3	.51	1.05	-3.17	4.07	0	1.40	14.91	L
	4	.00	.04	.01	.13	1	3.75	15.04	V
	5	-1.71	1.49	.81	4.72	0	1.85	10.16	L
	6	-2.89	.36	1.97	.38	1	11.02	12.49	T
	7	-13.91	1.98	.03	5.59	0	7.18	-5.33	S
	8	.55	-1.59	.40	-.05	0	2.04	.42	
	*	-.76	-1.47	-1.65	5.35	0	16.89	-.49	
	0	-1.40	-5.41	-5.95	.08	0	-16.27	2.37	
	Z	-4.76	-5.32	-5.20	-.08	0	-17.01	.45	
DEFENSE									
	L	9.66	3.65	-4.52	2.28	0	-11.59	9.55	
	E	6.09	1.96	-1.02	-.81	1	-.07	14.46	
	T	2.47	.09	-2.07	-2.38	1	-.63	10.75	
	L	.89	3.37	-1.06	-1.37	0	-13.30	1.30	
	V	.11	1.00	-.05	.15	1	-8.21	-6.57	
	S	.03	9.78	1.38	-.41	0	2.68	-1.70	
	L	-1.91	4.01	-1.16	1.48	0	.37	-.54	
	T	-2.33	1.10	1.78	1.74	1	16.37	-2.06	
	E	-6.34	.15	2.56	-3.81	0	14.34	.98	
	L	-11.63	1.16	1.14	-4.14	0	13.12	7.01	
	S	-14.46	7.61	2.88	.56	0	6.41	11.39	

Figure 5-10d. Predetermined Fullback vs. 50 Defense (Simulation Results)

EPOCH = 1.00 SECONDS

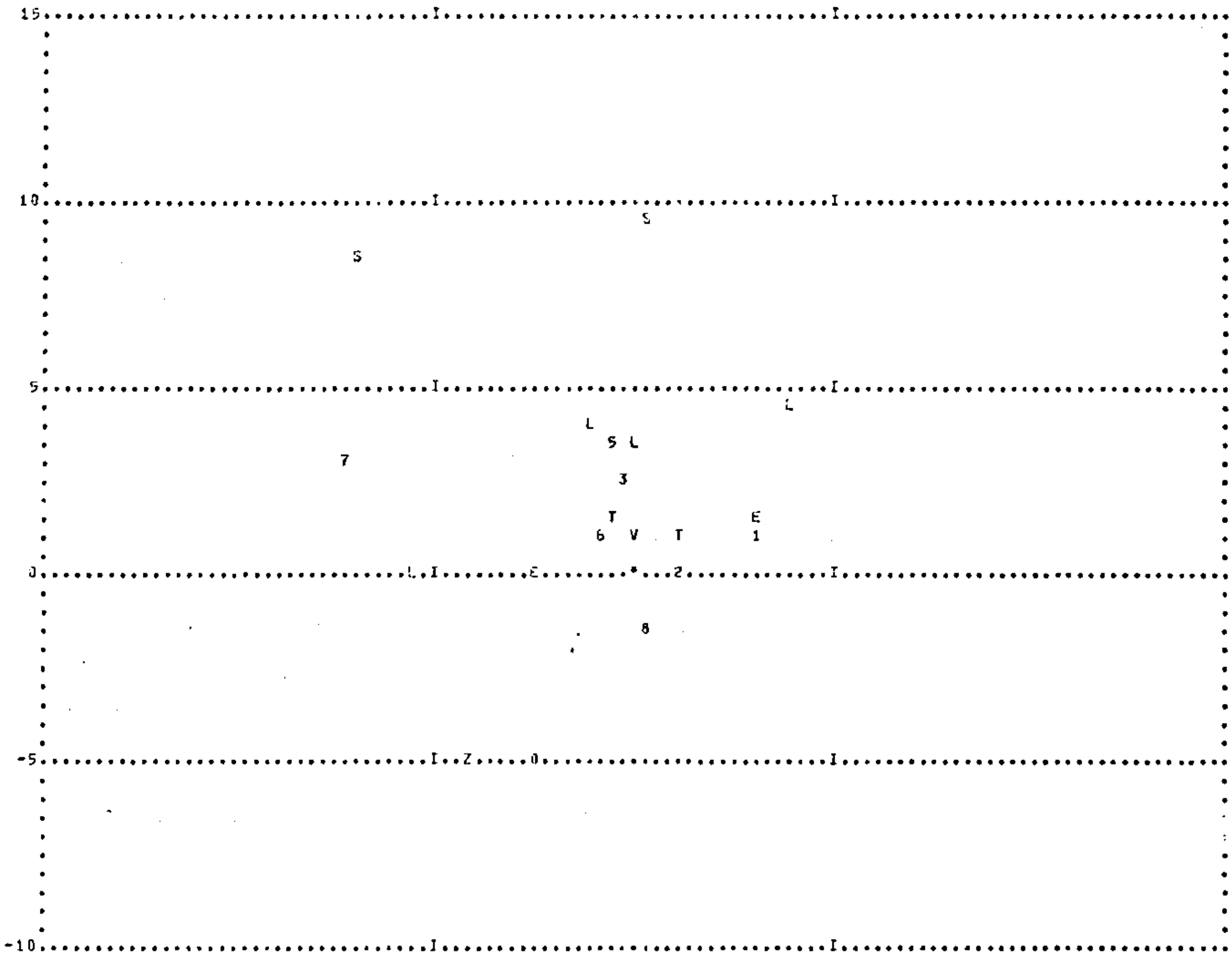


EPOCH = 1.00 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	6.00	1.10	-.54	.07	1	15.61	.35	E
	2	2.26	.20	-.02	.06	1	4.87	13.21	T
	3	-.05	2.19	-1.46	4.99	1	-11.32	-8.44	L
	4	.02	.09	-.22	.30	1	8.72	12.68	V
	5	-1.51	2.63	.79	4.46	0	1.25	10.05	L
	6	-2.38	.64	2.33	1.23	1	13.63	12.33	T
	7	-13.72	2.90	1.34	2.09	0	7.21	1.70	S
	8	.68	-1.59	.60	.05	0	.81	2.29	
	9	-.63	-.48	2.26	2.81	0	-.46	14.70	
	0	-2.93	-5.32	-6.26	.60	0	-18.36	-.74	
	Z	-6.35	-5.32	-6.54	.04	0	-15.00	-2.51	
DEFENSE									
	L	8.92	4.36	-4.63	3.12	0	-15.89	2.07	
	E	5.93	1.94	-.15	-.39	1	-7.93	15.90	
	T	2.47	1.00	.13	.09	1	-1.01	14.09	
	L	.32	3.06	-3.15	-.77	1	-12.55	-11.31	
	V	.08	1.06	.10	.18	1	4.50	4.62	
	S	.36	9.66	1.29	-.54	0	-1.81	1.72	
	L	-2.12	4.26	-.56	.70	0	7.11	-12.36	
	T	-1.81	1.38	2.19	1.36	1	13.37	.48	
	[illegible]	-5.53	-.53	3.56	-1.66	0	2.49	18.06	
	[illegible]	-11.06	.57	3.07	-.33	0	16.31	2.89	
	S	-13.70	8.51	3.13	2.43	0	4.99	14.74	

Figure 5-10e. Predetermined Fullback vs. 50 Defense (Simulation Results)

EPOCH = 1.25 SECONDS

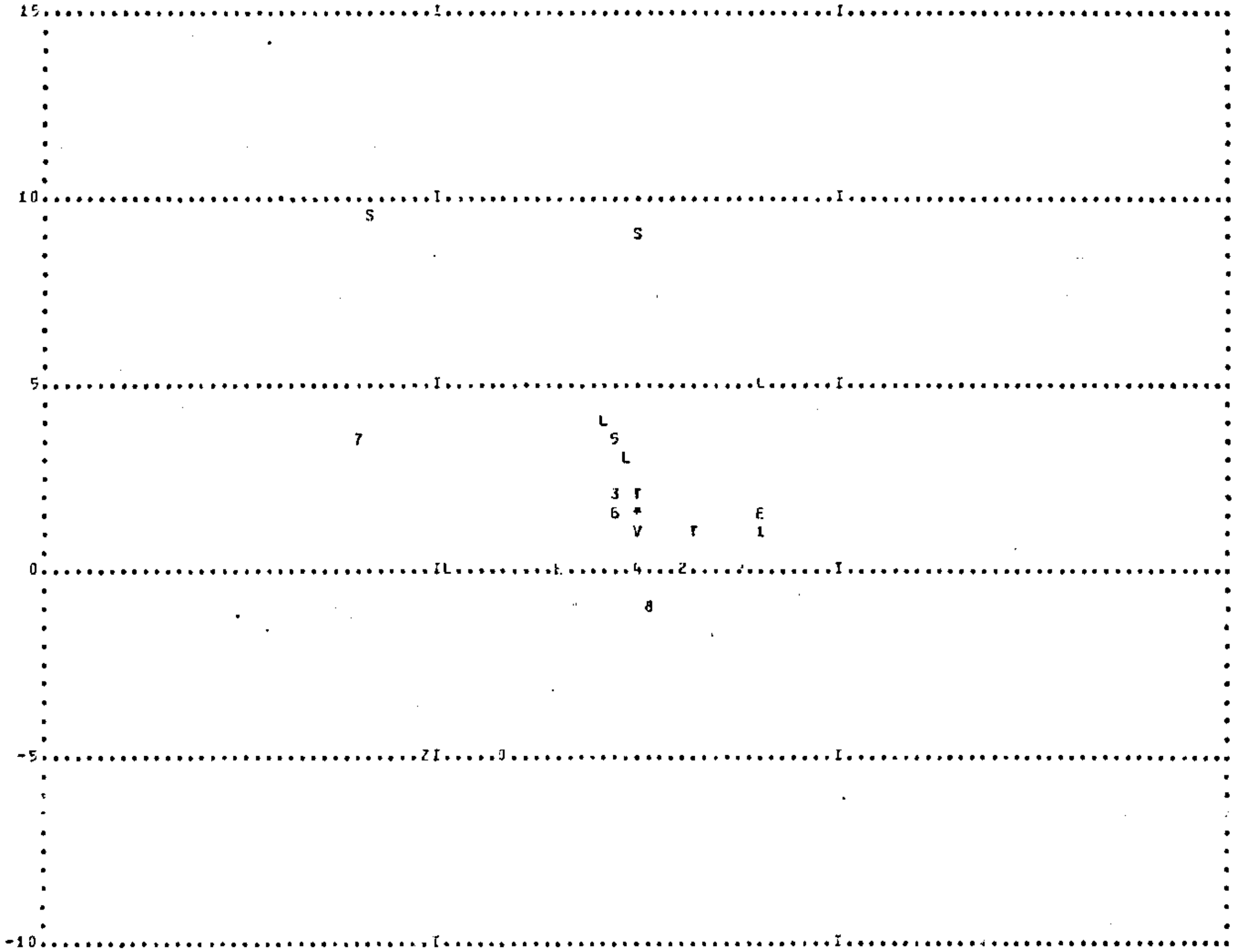


EPOCH = 1.25 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	5.96	1.11	-.58	.20	1	5.55	13.10	E
	2	2.28	.23	-.03	.30	1	3.25	12.00	T
	3	-.70	2.65	-2.73	1.65	1	-10.92	-8.61	L
	4	.04	.19	.12	.98	1	8.39	13.14	V
	5	-1.32	3.61	.45	1.92	1	1.51	10.47	L
	6	-1.76	1.06	2.68	1.79	1	10.70	7.61	T
	7	-13.29	3.34	2.03	1.45	0	4.33	7.48	S
	8	.61	-1.52	.48	.46	0	1.17	2.02	
	*	-.22	.42	1.14	4.27	0	-1.46	14.11	
	0	-4.57	-5.23	-6.82	.19	0	-17.29	1.33	
	Z	-7.96	-5.38	-6.35	-.45	0	-14.66	-.78	
DEFENSE									
	L	7.24	5.00	-5.48	2.08	0	-14.28	1.49	
	E	5.82	1.93	.11	-.10	1	-17.74	4.60	
	T	2.46	1.04	.11	.33	1	12.16	-2.16	
	L	-.31	3.50	-2.84	-2.73	1	3.62	-13.54	
	V	.09	1.16	.26	.28	1	15.76	1.43	
	S	.34	9.60	.34	.03	0	-7.13	-11.72	
	L	-2.07	4.24	1.07	.35	1	9.50	-11.87	
	T	-1.29	1.60	2.60	2.49	1	14.20	-10.54	
	E	-4.73	-.42	2.61	2.36	0	14.92	1.48	
	L	-10.09	.47	4.71	.04	0	12.33	.98	
	S	-12.99	8.85	2.63	4.07	0	14.91	.69	

Figure 5-10f. Predetermined Fullback vs. 50 Defense (Simulation Results)

EPOCH = 1.50 SECONDS

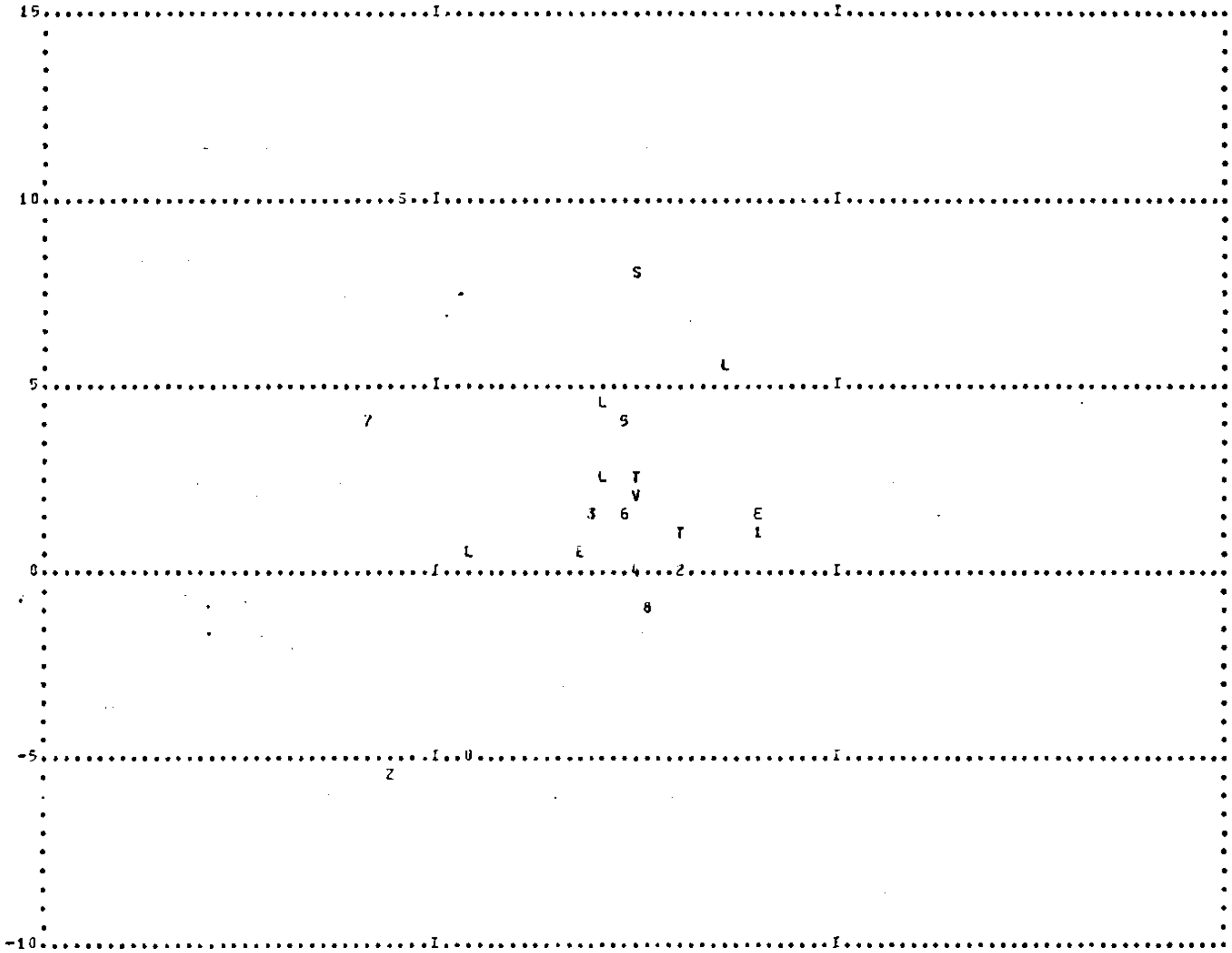


EPOCH = 1.50 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	5.91	1.17	-.84	.46	1	-3.84	15.76	E
	2	2.29	.33	.45	.30	1	3.66	13.61	T
	3	-1.48	2.47	-2.58	-.91	1	-15.65	-5.21	L
	4	.12	.34	.70	.73	1	3.70	14.34	V
	5	-1.08	3.94	.75	1.44	1	-.30	10.25	L
	6	-1.97	1.98	3.11	2.13	1	12.92	4.00	T
	7	-12.79	3.66	1.93	2.18	0	12.49	8.16	S
	8	.93	-1.39	.48	.63	0	-1.21	1.07	
	*	-.04	1.58	.35	4.95	0	6.70	15.72	
	0	-6.29	-5.16	-6.93	.35	0	-15.55	.75	
	Z	-9.53	-5.48	-6.18	-.39	0	-16.71	.74	
DEFENSE									
	L	5.85	5.42	-5.64	1.41	0	-14.12	4.62	
	E	5.70	1.97	-.11	-.03	1	-1.42	-1.50	
	T	2.53	1.12	.09	.12	1	-15.58	5.04	
	L	-.96	3.26	-3.29	-3.42	1	15.45	-1.79	
	V	.20	1.31	.51	.48	1	-8.65	9.49	
	S	.42	9.30	-1.15	-2.18	0	-5.07	-16.38	
	L	-1.89	4.43	1.11	.48	1	15.82	2.22	
	T	-.49	2.31	2.86	2.49	1	-10.93	-9.35	
	E	-3.86	.06	4.20	1.55	0	12.11	5.84	
	L	-6.89	.49	4.66	.18	0	16.95	3.11	
	S	-12.11	9.63	4.21	2.34	0	16.73	2.09	

Figure 5-10g. Predetermined Fullback vs. 50 Defense (Simulation Results)

EPOCH = 1.75 SECONDS

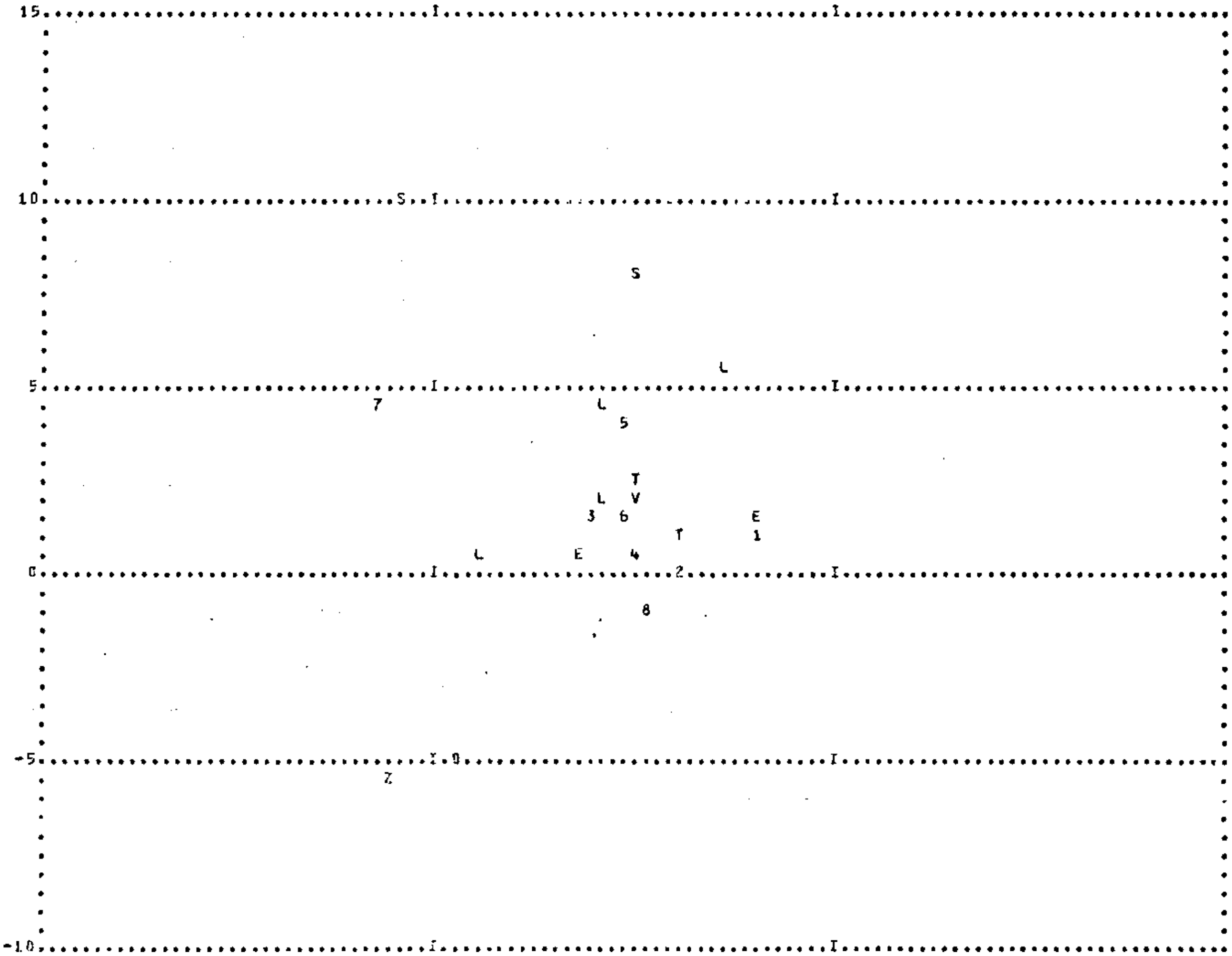


EPOCH = 1.75 SECONDS

	PLAYER	X	Y	XOUT	YOUT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	5.90	1.18	-.20	.16	1	-5.18	16.15	E
	2	2.24	.38	-.75	.33	1	5.20	12.66	T
	3	-2.26	1.96	-2.62	-1.96	1	-9.12	-9.75	L
	4	.25	.48	.38	.40	0	-.05	14.38	V
	5	-.80	4.23	1.21	1.74	1	1.01	10.78	L
	6	-.64	1.65	1.16	.77	1	9.57	11.13	T
	7	-12.10	4.42	3.38	2.71	0	13.83	1.58	S
	8	.99	-1.24	.03	.34	0	-1.32	3.21	
	*	.14	2.64	-.29	3.38	3	5.22	14.38	
	0	-7.99	-5.08	-6.66	.33	0	-15.83	-.88	
	Z	-11.11	-5.54	-6.47	-.07	0	-16.59	2.84	
DEFENSE									
	L	4.43	5.82	-5.70	1.06	0	-17.46	-.36	
	E	5.66	1.98	-.10	.09	1	-16.16	1.59	
	T	2.40	1.20	-.22	-.08	1	-12.35	8.83	
	L	-1.64	2.65	-3.37	-3.43	1	-9.22	.42	
	V	.18	2.07	.82	.71	3	2.22	-12.70	
	S	.06	8.38	-1.03	-4.52	3	-3.92	-17.28	
	L	-1.63	4.75	1.24	1.13	1	8.46	-12.95	
	T	-.12	2.01	1.39	1.37	1	12.26	-.90	
	S	-2.75	.59	4.54	1.33	0	11.75	9.99	
	L	-7.54	.61	5.80	.38	0	15.49	3.53	
	S	-10.90	10.12	5.41	1.66	0	15.55	-.08	

Figure 5-10h. Predetermined Fullback vs. 50 Defense (Simulation Results)

EPOCH = 1.60 SECONDS



EPOCH = 1.60 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	5.90	1.18	0.00	0.00	2	-5.18	16.15	E
	2	2.24	.38	0.00	0.00	2	5.20	12.66	T
	3	-2.28	1.96	0.00	0.00	2	-8.12	-9.75	L
	4	.27	.51	.33	1.03	0	-.05	14.38	V
	5	-.74	4.29	1.22	1.26	1	1.01	10.78	L
	6	-.58	1.89	1.03	.69	0	0.00	0.00	T
	7	-11.93	4.55	3.64	2.47	0	13.83	1.53	S
	8	.99	-1.21	-.03	.63	0	-1.32	3.21	
	*	.16	2.74	.42	2.90	3	5.22	14.38	
	0	-8.32	-5.06	-6.64	.25	0	-15.83	-.88	
	Z	-11.44	-5.54	-6.51	.07	0	-16.59	2.84	
DEFENSE									
	L	4.14	5.89	-5.87	1.45	0	-17.46	-.36	
	E	5.53	1.98	-.85	.16	0	-16.16	1.59	
	T	2.37	1.21	-.77	.35	0	-12.35	8.88	
	L	-1.81	2.49	-3.42	-3.01	0	-9.22	.42	
	V	.21	2.17	.28	.54	3	2.22	-12.70	
	S	-.02	8.15	-1.63	-4.81	0	-3.92	-17.28	
	L	-1.57	4.51	1.31	1.16	1	8.46	-12.95	
	T	-.04	2.67	1.31	1.17	3	12.26	-.95	
	E	-2.53	.60	4.57	2.18	0	11.75	9.93	
	L	-7.25	.65	5.86	.77	0	15.49	3.53	
	S	-10.62	10.19	5.52	1.46	0	15.55	-.08	

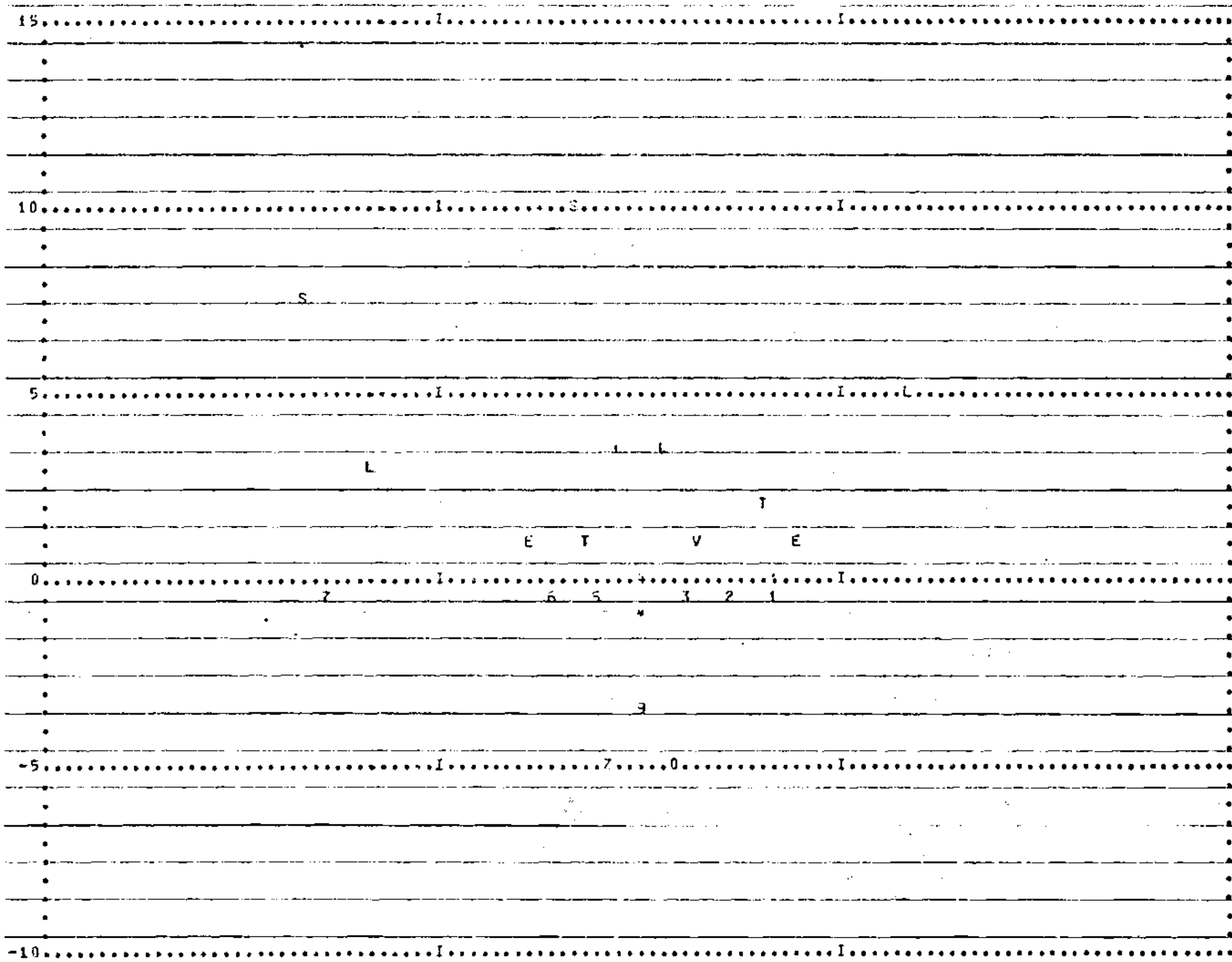
Figure 5-10i. Predetermined Fullback vs. 50 Defense (Simulation Results)

Table 5-4. Results of Predetermined Fullback vs. Split-Six

Run	Gain	Tackled By
1	2.21	8
2	3.14	7
3	2.11	8
4	2.00	7,8
5	2.93	4,7
6	3.82	7
7	2.41	7,9
8	3.76	7
9	3.25	7
10	6.29	4
11	2.67	7
12	6.93	1,6
13	2.89	7
14	3.22	4,7
15	2.93	7
16	2.27	7,8
17	50.00	-
18	3.72	4,7
19	2.19	7,8
20	3.37	4,7
21	2.01	7,8
22	5.27	4,7
23	1.89	7,8
24	2.18	8
25	2.08	7,9

The mean of the runs is 3.09 yards, not including the touchdown, and 4.97 including it. A sample play for the Predetermined Fullback versus the Split-Six Defense is given in Figure 5-11a through 5-11g.

EPOCH = 0.00 SECONDS

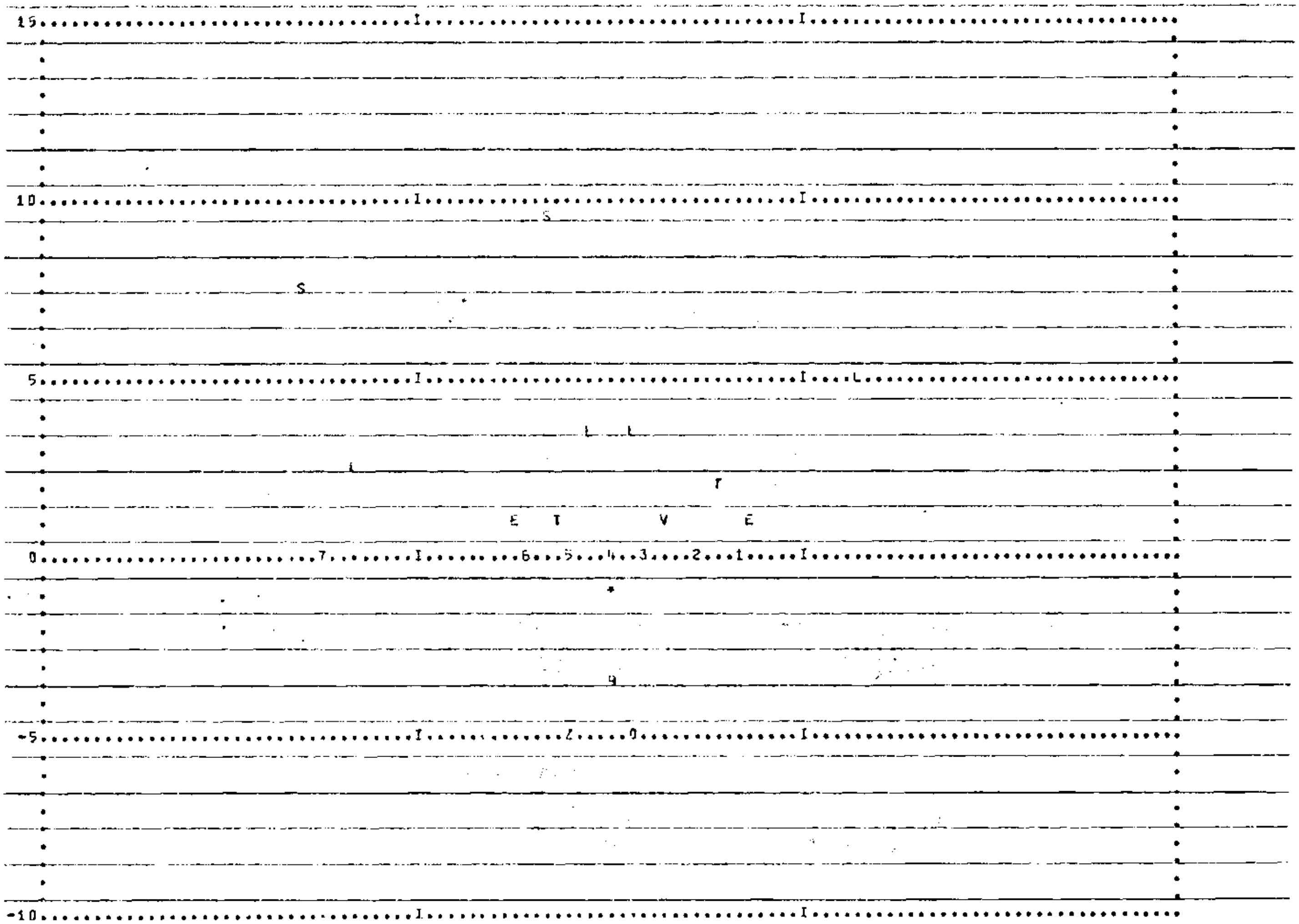


EPOCH = 0.00 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	6.00	-.60	0.00	0.00	0	2.47	15.75	E
	2	4.00	-.80	0.00	0.00	0	2.58	15.42	T
	3	2.00	-.80	0.00	0.00	0	-2.10	12.51	V
	4	0.00	-.60	0.00	0.00	0	-5.24	16.22	L
	5	-2.00	-.80	0.00	0.00	0	-5.10	13.97	T
	6	-4.00	-.60	0.00	0.00	0	-3.61	15.38	E
	7	-14.00	-.60	0.00	0.00	0	.49	17.73	S
	*	0.00	-1.30	0.00	0.00	0	6.69	-5.49	
	9	0.00	-3.90	0.00	0.00	0	-3.47	13.78	
	0	1.80	-5.30	0.00	0.00	0	-22.20	-.29	
	Z	-1.80	-5.30	0.00	0.00	0	-17.59	-.47	
DEFENSE									
	L	12.00	5.00	0.00	0.00	0	-16.23	.75	
	E	7.00	1.40	0.00	0.00	0	-.55	-.30	
	T	5.50	2.20	0.00	0.00	0	-.01	1.07	
	L	1.00	3.50	0.00	0.00	0	1.57	2.46	
	V	2.50	1.40	0.00	0.00	0	.03	-.29	
	S	-3.00	10.00	0.00	0.00	0	-2.18	-1.30	
	L	-1.00	3.50	0.00	0.00	0	-1.12	3.29	
	T	-2.50	1.40	0.00	0.00	0	-.07	.28	
	E	-5.00	1.40	0.00	0.00	0	1.02	-.66	
	L	-12.00	3.00	0.00	0.00	0	-.54	-17.44	
	S	-15.00	7.50	0.00	0.00	0	.32	.06	

Figure 5-11a. Predetermined Fullback vs. Split-Six Defense (Simulation Results)

EPOCH = .25 SECONDS

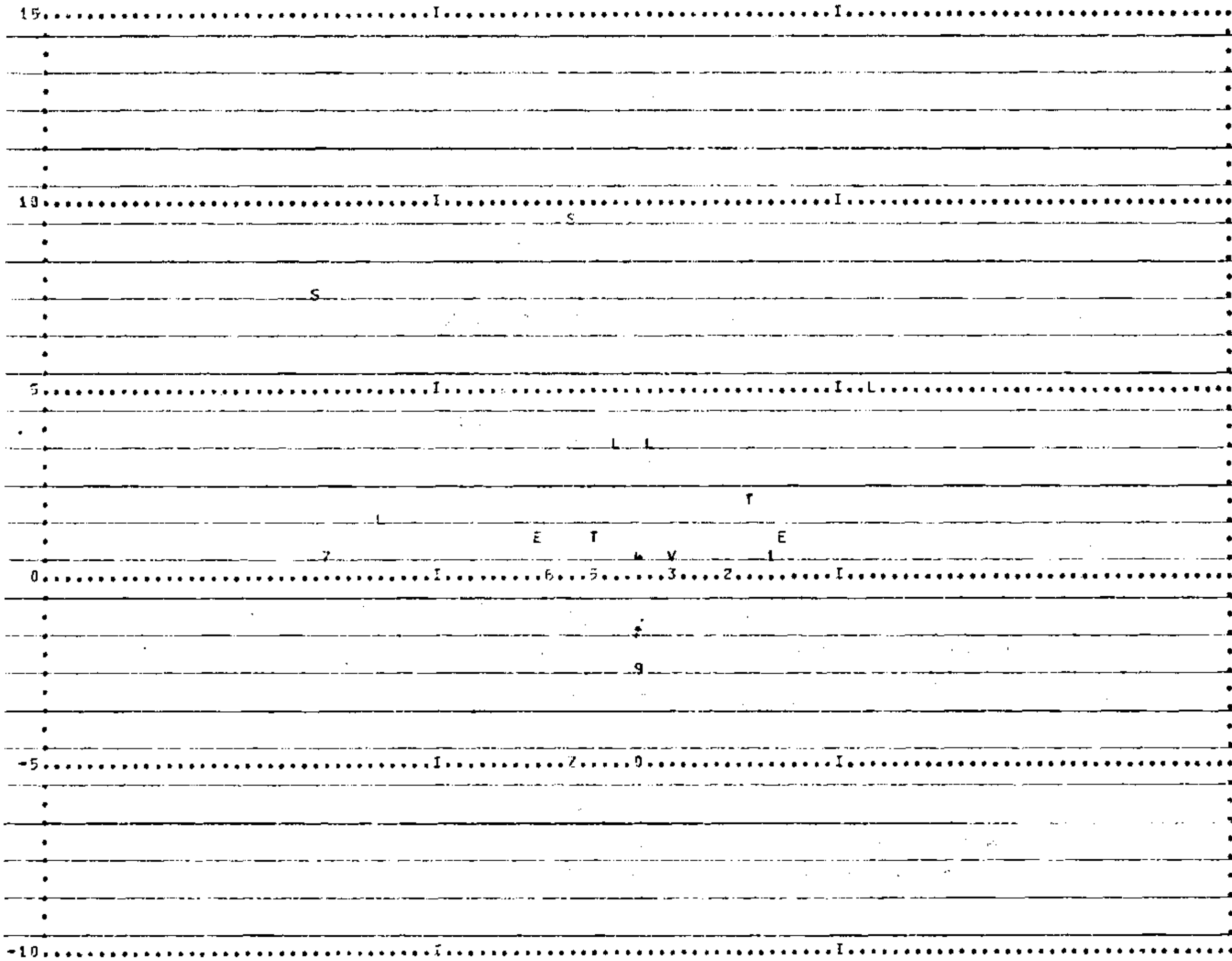


EPOCH = .25 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	6.06	-.39	.46	2.34	0	2.50	15.93	E
	2	4.07	-.40	.45	2.88	0	2.77	13.91	T
	3	1.95	-.46	-.39	2.34	0	2.55	12.10	V
	4	-.13	.02	-.98	3.03	0	-4.68	14.95	L
	5	-2.13	-.44	-.95	2.61	0	-.92	16.58	T
	6	-4.09	-.42	-.48	2.88	0	-4.32	13.67	E
	7	-13.59	-.34	.09	3.32	0	-2.18	16.95	S
	*	.17	-1.45	1.25	-1.03	0	-.65	1.97	
	9	-.09	-3.55	-.65	2.58	0	-3.67	16.69	
	0	1.23	-5.31	-4.15	-.95	0	-15.95	.00	
	Z	-2.25	-5.31	-3.29	-.99	0	-18.66	-1.46	
DEFENSE									
	L	11.58	5.92	-3.03	.14	0	-18.11	4.13	
	E	6.99	1.39	-.10	-.06	0	-17.87	-2.06	
	T	5.50	2.23	-.00	.20	0	-12.23	-7.11	
	L	1.04	3.56	.29	.46	0	-5.19	-3.00	
	V	2.50	1.39	.01	-.95	0	-10.55	-8.76	
	S	-3.06	9.97	-.41	-.24	0	.84	-.04	
	L	-1.03	3.53	-.21	.02	0	-7.89	1.44	
	T	-2.52	1.41	-.13	.05	0	5.10	-11.71	
	E	-4.97	1.38	.19	-.12	0	-.06	-5.05	
	L	-12.01	2.55	-.19	-3.26	0	5.19	-11.61	
	S	-14.53	7.59	.17	.01	0	3.36	1.00	

Figure 5-11b. Predetermined Fullback vs. Split-Six Defense (Simulation Results)

EPOCH = .50 SECONDS

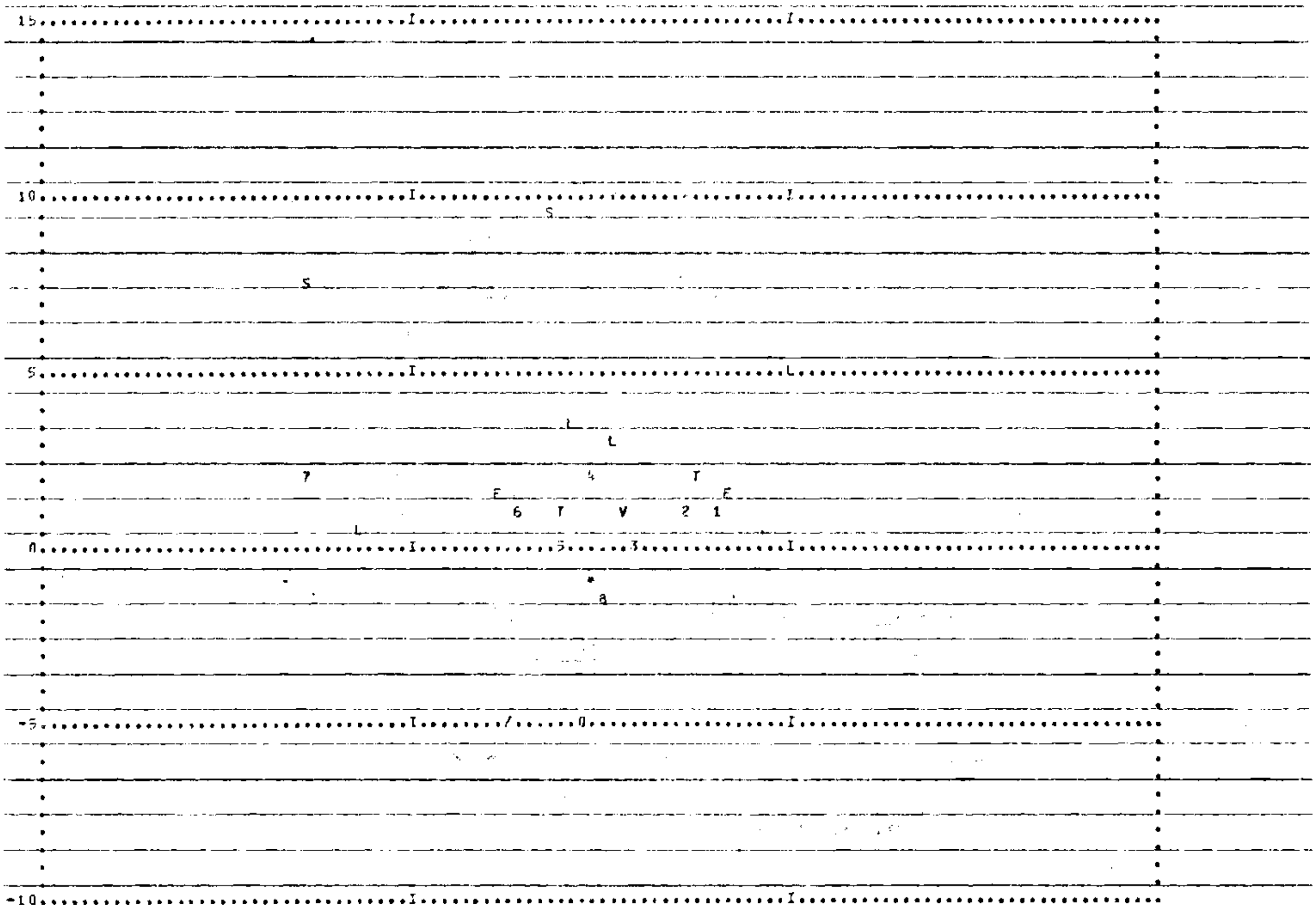


EPOCH = .50 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	6.21	.57	.72	-.57	1	-4.12	16.68	E
	2	4.23	.49	.78	-.15	0	-.52	14.62	T
	3	1.94	.27	.26	3.53	1	-11.66	11.27	V
	4	-.44	.97	-1.41	4.44	0	6.04	14.87	L
	5	-2.30	.31	.19	1.10	1	-1.03	13.78	T
	6	-4.33	.49	-1.17	4.12	1	-1.01	14.45	E
	7	-14.03	.71	-.36	4.97	0	-.52	16.97	S
	*	.39	-1.58	.56	-.19	0	-.01	.09	
	9	-.31	-2.53	-1.64	4.52	0	-3.97	16.69	
	0	.04	-5.32	-5.23	-.93	0	-16.11	1.73	
	Z	-3.35	-5.37	-5.27	-.32	0	-18.29	-.58	
DEFENSE									
	L	10.55	5.15	-5.05	.45	0	-14.61	4.22	
	E	6.51	1.23	-3.40	-.41	1	-15.51	1.77	
	T	5.18	2.08	-2.23	-1.22	0	-15.70	-.99	
	L	.95	3.57	-.81	-.31	0	2.09	-9.61	
	V	1.92	.20	-2.97	-2.43	1	-16.93	.61	
	S	-3.11	9.92	-.06	-.14	0	12.19	-3.73	
	L	-1.27	3.74	-1.59	.60	0	-6.45	5.03	
	T	-2.44	1.27	-.01	.06	1	-9.31	-3.75	
	E	-4.94	1.23	.09	-1.01	1	1.70	1.28	
	L	-11.90	1.54	.92	-3.34	0	3.40	-11.02	
	S	-14.56	7.53	.72	.19	0	14.17	2.80	

Figure 5-11c. Predetermined Fullback vs. Split-Six Defense (Simulation Results)

EPOCH = .75 SECONDS

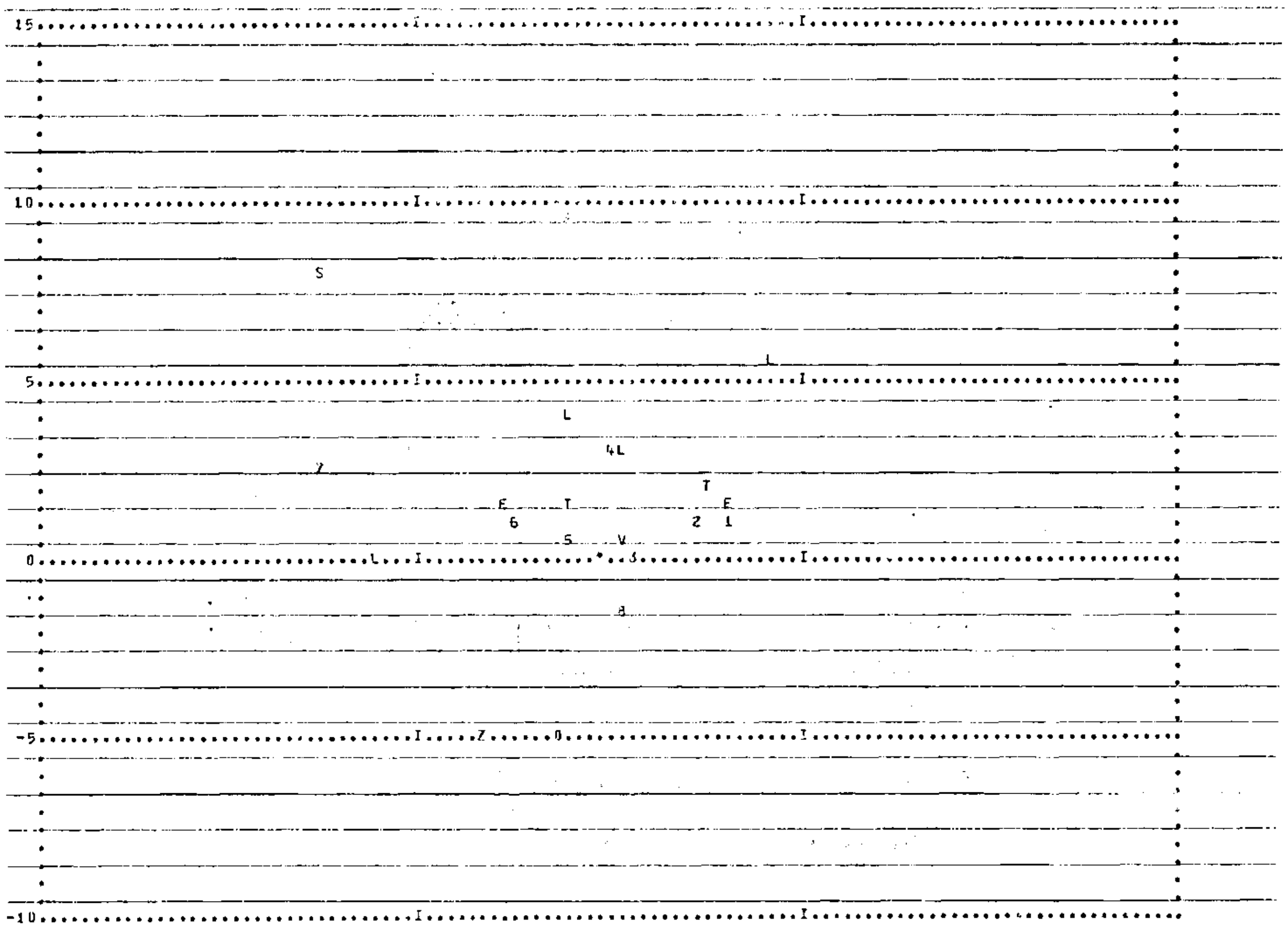


EPOCH = .75 SECONDS

PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE								
1	5.90	1.11	-2.00	2.36	1	4.19	-13.05	E
2	-.17	1.34	-1.75	1.63	1	3.19	-13.11	T
3	1.62	.41	-2.26	.75	1	-11.77	7.01	V
4	-.55	2.18	.37	5.19	0	13.21	7.78	L
5	-2.28	.47	-.32	.70	1	9.92	6.54	T
6	-4.53	1.03	-.83	2.34	1	-7.63	-10.33	E
7	-14.11	2.08	-.29	5.86	0	6.53	-7.11	S
8	.49	-1.62	.30	-.08	0	2.04	.42	
*	-.60	-1.36	-1.31	5.57	0	9.95	12.56	
0	-1.32	-5.28	-5.55	.31	0	-16.27	2.97	
Z	-4.80	-5.44	-6.28	-.28	0	-17.01	.45	
DEFENSE								
L	9.23	5.42	-5.46	1.25	0	-14.02	5.56	
E	6.07	1.91	-1.36	-.34	1	-.36	14.76	
T	4.58	2.11	-1.55	-1.43	1	-.63	10.75	
L	.86	3.27	-.05	-1.96	0	-13.30	1.30	
V	1.42	1.02	-1.51	-1.29	1	2.28	16.40	
S	-2.81	9.60	2.24	-.77	0	2.63	-1.70	
L	-1.73	4.00	-2.05	1.42	0	.77	-.54	
T	-2.46	1.42	.09	.10	1	16.37	-2.06	
E	-5.14	1.66	-.39	-.63	1	14.39	-2.31	
L	-11.64	.62	1.13	-4.19	0	11.74	3.29	
S	-14.35	7.64	1.34	.63	0	6.14	12.63	

Figure 5-11d. Predetermined Fullback vs. Split-Six Defense (Simulation Results)

EPOCH = 1.00 SECONDS

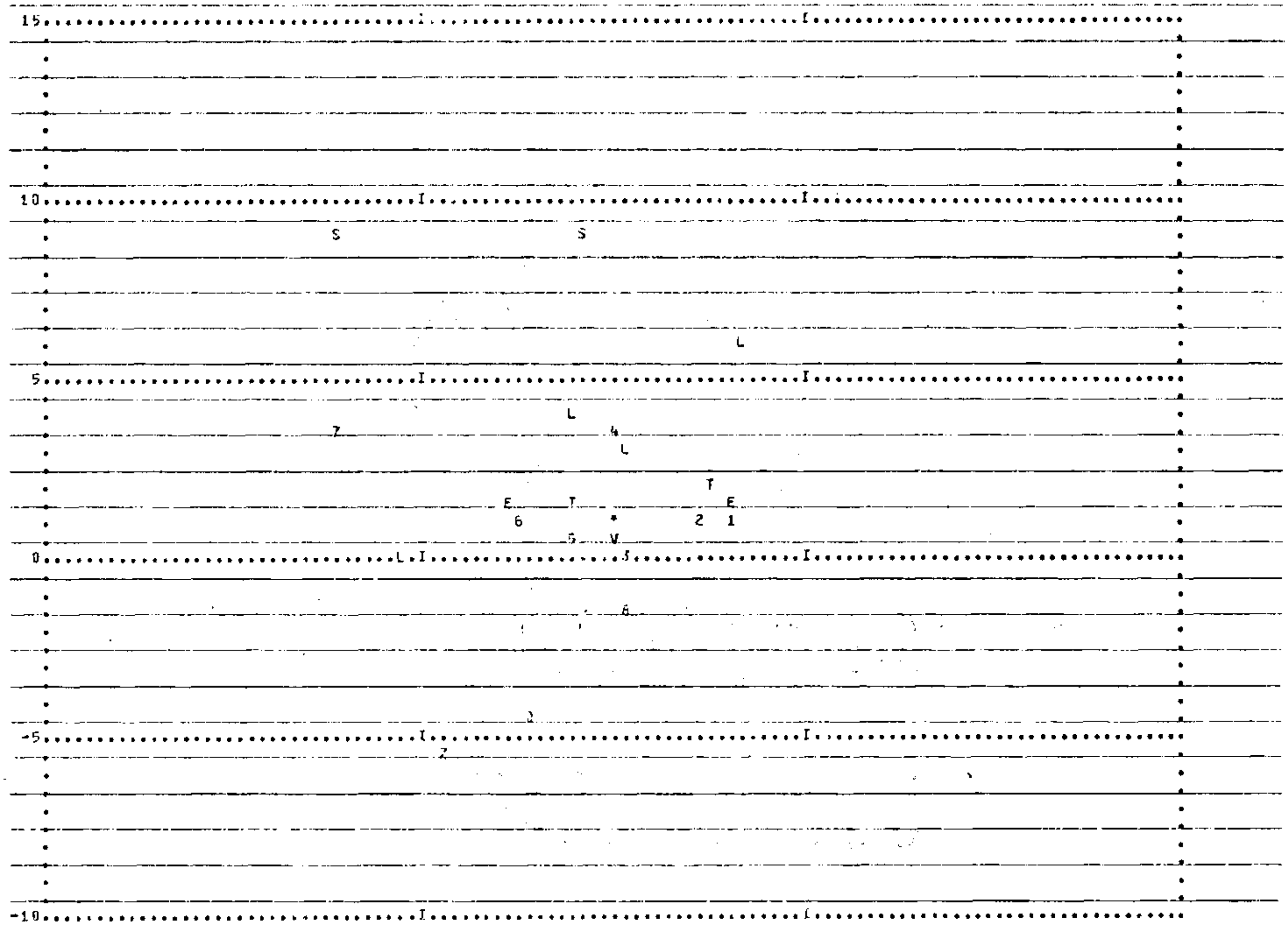


EPOCH = 1.00 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	5.85	1.16	-.68	.32	1	4.40	-12.47	E
	2	4.18	1.34	-.53	-.07	1	3.68	13.66	T
	3	1.10	.38	-1.75	.02	1	-14.00	6.38	V
	4	-.25	3.26	-.02	2.49	1	-13.50	8.10	L
	5	-2.27	.57	.35	.46	1	9.82	10.30	T
	6	-4.57	1.27	-.45	.77	1	-13.20	-3.08	E
	7	-13.84	2.99	1.46	1.85	0	11.63	8.83	S
	8	.60	-1.62	.54	.03	0	3.30	-.58	
	*	-.59	.00	1.15	5.37	0	5.86	12.62	
	0	-2.86	-5.14	-6.21	.72	0	-17.56	1.42	
	Z	-6.42	-5.45	-6.58	-.07	0	-16.93	-.94	
DEFENSE									
	L	7.64	5.65	-5.62	1.39	0	-17.32	3.37	
	E	5.85	1.95	-.33	-.51	1	-7.11	12.00	
	T	4.57	2.11	.13	.05	1	-.24	11.42	
	L	.62	3.04	.31	-.48	1	-14.95	.25	
	V	.56	.99	-2.33	-1.40	1	2.43	12.86	
	S	-2.32	9.61	1.72	-.74	0	1.72	-1.51	
	L	-2.11	4.33	-1.05	.67	0	11.70	-11.75	
	T	-2.44	1.33	.25	-.01	1	9.52	13.52	
	E	-5.30	1.90	-.59	-.74	1	11.43	-.30	
	L	-11.13	.06	2.81	-.54	0	17.73	4.36	
	S	-13.53	3.09	2.59	2.74	0	1.27	15.23	

Figure 5-11e. Predetermined Fullback vs. Split-Six Defense
(Simulation Results)

EPOCH = 1.25 SECONDS

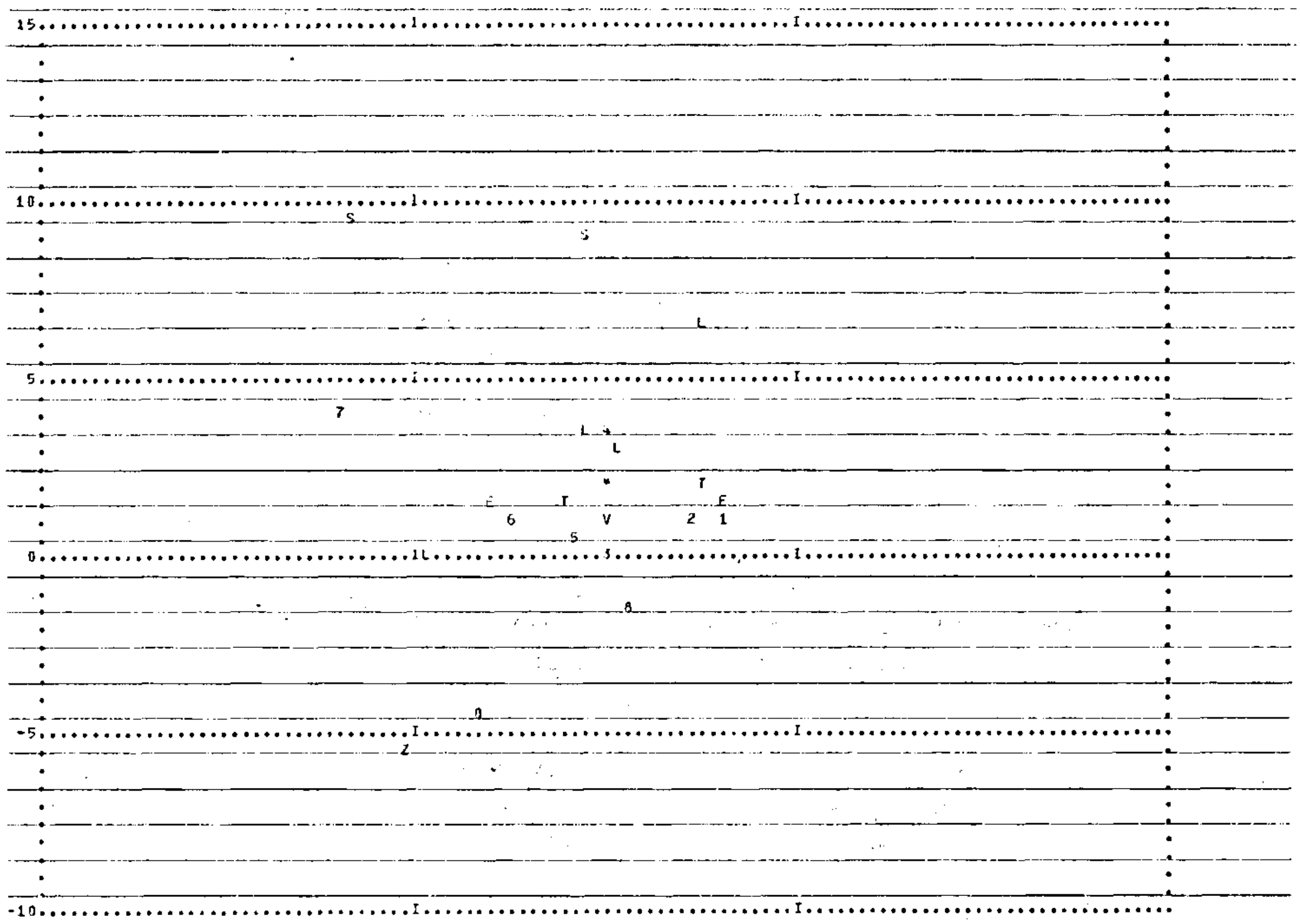


EPOCH = 1.25 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	5.56	1.16	-.22	.02	1	12.37	7.14	E
	2	4.19	1.38	.05	.34	1	-.54	13.79	T
	3	.54	.23	-2.09	-.43	1	-12.00	3.08	V
	4	-.28	3.62	-.57	1.23	1	12.34	-9.03	L
	5	-2.14	.60	.59	.31	1	8.26	9.73	T
	6	-4.70	1.29	-.42	-.06	1	-15.32	6.68	E
	7	-13.37	3.56	2.96	2.65	0	7.22	11.11	S
	8	.80	-1.63	1.02	-.09	0	3.54	-2.19	
	*	-.22	1.33	1.72	5.27	0	1.17	17.00	
	0	-4.46	-4.97	-6.65	.66	0	-15.06	-.78	
	Z	-8.13	-5.51	-7.11	-.14	0	-16.73	.21	
DEFENSE									
	L	6.31	6.30	-6.39	1.75	0	-18.46	-.11	
	E	5.83	1.95	.12	.91	1	-7.80	-2.50	
	T	4.56	2.16	.09	.39	1	-5.98	.84	
	L	.53	3.15	-.24	-.30	1	-6.95	-11.13	
	V	.62	.66	-2.21	-1.70	1	-13.07	10.11	
	S	-1.99	9.43	1.25	-.38	0	10.72	-10.15	
	L	-2.00	4.07	1.62	-1.34	0	17.93	-2.88	
	T	-2.33	1.63	.68	.70	1	-4.37	-1.42	
	E	-5.42	1.93	-.62	-.44	1	-8.79	.80	
	L	-10.15	.09	4.84	.92	0	15.43	1.61	
	S	-13.08	9.01	1.75	-.22	0	16.29	2.05	

Figure 5-11f. Predetermined Fullback vs. Split-Six Defense (Simulation Results)

EPOCH = 1.50 SECONDS

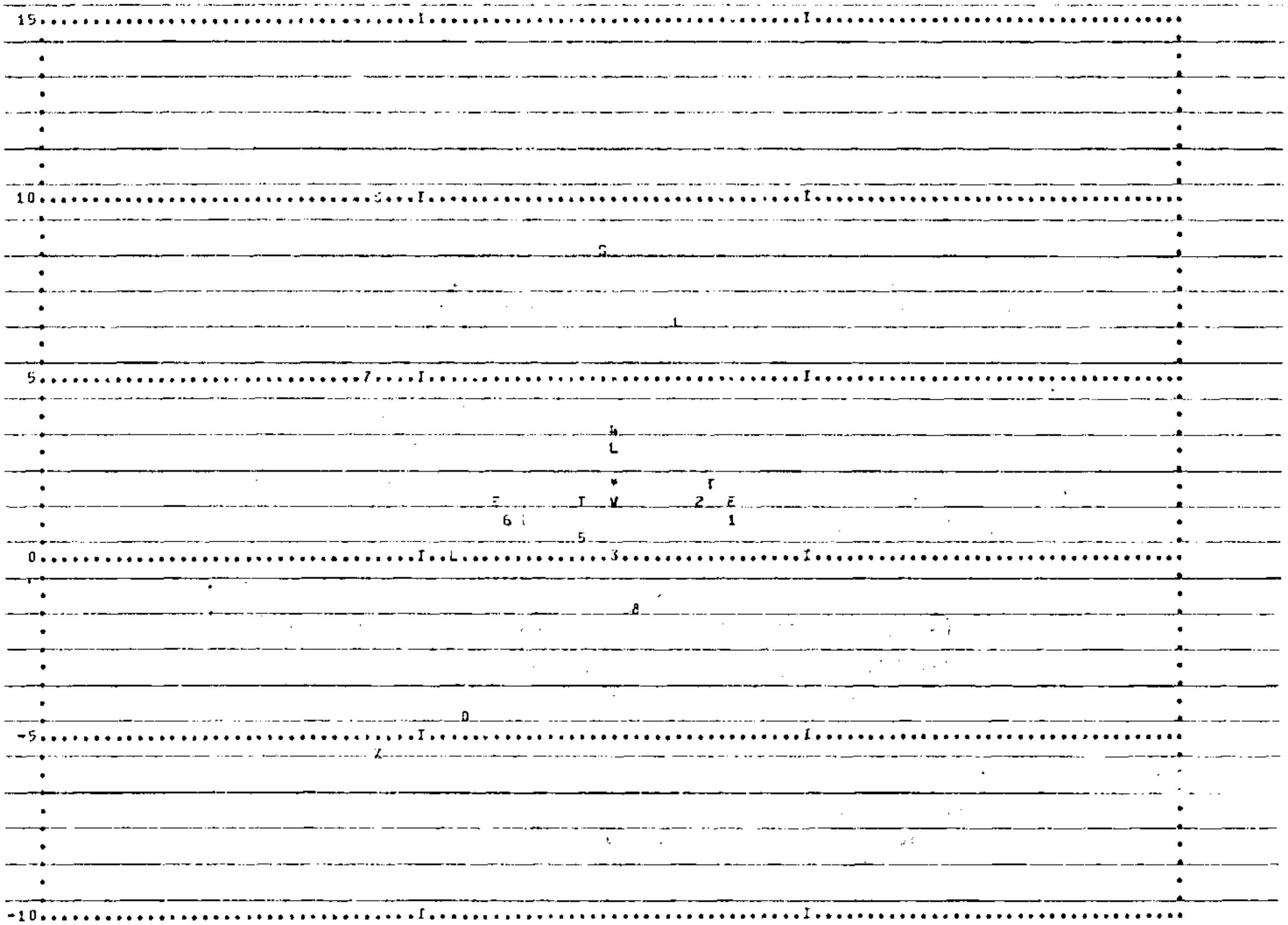


EPOCH = 1.50 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	5.80	1.15	-.47	-.07	1	-2.92	12.58	E
	2	4.20	1.50	-.02	.57	1	11.26	-8.53	T
	3	.15	.15	-1.13	-.23	0	-2.08	12.95	V
	4	-.27	3.61	.03	-.13	1	14.93	-1.95	L
	5	-1.94	.87	.49	.37	1	7.31	10.53	T
	6	-4.87	1.25	-.85	-.09	1	-10.96	9.72	E
	7	-12.63	4.35	2.06	-3.51	0	12.75	10.17	S
	8	1.09	-1.77	1.22	-.46	0	4.28	-3.36	
	*	-.15	2.24	-1.64	2.39	3	-5.89	11.93	
	Q	-6.11	-4.67	-6.42	.21	0	-15.87	-1.28	
	Z	-9.89	-5.53	-6.98	-.04	0	-15.24	-.03	
DEFENSE									
	L	4.61	6.59	-6.98	.81	0	-17.65	1.21	
	E	5.75	1.93	-.22	-.27	1	-17.23	.71	
	T	4.59	2.27	-.33	.27	1	4.35	3.05	
	L	.51	3.09	.15	-.31	1	-11.32	-5.27	
	V	.94	1.32	.05	-.62	3	.71	-17.31	
	S	-1.44	9.04	2.60	-2.27	0	15.56	.96	
	L	-1.24	3.65	-9.21	-1.53	0	9.85	-11.33	
	T	-2.19	1.61	.94	.25	1	14.52	3.45	
	E	-5.60	1.88	-.64	-.45	1	17.39	.58	
	L	-6.84	.23	5.51	.59	0	16.17	.77	
	S	-12.23	9.91	3.99	2.83	0	15.82	.90	

Figure 5-11g. Predetermined Fullback vs. Split-Six Defense (Simulation Results)

EPOCH = 1.70 SECONDS



EPOCH = 1.70 SECONDS

	PLAYER	X	Y	XDOT	YDOT	INTERACTION	CX	CY	MATCHUP
OFFENSE									
	1	5.74	1.1[illegible]	-.99	.08	1	-2.32	12.88	E
	2	4.22	1.57	.05	.31	1	11.26	-8.63	T
	3	-.07	.34	-1.02	1.31	0	-2.08	12.95	V
	4	-.30	3.56	-.40	-.23	1	0.00	0.00	L
	5	-1.94	.94	1.20	.53	1	7.31	10.53	T
	6	-5.04	1.21	-.68	-.22	1	-10.96	9.72	E
	7	-11.94	5.06	3.83	3.76	0	12.75	10.17	S
	8	1.35	-1.83	1.42	-.31	4	4.28	-3.36	
	*	-.36	2.41	-.83	.81	3	-9.39	11.99	
	0	-7.39	-4.86	-6.44	-.07	0	-15.37	-1.28	
	Z	-11.26	-5.56	-6.69	-.03	0	-15.24	-.03	
DEFENSE									
	L	3.21	6.73	-7.08	.67	0	-17.65	1.21	
	E	5.60	1.91	-.45	-.41	1	-17.23	.71	
	T	4.59	2.39	.25	-.34	1	4.35	3.05	
	L	.46	3.01	-.21	-.37	1	-11.32	-5.27	
	V	-.11	1.59	-1.12	-.39	3	.71	-17.31	
	S	-.76	8.70	-.07	-1.24	0	15.36	.96	
	L	-.41	3.21	4.14	-2.81	3	9.95	-11.83	
	T	-1.97	1.92	.79	.36	1	14.32	3.45	
	E	-5.75	1.66	-.87	-.64	1	17.39	.56	
	L	-7.70	.34	5.95	.48	0	10.17	.77	
	S	-11.43	10.57	1.95	1.56	0	15.42	.69	

Figure 5-11h. Predetermined Fullback vs. Split-Six Defense (Simulation Results)

CHAPTER VI

APPLICATIONS OF THE MODEL

The effort of Chapter III in developing and Chapter IV in validating the model leads to the natural question of what applications to football are feasible. This chapter discusses three areas of possible use. Two areas are for use in long-range planning with the remaining a short-range planning application.

Section 6.1 addresses itself to the problem of play selection. There is a natural application of game theory to the selection of offensive and defensive plays. This is discussed and the results of Chapter V are used in an example.

Long-range planning is discussed in Sections 6.2 and 6.3. In Section 6.2, five possible uses of the model are examined. These uses are concerned primarily with the evaluation of the effectiveness of different plays and player attributes. Section 6.3 deals with making blocking assignments. A method utilizing dynamic programming is presented to determine the optimal assignments. This could in turn be used, then, to design plays.

6.1 Play Selection

The choice of a play is perhaps the most basic and immediate decision that a coach is confronted with. These decisions are typically based on the requirements of the immediate play, the expectations of the opposition's actions, the capabilities of the players on the field, and past successes or failures of the possible offenses and defenses. In many cases, the choice is the culmination of a guessing game between opposing coaches. What this section does is to apply the methodology of game theory to the results from the football model to show how a coach could make these decisions.

Suppose the coach of the offensive team is to make a selection.* Taking the two offenses and two defenses described earlier as candidate plays**, the probability distributions (in discrete half-yard increments) are as follows:

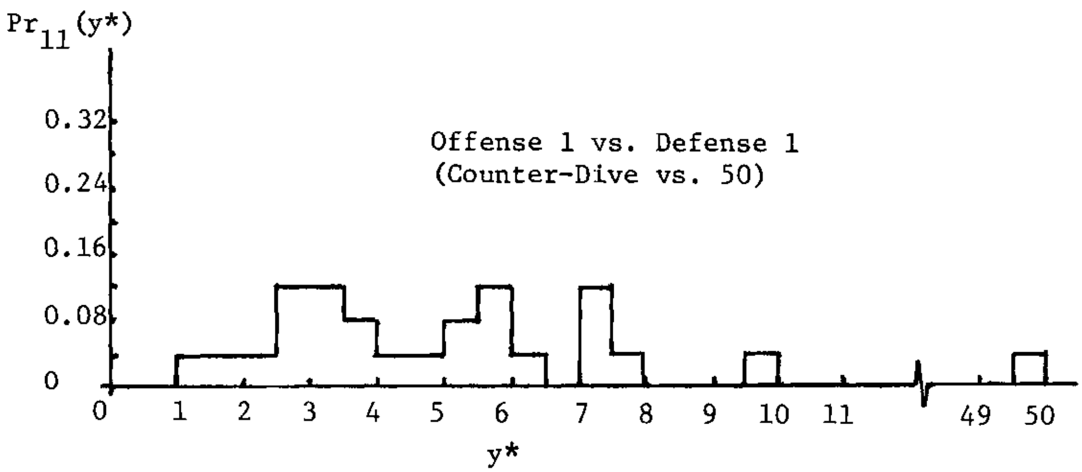


Figure 6-1. Gain Distribution, Offense 1 vs. Defense 1

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*The procedure here works equally well for defensive decision-making.

**In an actual game situation, the candidate offenses and defenses would number much more than two.

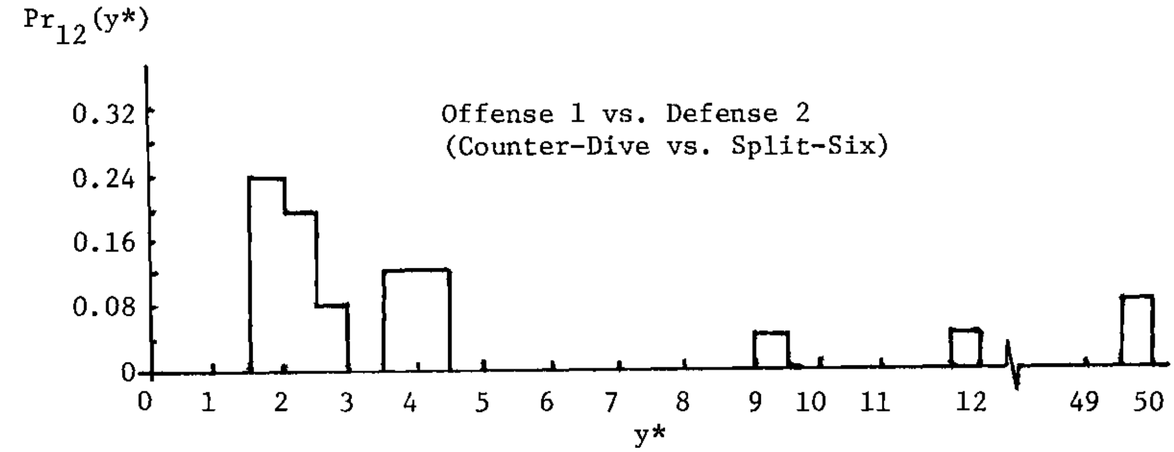

Figure 6-2. Gain Distribution, Offense 1 vs. Defense 2

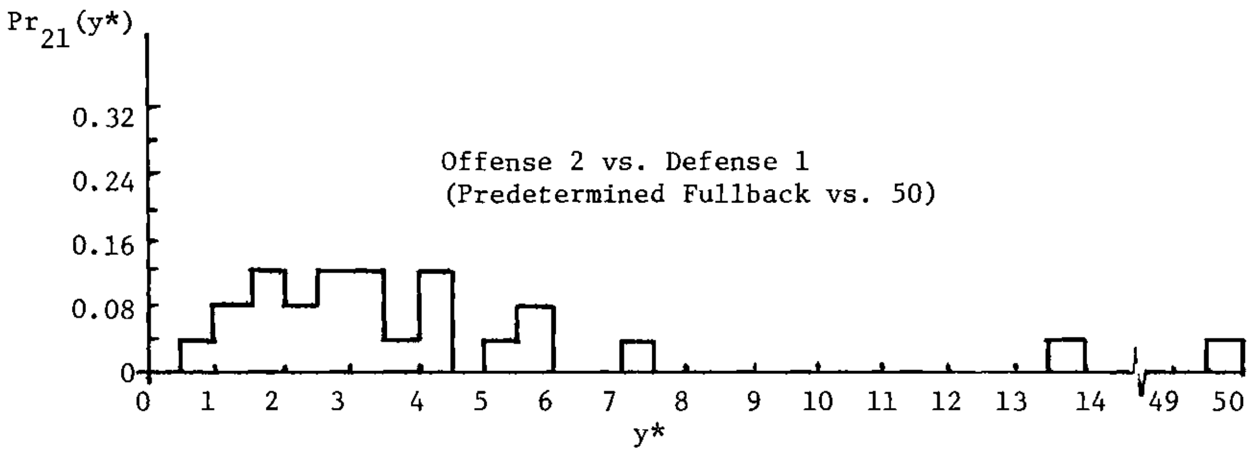

Figure 6-3. Gain Distribution, Offense 2 vs. Defense 1

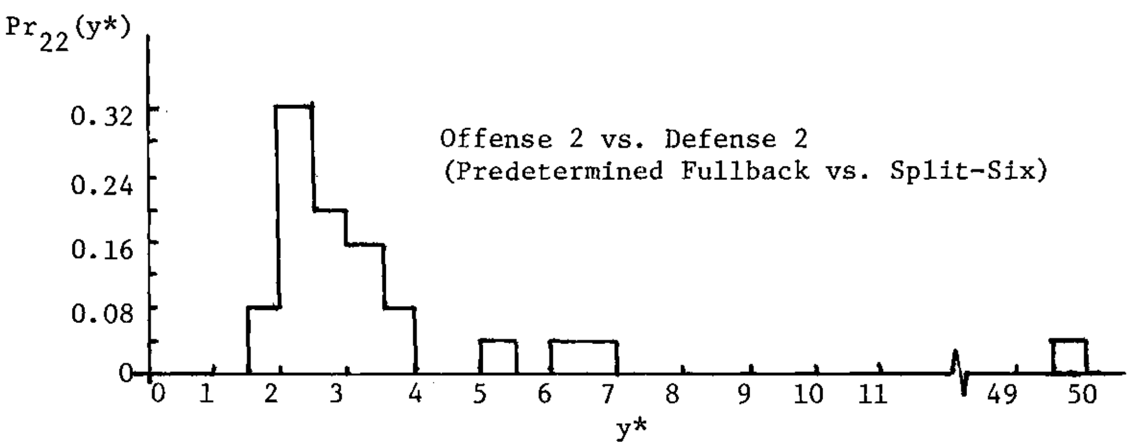

Figure 6-4. Gain Distribution, Offense 2 vs. Defense 2

The offensive coach must now select a utility function, U(y*), dictated by the game situation which reflects the relative values of the possible gains. Three examples are described below:

1. On a third down and three yards to go on the offense's own 30 yard line, near the end of the game and leading by three points, the utility function would be as follows:

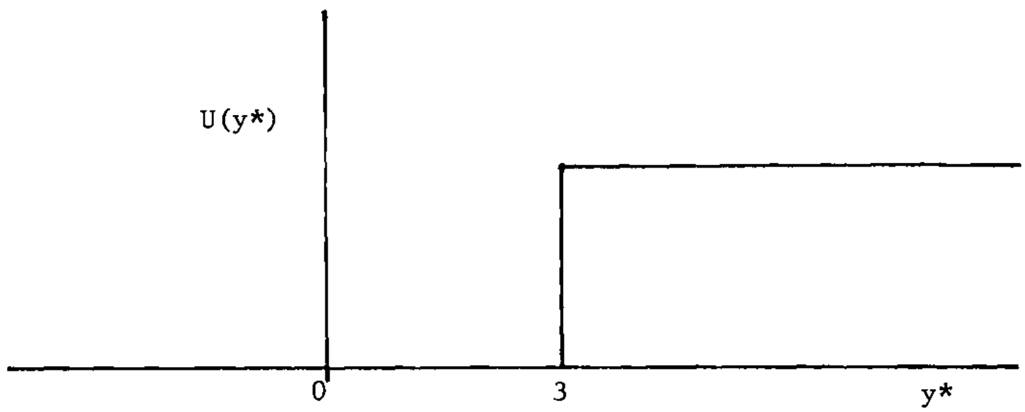

Figure 6-5. Utility Function: Third Down and Three Yards to Go

Any gain less than three yards does not meet the coach's requirements, i.e. keep possession of the ball. Likewise, the marginal value of a gain of four yards over a gain of three yards is negligible.*

2. On a first down and ten yards to go with the ball of the offense's two yard line, the objective function would look like

---

*It may be that the coach would wish to reduce the number of candidate plays based on game situations. For instance, in this example he may wish to consider only those plays which have a 0.9 or larger probability of ending with the ball-carrier inbounds (and thus keep the game clock moving).

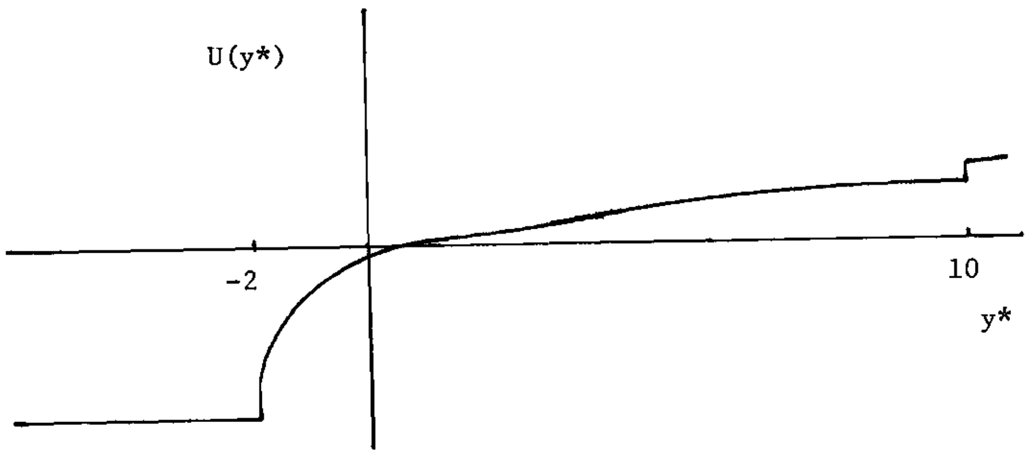

Figure 6-6. Utility Function, First and Ten on the Two Yard Line

Here, a loss of two or more yards carries a large penalty. Likewise the objective function jump at $y^* = 10$ yards reflects the value of a first down. The relatively flat shape of $U(y^*)$ at $0 \leq y^* \leq 3$ results from the fact that a small gain is not particularly attractive.

3. On a first down and ten yards to go on the offense's 20 yard line, the utility function might look like

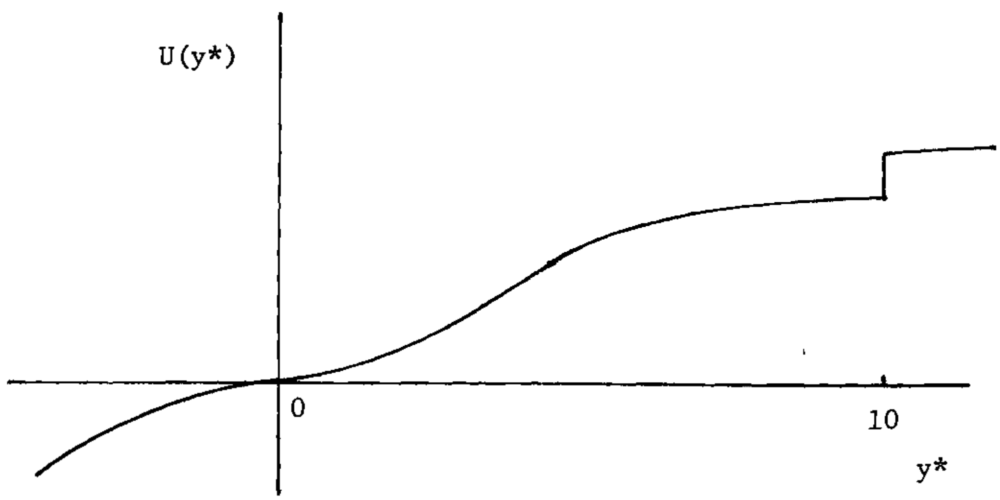

Figure 6-7. Utility Function, First and Ten on the Twenty Yard Line

> An inflection point at three yards indicates the increasing relative value placed on a gain over three yards; another inflection point at six yards shows a decrease in relative value for gains over six yards, since a first down in the next two plays becomes more likely. The jump at ten yards indicates the added value of obtaining a first down.

Define the payoff of a play as follows

$$G_{ij} = \int_{-\infty}^{\infty} U(y^*)Pr_{ij}(y^*)dy^* \tag{6.1}$$

where i is the offense selected, j is the defense selected, and G_{ij} is the payoff for the ij offense-defense pair. Equation (6.1) reduces to a summation if $Pr_{ij}(y^*)$ is defined (or calculated) only for discrete values of y*.

Using the third objective function* above and the two offenses and defenses in Chapter V, G_{ij} is calculated to be

$$G_{ij} = \begin{pmatrix} 2.626 & 2.642 \\ 2.718 & 1.770 \end{pmatrix}$$

If one row dominates another (if $G_{mj} \geq G_{nj}$ for all j, then row m dominates n) then for these circumstances the dominating row is clearly the superior offensive play. If a row dominates all other rows, then that play should be selected. If any one row does not

*Actually a piece-wise linear function between (0,0), (3,1), (6,3), (10,5 1/3), (10,6) and (50,16).

dominate all other rows, then one of two approaches may be chosen. The first possibility (call it the naive approach) is to assign a probability to each alternative available to the defense (Pr_j for each defense j). Defining

$$G_i = \sum_j Pr_j G_{ij} \tag{6.2}$$

where G_i is the expected payoff of play i given the probabilities of the various defenses, then play k should be selected such that

$$G_k \geq G_i \text{ for all } i \tag{6.3}$$

The practical problem of this approach is determining the Pr_j. This could be overcome by the use of frequency charts from past games (i.e. in previous games when faced with a third down and four yards, defense 1 was selected 47% of the time and defense 2 was selected 53% of the time. Hence on third down and four yards to go let $Pr_1 = 0.47$ and $Pr_2 = 0.53$).*

The second approach in selecting a play would be that of finite game theory. Some observations applicable to this particular instance are:

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*Bayesian statistics could be employed during the course of a game to update these probabilities depending on what defenses were actually selected.

1. This game is not a game of perfect information, since both coaches are required to choose a strategy while not knowing the other's choice.

2. This game, in general, will not possess a saddle point.

3. The optimum strategy is therefore in general a mixed strategy. These strategies can be solved using linear programming.

The linear program required to solve is

$$\text{minimize } z_1 = \sum_i x_i \tag{6.4}$$

$$\text{s.t. } \sum_i G^*_{ij} x_i \geq 1 \quad j = 1, 2, \ldots \tag{6.5}$$

$$x_i \geq 0 \tag{6.6}$$

where G^*_{ij} is equivalent to G_{ij} except that all dominated rows and columns have been deleted and all elements of G^*_{ij} have been made non-negative by the adding of a suitable constant to the corresponding element in G_{ij}. Letting z^*_1 and x^*_i denote the optimal objective and solution, then

$$Pr_i = x^*_i / z^*_1 \tag{6.7}$$

which represents the probabilities that the i'th offensive play should be selected.

Likewise, the dual of the above linear program is used to find the Pr_j:

$$\text{Maximize } z_2 = \sum_j y_j \tag{6.8}$$

$$\text{s.t. } \sum_j G^*_{ij} y_j \leq 1 \quad i = 1, 2, \ldots \tag{6.9}$$

$$y_j \geq 0 \tag{6.10}$$

and

$$Pr_j = y^*_j / z^*_2 \tag{6.11}$$

Note that by duality theory $z^*_1 = z^*_2$, and $1/z^*_1$ is also the value of the game (after subtracting the suitably chosen constant used in transforming G_{ij} to G^*_{ij}).

Which of the two approaches are to be preferred? The naive approach demonstrates a computational straightforwardness appealing for what would necessarily be a real-time system. Also, the optimal play selected would be just that, a play, unlike the game theory approach which would give the Pr_i's from which a play would then still have to be selected. The difficulty is, of course, that the determination of the Pr_j's is precisely the guessing game between coaches alluded to earlier. If a coach were able to detect a pattern in defensive strategy, then the naive approach is certainly justified. Barring this, however, he would be well advised to select the play randomly according to the distribution of Pr_i's generated in the game theory approach.

In the example based on the plays of Chapter V, the linear program of the game theory approach is only in two dimensions, so graphing the feasible space is the easiest method. All G_{ij} are positive, so no translation to G^*_{ij} is necessary.

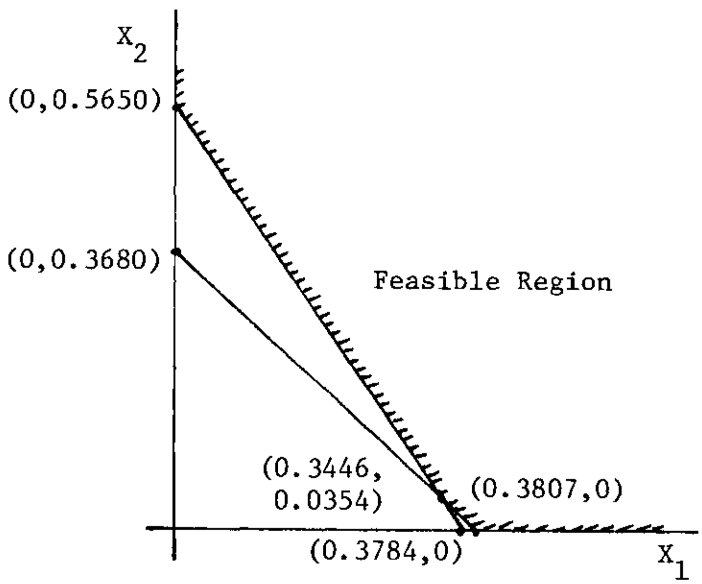

Figure 6-8. Graphical Solution to Game Theory Strategy Selection

The point (0.3446,0.0359) is optimal, giving a z^*_I of 0.3805. Thus $Pr_1 = 0.9058$ and $Pr_2 = 0.9042$. The corresponding value of the game is 2.6281. If a play were to gain 5.44 yards on each run, this corresponds to the value of the game above for the same utility function.

6.2 Evaluation of Plays and Players

The models of Chapters III and IV can be used in a number of ways to make long-range planning decisions. A by-no-means complete list follows:

1. Selection of plays for practice. Football teams usually prepare for a game in the week (and occasionally for a few weeks) prior to the game. In the practice sessions the team concentrates on specific requirements (including offensive and defensive plays) that the coaches have identified to be necessary. By evaluating the many candidate plays in the team's repertoire against a specific opponent, the model could identify which plays would be effective and which should be temporarily discarded. The selected plays could then be incorporated into the practice schedule.

2. Analysis of the effect of where the ball is spotted. The distance from the sideline to where the ball is when the play begins certainly affects the ball-carrier's actions and the defensive reactions. Thus it also should have an effect on the gain of the play. The sensitivity of this factor could be found by running the model with varying x-values of the initial positions of the players.

3. Analysis of the marginal effect of a player's (or players') attributes. If a player's speed, weight or strength were changed, this would change the gain distribution. In some cases the change would be significant, in others it would not. Identifying which players are critical to the play would help in determining the effectiveness of the play given the team's total player resources.

4. Use as a learning tool. The model could be used to show the interactions and relationships to the players who would have to carry out the play. By showing the whole play unfolding epoch by epoch, the model may be used as an illustration to the normal play diagrams.

5. Play selection frequency table generation. In lieu of a real-time information system, a table of play frequencies could be generated before the game based on the game theory considerations of the previous section. Also, strategy sessions involving coaches who must make play selections could be done by simulating actual game situations and then analyzing the play selection.

6.3 Play Design

In Section 3.5.1 the method for finding $\vec{C}^E$ for a ball-carrier taking into account the efforts of offensive blockers is given. The decision rule for the ball-carrier was to maximize his expected gain at tackle (or going out-of-bounds) based on some computed probabilities of successful blocks, the values of C^{E_i} and C^{P_i}, his own C^{E_o}, the position and velocities of all the players in the play, and the assignments of the offensive blockers. It is this last item, the assignment of blockers to defensive tacklers, which this section studies.

There is a very large number of possible blocking assignments for a football team (11! = 39,916,800 if only one-on-one blocking is allowed; 3,073,593,600 if double-teaming is also allowed). Although many of these assignments make no sense in a given circumstance, the remaining number of possible combinations is still so large as to make enumeration methods impractical. The problem is therefore to find an efficient procedure to find the optimal blocking assignment.*

Let M denote the number of blockers to be assigned. As in Section 3.5.1, for each trial value of θ, y^j corresponding to the first-time interaction for P^j are calculated. As before y^0 is the y-value corresponding to E_o going out-of-bounds. The y^j and y^0 again are ordered so that $y^1 \leq y^2 \leq \ldots \leq y^N \leq y^o$ (note that this defines N, the number of defensive tacklers. Also note that N depends on θ). Using (3.72), (3.73), (3.75), and (3.81) the matrix

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*The following procedure also finds the optimal θ as a by-product.

$Pr_{ij}(1 \leq i \leq M,\ 1 \leq j \leq N)$, the probability that E_i can successfully block P_j is calculated.

Define a decision variable x_{ij} so that if $x_{ij} = 1$ then E_i is assigned to P_j, and if $x_{ij} = 0$, then E_i is not assigned to P_j. Clearly, a requirement is

$$\sum_j x_{ij} = 1 \quad i = 1, 2, \ldots, M \tag{6.12}$$

since a player can be assigned only to one defensive tackler and it is not optimal to have him stand idly, not blocking anyone. Since only double-teaming is allowed,

$$\sum_i x_{ij} \leq 2 \quad j = 1, 2, \ldots, N \tag{6.13}$$

Equations (6.12) and (6.13), along with $x_{ij} = 0, 1$ form the constraints of the problem. The objective function takes the form

$$E[y^*] = y_1 \prod_{i=1}^{M}(1-Pr_{i1}x_{i1}) + y_2[1-\prod_i(1-Pr_{i1}x_{i1})] \prod_i(1-Pr_{i2}x_{i2})$$

$$+ \ldots + y_N \prod_{j=1}^{N-1} [1-\prod_i(1-Pr_{ij}x_{ij})] \prod_i(1-Pr_{iN}x_{iN}) \tag{6.14}$$

$$+ y_0 \prod_j[1-\prod_i(1-Pr_{ij}x_{ij})]$$

which can be simplified considerably by noting that

$$\prod_i (1-Pr_{ij}x_{ij}) = 1 - \sum_i Pr_{ij}x_{ij} + \sum_{i=1}^{M} \sum_{\substack{K=1 \\ K\neq i}}^{M} Pr_{ij}Pr_{Kj}x_{ij}x_{Kj}$$

$$- \sum_{i=1}^{M} \sum_{\substack{K=1 \\ K\neq i}}^{M} \sum_{\substack{M=1 \\ M\neq i,K}}^{M} Pr_{ij}Pr_{Kj}Pr_{Mj}x_{ij}x_{Kj}x_{Mj} + \cdots \tag{6.15}$$

But the constraints of $x_{ij} = 0,1$ and (6.12) require that at most two x_{ij} may be strictly positive for any j. Thus any term of (6.15) with three or more x_{ij} is zero and (6.14) becomes

$$\prod_i (1-Pr_{ij}x_{ij}) = 1 - \sum_i Pr_{ij}x_{ij}$$

$$+ \sum_{i=1}^{M} \sum_{\substack{K=1 \\ K\neq i}}^{M} Pr_{ij}Pr_{Kj}x_{ij}x_{Kj} \tag{6.16}$$

Defining $\vec{x}_j$, a column vector consisting of

$$\vec{x}_j = (x_{1j}, x_{2j}, \ldots, x_{Mj})^t \tag{6.17}$$

and $\vec{D}(x_j)$ as

$$D(\vec{x}_j) = \sum_i Pr_{ij}x_{ij} - \sum_{i=1}^{M} \sum_{\substack{K=1 \\ K\neq i}}^{M} Pr_{ij}Pr_{kj}x_{ij}x_{kj} \tag{6.18}$$

Then (6.13) can be written as

$$\begin{aligned} E[y^*] = {} & y^1[1-D(\vec{x}_1)] + y^2 D(\vec{x}_1)[1-D(\vec{x}_2)] \\ & + \ldots + y^N D(\vec{x}_1)D(\vec{x}_2) \ldots D(\vec{x}_{N-1})[1-D(\vec{x}_N)] \\ & + y^0 \, D(\vec{x}_1) \, D(\vec{x}_2) \ldots D(\vec{x}_N) \end{aligned} \tag{6.19}$$

or

$$\begin{aligned} E[y^*] = {} & y^1 + (y^2-y^1) \, D(\vec{x}_1) + (y^3-y^2) \, D(\vec{x}_1) \, D(\vec{x}_2) \\ & + \ldots + (y^0-y^N) \, D(\vec{x}_1) \, D(\vec{x}_2) \ldots D(\vec{x}_N) \end{aligned} \tag{6.20}$$

A perhaps more enlightening form of (6.19) is

$$E[y^*] = \prod_{K=0}^{N-1} f(K) \tag{6.21}$$

where

$$f(K) = \begin{cases} (y^0 - y^N)\ D(\vec{x}_N) & K = 0 \\ \left[f(K+1) + (y^{N-K+1} - y^{N-K})\ D(\vec{x}_{N-K})\right] & K \neq 0 \end{cases} \tag{6.22}$$

and for convention $D(\vec{x}_o) \equiv 1$ and y^{N-K} for $N = K^*$ is zero.

Equation (6.21) suggests an N-stage dynamic programming procedure**. The states are $\sum_j x_{ij}$ for $i = 1, 2, \ldots, M$ which indicate whether blocker i has been assigned to a defensive player. The number of states is thus 2^M. The recursion relation is

$$f(K) = \text{MAX} \left\{\left[f(K+1) + (y^{N-K+1} - y^{N-K})\right] D(\vec{x}_K)\right\} \tag{6.23}$$

where the maximization is over the states allowed by the constraints.

As an example, suppose $N = 2$, $M = 3$, Pr_{ij} is as below:

$$Pr_{ij} = \begin{pmatrix} 0.5 & 0 \\ 0.4 & 0.4 \\ 0.2 & 0.6 \end{pmatrix}$$

and $y^1 = 10$, $y^2 = 15$, and $y^0 = 20$. Let a three-component binary vector indicate whether E_i has been assigned ((1,1,0) indicates E_1 and E_2 are

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*This is not the same as y^0, the y-value where the ball-carrier goes out-of-bounds.

**The form is similar to many quality control problems.

assigned and E_3 is not). Then the blocking assignment problem is solved as follows:

Stage 2

Input	Allowable x_2	$f(2) = \max\ (y^0-y^2)D(\vec{x}_2)$
(0,1,1)	(1,0,0)	(5)(0) = 0
(1,0,1)	(0,1,0)	(5)(0.4) = 2.0
(1,1,0)	(0,0,1)	(5)(0.6) = 3.0
(0,0,1)	(1,1,0)	(5)(0.4) = 2.0
(0,1,0)	(1,0,1)	(5)(0.6) = 3.0
(1,0,0)	(0,1,1)	(5)(0.76) = 3.8

Stage 1

Input	Allowable x_2	$f(1) = \max\ f(2) + (y^2-y^1)D(\vec{x}_1)$
(0,0,0)	(0,0,1)	(2.0+5)(0.2) = 0.5
	(0,1,0)	(3.0+5)(0.4) = 1.4
	(0,1,1)	(0.0+5)(0.52) = 2.6
	(1,0,0)	(3.8+5)(0.5) = 4.4
	(1,0,1)	(2.0+5)(0.6) = 1.5
	(1,1,0)	(3.0+5)(0.7) = 5.6 optimal

Stage 0

Input	$f(0) = f(1) + y^1$
(0,0,0)	5.6 + 10 = 15.6

Thus E_1 and E_2 are assigned to P_1 and E_3 is assigned to P_2, with an expected gain of 15.6 yards.

Several special cases of the problem can be identified if the objective is instead to maximize the probability of a touchdown.* The objective function is then

$$E[y^*] = D(\vec{x}_1)\ D(\vec{x}_2)\ \ldots\ D(\vec{x}_N) \tag{6.24}$$

and f(K) takes the form

$$f(K) = f(K+1)\ D(\vec{x}_{N-K}) \tag{6.25}$$

with $f(1) \equiv 1$. In the example given above:

Stage 2

Input	Allowable x_2	$f(2) = \max D(x_2)$
(0,1,1)	(1,0,0)	0
(1,0,1)	(0,1,0)	0.4
(1,1,0)	(0,0,1)	0.6
(0,0,1)	(1,1,0)	0.4
(0,1,0)	(1,0,1)	0.6
(1,0,0)	(0,1,1)	0.76

*The same result is achieved by taking y^0 large in the previous procedure.

Stage 1

Input	Allowable x_1	$f(1) = \max f(2)D(x_1)$
(0,0,0)	(0,0,1)	(0.2)(0.4) = 0.08
	(0,1,0)	(0.4)(0.6) = 0.24
	(0,1,1)	(0)(0.72) = 0
	(1,0,0)	(0.5)(0.76) = 0.38
	(1,0,1)	(0.6)(0.4) = 0.24
	(1,1,0)	(0.7)(0.6) = 0.42 optimal

Again E_1 and E_2 are assigned to P_1 and E_3 is assigned to P_2, with a probability of 0.42 of both P_1 and P_2 being blocked successfully.

Another interesting "touchdown" variation is when M = N. This case results in one-on-one blocking only, since each defensive player must be blocked (in this model--not in the game) to insure any chance of a touchdown. Thus (6.17) becomes

$$D(x_j) = \sum_i Pr_{ij}x_{ij} \qquad j = 1, 2, \ldots, M \tag{6.26}$$

and the problem becomes

$$\text{Maximize } \prod_i \left[\sum_j Pr_{ij}x_{ij}\right] \tag{6.27}$$

$$\text{s.t. } \sum_i x_{ij} = 1 \qquad j = 1, 2, \ldots, M \tag{6.28}$$

$$\sum_j x_{ij} = 1 \quad i = 1, 2, \ldots M \tag{6.29}$$

$$x_{ij} = 0, 1 \tag{6.30}$$

Considering the linear program*

$$\text{Maximize } \sum_i \sum_j Pr'_{ij} x_{ij} \tag{6.31}$$

$$\text{s.t. } \sum_i x_{ij} = 1 \quad j = 1, 2, \ldots, M \tag{6.32}$$

$$\sum_j x_{ij} = 1 \quad i = 1, 2, \ldots, M \tag{6.33}$$

$$x_{ij} \geq 0 \tag{6.34}$$

then by the unimodularity of the constraint matrix, at optimum $x_{ij} = 0$ or 1. Thus

$$\text{Max } \sum_i \sum_j Pr'_{ij} x_{ij} = Pr'_{i_1 1} + Pr'_{i_2 2} + \ldots + Pr'_{i_M M} \tag{6.35}$$

---

*This is the definition of the Assignment Problem, the solution for which exist some very efficient algorithms.

for some $i_1, i_2, \ldots, i_M$ where $i_K \neq i_1$ for $K \neq 1$. If

$$Pr'_{ij} = \log Pr_{ij} \tag{6.36}$$

then

$$\text{Max} \sum_i \sum_j Pr'_{ij} x_{ij} = \max \log \left[Pr_{i_1 1} Pr_{i_2 2} \cdots Pr_{i_M M} \right] \tag{6.37}$$

and if the logarithm of a product is maximized, then the product itself is maximized. Hence this special case reduces to the solution of an Assignment Problem.

The selected procedure is repeated for all trial values of θ and the optimal assignments and θ are chosen based on the value of the objective function.

CHAPTER VII

RECOMMENDATIONS AND CONCLUSIONS

Mathematical models are becoming increasingly valuable to managers of complex systems. Nevertheless, these types of models are not presently being utilized by the managers of sports-systems; this can probably be attributed to a lack of the realistic and flexible models necessary for such applications. The preceding pages present an initial attempt to establish such a model for the game of football.

It is logical to build a football model at the level of the play, for from this foundation many of the game's important decisions are made. A football play lends itself to a simulation model and the method it used to treat the players' actions and reactions as physical processes.

7.1 Conclusions

Operational and physiological constraints were combined with the kinematic equations to produce the foundation of the model. The strategy of the ball-carrier was constructed to take into account the possible blocks that his teammates may affect. The blockers and defenders select their strategies in remarkably similar fashion to accomplish their respective objectives. Blocking and tackling models were developed to handle the necessary interactions between opposing players.

The methods of validating the model were discussed. The data for the validation was taken off game films, every sixth frame (0.25 seconds) being photocopied. The players' position was then taken from the film in the form of x-y coordinates. Game film data was also used to estimate the parameters required by the model. With these parameters, the play was simulated twenty-five times. The game film data was then compared with data from the simulation. The hypothesis of the validity of the results was accepted at the five percent level.

Example plays were modeled and the results given in Chapter V. Possible uses of the results are given. These include selecting particular plays for game situations, evaluation of the effectiveness of a given play, and play design.

It is clear than an expanded model of this type could be of beneficial use to a coach in a number of ways; the more obvious are listed in Chapter VI. The next section contains the author's recommendations for such expansion.

7.2 Recommendations

In the running, blocking and tackling portions of the model, physical laws are invoked to derive results for the players' position and velocity. Applying some additional mathematical concepts to determine the individual players' strategies, and making a number of simplifying assumptions to keep the model manageable, the resulting model is physically-oriented and scientifically sound (at least within the scope of the model and limitations of the assumptions).

Whereas it is intriguing to get consistent results from a model in the same manner that one might for, say, nuclear fusion, it is more important to recognize the implications in these successes for further study. There exist three major avenues for such study.

The first area of continued study is to attempt to reduce by the application of the physical sciences the necessity for the assumptions contained in Chapter III. In regards to human motion and reaction (to which many of these assumptions apply), the methods of kinesiology would be required. Possible research would include the following:

1. Analysis of blocking and tackling techniques to derive a more precise model in both the time delay and resultant motion portions.

2. Analysis of the values of the constant and determination if better measures of player attributes exist. (For instance, blocking-sled data may be preferable to bench-press data and the C_3 constant.) A sensitivity analysis for a player-dependent C_1 constant (in effect including it as an attribute) may be advisable.

3. As indicated by the limitations in Chapter II, the model assumes that every player knows where every other player is at the beginning of each epoch. The necessity of this assumption and the impact of allowing limited or faulty knowledge could be investigated. This in turn suggests some study into the methods by which football players become aware of other players' presence.

4. Analysis of the mechanics of ball handling, including the center's snap to the quarterback, the quarterback's handoffs to the running backs, and the running backs' reactions to being tackled.

A second area of research would be in the application of the results of the model (i.e., the distribution of the yards gained for the offense-defense pairs). For example, it may be indicated that play selection may be better modeled by a three- or four-move game, rather than the one-move game described in Chapter VI. Use of such a game would make the selection of a utility function considerably easier. (The use of the utility function in the discussion also assumes that any outcome of any play can be adequately described by a single number--a result which also would need further analysis.)

The final area of suggested research is that of evaluation and selection of various defensive objectives. It is interesting to note that the effect of the research would be to expand the scope of the research and enable the model to realistically be extended to all passing and running plays. In the model described in the body of the thesis, a single defensive objective is given to select a strategy in reaction to a ball-carrier's running threat. This single objective is not applicable, however, when two or more potential ball-carriers are threats to the defender (in football, this situation is called an option). It is surmised that what would be necessary to model defenders against an option is to generate general areas of responsibilities (based on the offensive players' positions and velocities). A methodology to evaluate threats within these areas of responsibilities would then enable the choice of the appropriate strategies for the defenders. The bonus for a research of this type would be that the identical technique could be used to model zone-coverage on pass plays. Man-to-man coverage could then be modeled as a degenerate zone-coverage and a pass defense thus completed. With some additional work on ball-handling models (pitches and passes) and catching models, a unified and complete football model could be constructed.

From the preceding paragraphs, it is clear that the model as presented here is incomplete. Nevertheless it is felt that the results demonstrate that the framework of a physical simulation model is viable, and that this research is a first step to a realistic and useful football model.

APPENDIX A

This appendix lists the program used to generate the graph presented as Figure 4-1 and used to determine σ_C.

Two library routines called by the program are not universal to all FORTRAN compilers:

1. RANSET(X). This subroutine sets the random number generator seed.

2. RANF(X). This function returns a pseudo-random uniformly-distributed between 0 and 1 exclusive.

```
      PROGRAM EAT1(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      DATA E/6HAAAAAA/
2     READ(5,*)CX
      SIG=0.5
      C1=2.+5
      CALL RANSET(E)
5     A=0.
      B=0.
      DO 60 I=1,200
      X=0.
      XDOT=0.
      T2=0.
      DO 40 J=1,100
      Y=RANF(E)
      Z=1.
      IF(Y.LT.0.5)GO TO 10
      Y=1.-Y
      Z=-1.
10    T=SQRT(ALOG(1./Y/Y))
      T1=T-(2.30753+0.27061*T)/(1.+0.99229*T+0.04481*T*T)
      T1=T1*Z*SIG+CX
      IF(X.LT.35.)GO TO 30
      DO 20 K=1,5
      T2=T2+0.05
      X1=X+T1*(K*0.05/C1-(1.-EXP(-C1*0.05*K))/C1/C1)+XDOT*(1.-
     1EXP(-C1*0.05*K))/C1
      IF(X1.GE.40.)GO TO 50
20    CONTINUE
      X=X1
      GO TO 40
30    T2=T2+0.25
      X=X+T1*(0.25/C1-(1.-EXP(-C1*0.25))/C1/C1)+XDOT*(1.-
     2EXP(-C1*0.25))/C1
40    XDOT=T1*(1.-EXP(-C1*0.25))/C1+XDOT*EXP(-C1*0.25)
50    A=A+T2
60    B=B+T2*T2
      C=A/200.
      D=SQRT(B/200.-C*C)
      WRITE(6,100)C,D
100   FORMAT(2F8.3)
      SIG=SIG+0.25
      IF(SIG.LE.3.)GO TO 5
      READ(5,*)J
      IF(J.EQ.1)GO TO 2

      STOP
      END
```

APPENDIX B

This appendix lists the routines used to generate the results presented in Chapter V. The input to the program is as follows:

1st card: NN,C1,C2,C3,MM,PP,

2nd through 12th card: C^{E_i}, W^{E_i}, S^{E_i},

13th through 23rd card: C^{P_i}, W^{P_i}, S^{P_i}

where

NN = epoch length in hundredths of seconds

MM = maximum number of epochs to be calculated by the program

PP = printer control

1: print player statistics and plot positions

0: print player statistics only

A brief description of the routines follow:

1. CONTROL. This program is the main-line routine which calls all subsequent subroutines.

2. OFFBA. This subroutine calculates the proper strategy ($\vec{C}^E$) for the ball carrier so that the expected gain (for the worst-case defensive strategies) are maximized.

3. OOB. This subroutine calculates the time and y-value when an offensive ball carrier goes out-of-bounds for a given strategy.

4. TIME. This subroutine calculates the first possible time a given defensive player can move to within one yard of a ball carrier.

5. OFFLIN. This subroutine provides the rotation and de-rotation of the coordinates so that the logic of the defensive players can be used for offensive blockers.

6. DEFBA. This subroutine is the main calling subroutine for the defensive strategy selection.

7. OPTIM. This subroutine calculates the $C_x^{P_i}$ such that the defensive player's position and velocity are equal to the offensive ball carrier's at the first possible time of intercept.

8. CONTA. This subroutine determines whether (and by how much) the offensive player can "beat" the defensive player for a given $C_y^{P_i}$ for $C_x^{P_i} = \pm \sqrt{(C^{P_i})^2 - (C_y^{P_i})^2}$.

9. MAXIM. This subroutine calculates the minimum $C_y^{P_i}$ so that the defensive player cannot be beaten.

10. RANDOM. This subroutine generates a random normally-distributed variable with zero mean and variance.

11. GRIDIR. This subroutine prints the player statistics and plots the player's position in the field. There is an interaction code which is printed as follows:

    a. 0: no interaction, i.e. the player is free to run according to the selected strategy

    b. 1: the player is being blocked

    c. 2: the player is on the ground with zero velocity and thus constant position

    d. 3: the player is tackling the ball-carrier (for defensive players) or being tackled (for the offensive ball carrier)

    e. 5: the player is out-of-bounds

12. BLOCK. This subroutine determines if a block continues. If it does, then the routine calculates the translational and rotational constants to be used in the updating routine.

13. UPDATE. This subroutine updates the players' positions and velocities to the beginning of the next epoch. The routine checks the players' positions during the epoch to see if an interaction begins. If it does, the subroutine calculates the resultant position and velocities for the end of the epoch.

14. TACKLE. This subroutine is for tackling what the BLOCK subroutine is for blocking.

15. OFFPLAY. This subroutine sets the offensive blocking assignments, and starts the play in motion. For the first few epochs, the players' strategies are tightly determined. As the play develops, the players are let loose to determine their own strategies.

16. DEFPLAY. This subroutine is the defensive equivalent of the OFFPLAY subroutine.

One final note: it is not the author's intention to demonstrate impressive programming technique. There are certain constants used in the root-finding portions that may be too tight to allow efficient run times or too loose to ensure accurate results. This is one more item that is left to further study.

For each epoch, the following are output:

1. A plot of the players' positions on the football field (this is only output if PP = 1)

2. Under the column "PLAYER," a list of the symbols which identify the players. The offensive players are numbered (with the exception of the ball carrier who is denoted by a *), and the defensive players are denoted by letters

3. Under the columns "X" and "Y," the players' positions relative to the center of the playing field and the line of scrimmage

4. Under the columns "XDOT" and "YDOT," the players' velocities

5. Under the column "INTERACTION," a code which denotes:

    a. 0: the player is free to run without restraint

    b. 1: the player is involved in a blocking interaction

    c. 2: the player is on the ground

    d. 3: the player is tackling the ball carrier (defense) or is being tackled (ball carrier)

    e. 5: the player is out-of-bounds

6. Under the column "CX" and "CY," the vector strategy chosen by the players.

7. Under the column "MATCHUP," the list of defensive players assigned to the various offensive blockers.

```
      PROGRAM CONTROL(OUTPUT,INPUT,TAPE6=OUTPUT,TAPE5=INPUT)
      DIMENSION AX(11),AY(11),AXD(11),AYD(11),BX(11),BY(11),
     1CA(11),CB(11),IMATCH(11),CXA(11),CYA(11),CXB(11),
     2CYB(11),IFREEO(11),IFREED(11),BXD(11),BYD(11),WA(11),WB(11),
     3SA(11),SB(11),ITIME(11)
      DATA AX/12.,7.,3.,1.5,0.,0.,-1.5,-3.,-7.,-12.,-15./
      DATA AY/3.,1.4,1.4,3.5,1.,10.,3.5,1.4,1.4,3.5,7.5/
      DATA BX/6.,4.,2.,0.,-2.,-4.,-14.,0.,0.,1.8,-1.8/
      DATA BY/-.3,-.5,-.8,-.4,-.8,-.8,-.3,-1.3,-3.9,-5.3,-5.3/
      DATA AXD/0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0./
      DATA AYD/0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0./
      DATA BXD/0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0./
      DATA BYD/0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0./
      READ(5,*)N,C1,C2,C3,N2,LMN,SIG
      DO 10 I=1,11
      IFREEO(I)=0
   10 READ(5,*)CB(I),WB(I),SB(I)
      DO 20 I=1,11
       IFREED(I)=0
   20 READ(5,*)CA(I),WA(I),SA(I)
      READ(5,100)SEED
  100 FORMAT(A6)
      CALL RANSET(SEED)
      T=0.
      L1=0
      L=0
      N1=1
      IOFF=11
      IDEF=11
   35 DO 45 I=1,11
   45 ITIME(I)=0
      CALL OFFPLAY(AX,AY,AXD,AYD,BX,BY,BXD,BYD,CA,CB,CXB,CYB,
     1IMATCH,IFREED,IFREEO,IBALL,N1,C1,N)
      CALL DEFPLAY(AX,AY,AXD,AYD,BX,BY,BXD,BYD,CA,CB,CXA,CYA,
     1IMATCH,IFREEO,IFREED,IBALL,N1,C1,N)
      DO 73 I=1,IDEF
      CALL RANDOM(CXA(I),SIG)
      CALL RANDOM(CYA(I),SIG)
   73 CONTINUE
      DO 75 I=1,IOFF
      CALL RANDOM(CXB(I),SIG)
      CALL RANDOM(CYB(I),SIG)
   75 CONTINUE
   80 CALL GRIDIR(AX,AY,AXD,AYD,BX,BY,BXD,BYD,T,IOFF,IDEF,
     7IFREEO,IFREED,IBALL,CXB,CYB,CXA,CYA,IMATCH,LMN)
      IF(L.EQ.1)STOP
      CALL UPDATE(AX,AY,AXD,AYD,BX,BY,BXD,BYD,IOFF,IDEF,IFREEO,
     8IFREED,IBALL,IMATCH,CXA,CYA,CXB,CYB,L,N,K,C1,C2,C3,CA,CB,
     9WA,WB,SA,SB)
      T=T+K*0.01
```

```
      N1=N1+1
      IF(IFREEC(IBALL).EQ.3.AND.BYC(IBALL).LE.0.)L1=L1+1
      IF(IFREEC(IBALL).NE.3)L1=0
      IF(L1.GE.2)L=1
      IF(L.EQ.1)GO TO 80
      IF(N1.GT.N2)STOP
      GO TO 35
```

```
      SUBROUTINE OFFBA(CA,CB,C1,CXB,CYB,AX,AY,AXD,AYD,BX,BY,
     1 BXC,BYC,IOFF,IDEF,IFREEC,IFREEC,IBALL,IMATCH)
      DIMENSION CA(11),CB(11),AX(11),AY(11),AXD(11),AYD(11),
     2 BX(11),BY(11),BXD(11),BYD(11),IFREEO(11),IFREED(11),
     3 IMATCH(11),YMAXP(11),PROB(11)
      YMAX=-1000.
      DO 70 I=1,181,2
      CX=CB(IBALL)*COS((I-1)*3.14159/180.)
      CY=CB(IBALL)*SIN((I-1)*3.14159/180.)
      CALL DOB(CX,CY,BX(IBALL),BY(IBALL),BXD(IBALL),BYD(IBALL),
     1 YTH,TSTAR,C1)
      K2=0
      DO 40 J=1,IDEF
      YMAXP(J)=-999.
      PROB(J)=0.
      IF(IFREED(J).GT.2)GO TO 40
      X=AX(J)-BX(IBALL)
      Y=AY(J)-BY(IBALL)
      XDOT=AXD(J)-BXD(IBALL)
      YDOT=AYD(J)-BYD(IBALL)
      Z7=CA(J)
      IF(IFREED(J).NE.0)Z7=0.8*Z7
      CALL TIME(X,Y,XDOT,YDOT,T,Z7,CX,CY,C1)
      IF(T.GT.TSTAR)GO TO 40
      Z1=1.
      IF(C1*T.LT.10.)Z1=1.-EXP(-C1*T)
      Y1=BY(IBALL)+CY*T/C1-(CY-C1*BYD(IBALL))/C1/C1*Z1
      K=0
      DO 30 J1=1,IOFF
      IF(IMATCH(J1).NE.J)GO TO 30
      Z2=C1*C1*(BX(IBALL)-BX(J1))+C1*(BXD(IBALL)-BXD(J1))*Z1
      Z3=Z1-T*C1
      Z4=Z2/Z3-CX
      Z2=C1*C1*(BY(IBALL)-BY(J1))+C1*(BYD(IBALL)-BYD(J1))*Z1
      Z5=Z2/Z3-CY
      Z6=SQRT(Z4*Z4+Z5*Z5)
      IF(Z6.GT.CB(J1))GO TO 30
      K=1
```

```
                    K2=1
                    YMAXP(J)=Y1
                    PROB(J)=0.5+0.5*(1.-Z6/CB(J1))
                    GO TO 35
              30    CONTINUE
              35    IF(K.EQ.0.AND.Y1.LT.YTH)YTH=Y1
              40    CONTINUE
                    IF(K2.EQ.1.AND.ABS(BX(IBALL)).LT.20.67)YTH=YTH-2.
                    Y=0.
                    P=1.
                    Y1=100.
                    DO 60 J2=1,ICEF
                    K1=0
                    DO 50 J=1,ICEF
                    IF(YMAXP(J).EQ.-999.)GO TO 50
                    IF(YMAXP(J).GE.Y1)GO TO 50
                    K1=K1+1
                    Y1=YMAXP(J)
                    J1=J
                    IF(K1.EQ.0)GO TO 65
  60                IF(Y1.GT.YTH)GO TO 65
                    Y=Y+Y1*P*(1.-PROB(J1))
                    P=P*PROB(J1)
                    YMAXP(J1)=-999.
                    Y1=100.
  65          60    CONTINUE
              65    YTH=Y+YTH*P
                    IF(YTH.LE.YMAX)GO TO 70
                    YMAX=YTH
                    CXB=CX
  70                CYB=CY
              70    CONTINUE
                    RETURN
                    END


                    SUBROUTINE CCB(CX,CY,BX,BY,BX0,BY0,YSTAR,TSTAR,C1)
                    DELTA=1.
                    T1=0.
                    X1=BX
              10    IF(ABS(X1).GT.16.67)DELTA=0.1
                    T2=T1+DELTA
                    IF(T2.GT.10.)GO TO 35
                    Z1=1.
                    IF(C1*T2.LT.10.)Z1=1.-EXP(-C1*T2)
                     X2=BX+CX*T2/C1-(CX-BX0*C1)/C1/C1*Z1
                    IF(ABS(X2).LT.26.67)GO TO 50
              20    T3=T1-(ABS(X1)-26.67)/(ABS(X2)-ABS(X1))*(T2-T1)
```

```
      Z1=1.
      IF(C1*T3.LT.10.)Z1=1.-EXP(-C1*T3)
      X3=BX+CX*T3/C1-(CX-BXD*C1)/C1/C1*Z1
      IF(ABS(ABS(X3)-26.67).LT.0.1)GO TO 40
      IF(ABS(X3).LT.26.67)GO TO 30
      T2=T3
      X2=X3
      GO TO 20
30    T1=T3
       X1=X3
      GO TO 20
35    T3=T2
40    TSTAR=T3
      YSTAR=BY+CY*T3/C1-(CY-BYD*C1)/C1/C1*Z1
      RETURN
50    T1=T2
      X1=X2
      DELTA=1.
      GO TO 10
      END
```

```
      SUBROUTINE TIME(X,Y,XDOT,YDOT,T,CA,CX,CY,C1)
      DO 10 J=1,200
      T=0.01+(J-1)*0.05
      Z1=1.
      IF(C1*T.LT.10.)Z1=1.-EXP(-C1*T)
      Z2=C1*C1*X+C1*XDOT*Z1
      Z3=Z1-C1*T
      CX1=Z2/Z3+CX
      Z2=C1*C1*Y+C1*YDOT*Z1
      CY1=Z2/Z3+CY
      Z4=SQRT(CX1*CX1+CY1*CY1)-C1*C1/Z3
      IF(Z4.LT.CA)GO TO 20
      T1=T
10    Z=Z4
      RETURN
20    IF(T.LE.0.01)RETURN
      T2=T
30    T=(T2-T1)*(Z-CA)/(Z-Z4)+T1
      IF(T*C1.LT.10.)Z1=1.-EXP(-C1*T)
      Z2=C1*C1*X+C1*XDOT*Z1
      Z3=Z1-T*C1
      CX1=Z2/Z3+CX
      Z2=C1*C1*Y+C1*YDOT*Z1
      CY1=Z2/Z3+CY
      Z5=SQRT(CX1*CX1+CY1*CY1)-C1*C1/Z3
      IF(ABS(Z5-CA).LT.0.05)RETURN
```

```
      IF(Z5.LT.CA)GO TO 40
      T1=T
      Z=Z5
30    GO TO 30
   40 Z4=Z5
      T2=T
      GO TO 30
      END
```

```
      SUBROUTINE OFFLIN(CA,CB,C1,CXB,CYB,AX,AY,AXD,AYD,BX,BY,
     1 BXD,BYD,EX,EY)
      D=SQRT((AX-EX)**2+(AY-EY)**2)
      DX=(EX-AX)/D
      DY=(EY-AY)/D
      RAY=0.
      RAX=0.
      RAXD=AXD*DY-AYD*DX
      RAYD=AXD*DX+AYD*DY
      RBX=(BX-AX)*DY-(BY-AY)*DX
      RBY=(BX-AX)*DX+(BY-AY)*DY
      RBXD=(BXD-AXD)*DY-(BYD-AYD)*DX
      RBYD=(BXD-AXD)*DX+(BYD-AYD)*DY
      CALL DEFBA(CB,CA,C1,RBX,RBY,RBXD,RBYD,RAX,RAY,RAXD,RAYD,
     2 CX,CY,I,181)
      CXB=CX*CY+CY*DX
      CYB=-CX*DX+CY*DY
      RETURN
      END
```

```
      SUBROUTINE DEFBA(CA,CB,C1,AX,AY,AXD,AYD,BX,BY,BXD,BYD,
     1 CXA,CYA,I1,I2)
      D=SQRT((AX-BX)**2+(AY-BY)**2)
      CYA=0.
      Z=0.
      ZZ=CB*COS((I1+I2-2)*3.14159/360.)
       IF(AY.LE.BY)GO TO 50
      CALL CONTA(CA,CB,CYA,C1,AX,AY,AXD,AYD,BX,BY,BXD,BYD,TSTAR,
     2 COV1,COV2,I1,I2)
      Z1=COV1
      Z2=COV2
      IF(COV1.LT.0.)GO TO 30
      IF(COV2.LT.0.)GO TO 30
      X=AX-BX
      XDOT=AXD-BXD
      CALL OPTIM(CA,CXA,0.,C1,X,XDOT,TSTAR,AX,AXD,D,ZZ)
```

```
      IF(ABS(CXA).NE.CA)GO TO 10
      CYA=0.
      RETURN
 10   CYA=-SQRT(CA*CA-CXA*CXA)
      CALL CONTA(CA,CB,CYA,C1,AX,AY,AXD,AYD,BX,BY,BXD,BYD,
     3TSTAR,COV1,COV2,I1,I2)
      IF(COV1.LT.0.)GO TO 20
      IF(COV2.LT.0.)GO TO 20
      RETURN
 20   CALL MAXIM(CA,CB,C1,AX,AY,AXD,AYD,BX,BY,BXD,BYD,CYA,Z,
     4COV1,COV2,Z1,Z2,C,TSTAR,I1,I2)
      CYA=C
      CALL OPTIM(CA,CXA,CYA,C1,X,XDOT,TSTAR,AX,AXD,D,ZZ)
      RETURN
 30   X=AX-BX
      XDOT=AXD-BXD
      DO 40 I=1,91,5
      CYA= CA*SIN((I-1)*3.14159/180.)
      CALL CONTA(CA,CB,CYA,C1,AX,AY,AXD,AYD,BX,BY,BXD,BYD,
     5TSTAR,COV1,COV2,I1,I2)
      IF(COV1.LT.0.)GO TO 31
      IF(COV2.LT.0.)GO TO 31
      GO TO 50
 31   Z1=COV1
      Z2=COV2
 40   Z3=CYA
      CXA=0.
      RETURN
 50   CALL MAXIM(CA,CB,C1,AX,AY,AXD,AYD,BX,BY,BXD,BYD,Z3,CYA,
     6Z1,Z2,COV1,COV2,C,TSTAR,I1,I2)
      CYA=C
      CALL OPTIM(CA,CXA,CYA,C1,X,XDOT,TSTAR,AX,AXD,D,ZZ)
      RETURN
 60   CXA=-CA*(AX-BX)/D
      CYA=-CA*(AY-BY)/D
      RETURN
      END
```

```
      SUBROUTINE OPTIM(CA,CXA,CYA,C1,X,XDOT,TSTAR,AX,AXD,D,ZZ)
      Z1=1.
      IF(C1*TSTAR.LT.10.)Z1=1.-EXP(-C1*TSTAR)
      Z1=Z1/C1
      Z2=(TSTAR-Z1)/C1
      Z4=(-TSTAR/C1+Z1/C1+0.5*TSTAR*TSTAR)/C1
      Z5=-XDOT*(1.-Z1)*C1
      Z6=-X-XDOT*Z1
```

```
      Z7=Z5*Z4-Z6*Z2
      Z8=Z1*Z4-Z2*Z2
      Z3=Z6*Z1-Z2*Z5
      IF(Z8.NE.0.)CXA=Z7/Z8+Z3/Z8*0.125+ZZ
      IF(Z8.NE.0.)GO TO 5
      IF(Z7.NE.0.)CXA=Z7/ABS(Z7)*SQRT(CA*CA-CYA*CYA)
      IF(Z7.EQ.0.)CXA=0.
5     IF(ABS(CXA).GT.SQRT(CA*CA-CYA*CYA))CXA=SQRT(CA*CA-CYA*CYA)*
     1CXA/ABS(CXA)
      X1=AX+CXA*0.25/C1-(CXA-C1*AXD)/C1*EXP(-0.25*C1)/C1
      Z1=EXP(-C1*0.25)/C1
      Z2=(0.25-Z1)/C1
      Z7=1.
      IF(D.LT.1.)Z7=0
      IF(X1.GT.-26.67)GO TO 10
      Z8=-26.67-AX+Z7-XDOT*Z1
      IF(Z2.NE.0.)CXA=Z8/Z2+ZZ
      GO TO 20
10    IF(X1.LT.26.67)GO TO 20
      Z8=26.67-AX-Z7-XDOT*Z1
      IF(Z2.NE.0.)CXA=Z8/Z2+ZZ
20    IF(ABS(CXA).GT.SQRT(CA*CA-CYA*CYA))CXA=SQRT(CA*CA-CYA*CYA)
     2*CXA/ABS(CXA)
      RETURN
      END
```

```
      SUBROUTINE CCNTA(CA,CB,CYA,C1,AX,AY,AXD,AYD,BX,BY,BXD,
     1BYD,TSTAR,COV1,COV2,I1,I2)
      TSTAR=1000.
      COV1=54.
      COV2=54.
      Y1=AY-BY
      CXA1=SQRT(CA*CA-CYA*CYA)
      CXA2=-CXA1
      DO 50 I=I1,I2,2
      Z1=(I-1)*3.14159/180.
      CXB=CB*COS(Z1)
      CYB=CB*SIN(Z1)
5     Y2=(CYA-CYB)/C1
      Y3=(AYD-BYD)/C1-Y2/C1
      IF(Y2.NE.0.)GO TO 10
      IF(Y3.EQ.0.)GO TO 50
      IF((1.+Y1/Y3).LT.0.)GO TO 50
      T=-ALOG(1.+Y1/Y3)/C1
      GO TO 30
10    T=-(Y1+Y3)/Y2
```

```
      IF(ABS(C1*T).GT.10.)GO TO 30
      IF(T.LT.0.)GO TO 50
      K=0
20    Z5=1.
      IF(T.LT.0.)GO TO 50
      IF(C1*T.LT.10.)Z5=1.-EXP(-C1*T)
      Z1=Y1+Y2*T+Y3*Z5
      Z2=Y2+C1*Y3*(1.-Z5)
      IF(Z2.EQ.0.)GO TO 50
      Z3=-Z1/Z2
     T=T+Z3
      IF(ABS(Z3).LT.0.001)GO TO 30
      K=K+1
      IF(K.GT.20)GO TO 50
      GO TO 20
30    IF(T.LT.0.)GO TO 50
      IF(T.LT.TSTAR)TSTAR=T
      Z4=1.
      IF(C1*T.LT.10.)Z4=1.-EXP(-C1*T)
      BX1=BX+CXB*T/C1-(CXB-T*BXD)/C1/C1*Z4
      AX1=AX+CXA1*T/C1-(CXA1-T*AXD)/C1/C1*Z4
      AX2=AX+CXA2*T/C1-(CXA2-T*AXD)/C1/C1*Z4
      IF(BX1.GT.26.67)GO TO 50
      Z1=AX1-BX1
      IF(COV1.GT.Z1)COV1=Z1
40    IF(BX1.LT.-26.67)GO TO 50
      Z1=BX1-AX2
      IF(COV2 .GT.Z1)COV2=Z1
50    CONTINUE
      COV1=COV1+1.
      COV2=COV2+1.
      RETURN
      END


      SUBROUTINE MAXIM(CA,CB,C1,AX,AY,AXD,AYD,BX,BY,BXD,BYD,
     1CYA1,CYA2,COV11,COV12,COV21,COV22,C,TSTAR,I1,I2)
      K=0
      J=0
      IF(COV11*COV21.GT.0.)GO TO 31
      IF(COV12*COV22.GT.0.)GO TO 10
      K=1
      C5=CYA1
      C6=CYA2
10     Z1=COV11/(COV11-COV21)*(CYA2-CYA1)
      C=CYA1+Z1
      J=J+1
      IF(J.GT.20)GO TO 100
      IF(ABS(Z1).LT.0.01*(CYA2-CYA1).OR.ABS(Z1).GT.0.99*(CYA2-
```

```
     2 CYA1))GO TO 100
       CALL CONTA(CA,CB,C,C1,AX,AY,AXC,AYD,BX,BY,BXC,BYC,TSTAR,
     3 C2,C3,I1,I2)
       IF(C2*COV11.LT.0.)GO TO 20
       COV11=C2
       CYA1=C
       IF(ABS(C2).LT.0.1)GO TO 100
       GO TO 10
 20    COV21=C2
       CYA2=C
       IF(ABS(C2).LT.0.1)GO TO 100
       GO TO 10
 30    CYA1=C5
       CYA2=C6
 31    Z1=COV12/(COV12-COV22)*(CYA2-CYA1)
       C=CYA1+Z1
       J=J+1
       IF(J.GT.20)GO TO 110
       IF(ABS(Z1).LT.0.01*(CYA2-CYA1).OR.ABS(Z1).GT.0.99*(CYA2-
     4 CYA1))GO TO 110
       CALL CONTA(CA,CB,C,C1,AX,AY,AXC,AYD,BX,BY,BXC,BYC,TSTAR,
     5 C2,C3,I1,I2)
       IF(C3*COV12.LT.0.)GO TO 40
       COV12=C3
        IF(ABS(C3).LT.0.1)GO TO 110
       CYA1=C
       GO TO 31
 40    COV22=C3
       CYA2=C
       IF(ABS(C3).LT.0.1)GO TO 110
       GO TO 31
 100   IF(K.EQ.0)RETURN
       C4=C
       J=0
       GO TO 30
 110   IF(K.EQ.0)RETURN
       IF(C4.LT.C)C=C4
       RETURN
       END
```

```
      SUBROUTINE RANDOM(CXA,SIGA)
      Z=1.
      X=RANF(A)
      IF(X.LT.0.5)GO TO 10
      X=1.-X
      Z=-1.
10    T=SQRT(ALOG(1./X/X))
      T1=T-(2.30753+0.27061*T)/(1.+0.99229*T+0.04481*T*T)
      T1=Z*SIGA*T1
      CXA=CXA+T1
      RETURN
      END
```

```
      SUBROUTINE GRIDIR(AX,AY,AXD,AYD,BX,BY,BXD,BYD,T,IOFF,
     1ICEF,IFREEO,IFREED,IBALL,CXB,CYB,CXA,CYA,IMATCH,LMN)
      DIMENSION AX(11),AY(11),AXD(11),AYD(11),BX(11),BY(11),
     2BXD(11),BYD(11),IFREEO(11),IFREED(11),IAX(11),IAY(11),
     3IBX(11),IBY(11),ZA(11),ZB(12),ZPRINT(107),CXA(11),
     4CYA(11),CXB(11),CYB(11),IMATCH(11),ZC(12)
       DATA ZA/1HL,1HE,1HT,1HL,1HV,1HS,1HL,1HT,1HE,1HL,1HS/
       DATA ZC/1H1,1H2,1H3,1H4,1H5,1H6,1H7,1H8,1H9,1H0,1HZ,1H*/
      DATA Z1/1H./,Z2/1H /,Z3/1HI/
      DO 10 I=1,ICFF
      IF(I.EQ.IBALL)ZB(I)=ZC(12)
      IF(I.NE.IBALL)ZB(I)=ZC(I)
      IBX(I)=IFIX(2.*BX(I))+54
10    IBY(I)=-IFIX(2.*BY(I))+31
      IF(LMN.EQ.0)GO TO 91
      DO 20 I=1,ICEF
      IAX(I)=IFIX(2.*AX(I))+54
20    IAY(I)=-IFIX(2.*AY(I))+31
      WRITE(6,100)T
100   FORMAT(1H1,10X,7HEPOCH =,F6.2,8H SECONDS,//)
      I=15
      K=0
30    DO 90 J=1,51
      IF((J-1).EQ.(10*K))GO TO 40
      ZPRINT(I)=Z1
      ZPRINT(107)=Z1
      DO 35 J1=2,106
35    ZPRINT(J1)=Z2
      GO TO 50
40    DO 45 J1=1,107
45    ZPRINT(J1)=Z1
      ZPRINT(72)=Z3
      ZPRINT(36)=Z3
50    DO 60 J1=1,ICFF
```

```
      IF(IFY(J1).NE.J)GO TO 60
      J2=IBX(J1)
      ZPRINT(J2)=ZB(J1)
60    CONTINUE
      DO 70 J1=1,IDEF
      IF(IAY(J1).NE.J)GO TO 70
      J2=IAX(J1)
      ZPRINT(J2)=ZA(J1)
70    CONTINUE
      IF((J-1).EQ.(10*K))GO TO 80
      WRITE(6,101)(ZPRINT(J3),J3=1,107)
101   FORMAT(4X,107A1)
      GO TO 90
80     K=K+1
      WRITE(6,102)I,(ZPRINT(J3),J3=1,107)
      I=I-5
102   FORMAT(I4,107A1)
90    CONTINUE
91    WRITE(6,100)T
      WRITE(6,103)
103   FORMAT(10X,6HPLAYER,7X,1HX,7X,1HY,4X,4HXDOT,4X,4HYDOT,
     5 2X,11HINTERACTION,5X,2HCX,6X,2HCY,2X,7HMATCHUP,/,
     6 8H OFFENSE)

      DO 120 J=1,IOFF
      IF(J.EQ.IBALL)GO TO 105
       IF(IFREEO(J).EQ.5)GO TO 110
      I1=IMATCH(J)
      IF(I1.EQ.0)GO TO 105
       WRITE(6,104)ZB(J),BX(J),BY(J),BXD(J),BYD(J),IFREEO(J),CXB(J),
     7 CYB(J),ZA(I1)
      GO TO 120
105   IF(IFREEO(J).EQ.5)GO TO 106
      WRITE(6,104)ZB(J),BX(J),BY(J),BXD(J),BYD(J),IFREEO(J),CXB(J),
     8 CYB(J)
      GO TO 120
106   WRITE(6,109)ZB(J),BX(J),BY(J),BXD(J),BYD(J),IFREEO(J)
      GO TO 120
110   I1=IMATCH(J)
      WRITE(6,109)ZB(J),BX(J),BY(J),BXD(J),BYD(J),IFREEO(J),ZA(I1)
104   FORMAT(15X,A1,4F8.2,11X,I2,2F8.2,8X,A1)
109   FORMAT(15X,A1,4F8.2,11X,I2,24X,A1)
120   CONTINUE
      WRITE(6,107)
107   FORMAT(1H0,7HDEFENSE)
      DO 140 J1=1,IDEF
      IF(IFREEO(J1).EQ.5)GO TO 130
      WRITE(6,104)ZA(J1),AX(J1),AY(J1),AXD(J1),AYD(J1),IFREEO(J1),
```

```
     9  CXA(J1),CYA(J1)
        GO TO 140
130     WRITE(6,109)ZA(J1),AX(J1),AY(J1),AXD(J1),AYD(J1),IFREED(J1)
140     CONTINUE
        RETURN
        END
```

```
        SUBROUTINE BLOCK(IO,IC,ID,AX,AY,AXD,AYD,BX,BY,BXD,BYD,
     1  CX,CY,CXD,CYD,WA,WB,WC,SA,SB,SC,CA,CB,CC,CXA,CYA,CXB,CYB,
     2  CXC,CYC,C4,C2,C3,K1,X,Y,THED,THED1,XDOT1,YDOT1,XDOT,YDOT)
        A=SQRT(AXD*AXD+AYD*AYD)
        B=SQRT(BXD*BXD+BYD*BYD)
        C=SQRT(CXD*CXD+CYD*CYD)
        B1=AXD*BXD+AYD*BYD
        C1=AXD*CXD+AYD*CYD
        K=K1
        IF(A.NE.0..AND.B.NE.0.)B1=B1/A/B
        IF(A.NE.0..AND.C.NE.0.)C1=C1/A/C
        IF(K.EQ.2)GO TO 30
        X=RANF(C)
        IF(X.GE.(0.015*(SB/SA+(1.+B)/(1.+A)+1.+B1)))GO TO 10
        ID=2
        IO=2
        BXD=0.
        BYD=0.
        AXD=0.
        AYD=0.
        RETURN
10      IF(X.LE.(1.-0.04*(SA/SB+(1.+A)/(1.+B)+1.-B1)))GO TO 20
        IC=0
        IC=2
        BXD=0.
        BYD=0.
        RETURN
20      IF(A.EQ.0.)A=1.
        IF(B.EQ.0.)B=1.
        IF(C.EQ.0.)C=1.
        XDOT=(WA*AXD+WB*BXD+WC*CXD)/(WA+WB+WC)
        XDOT1=C2*(SA*CXA/CA+SB*CXB/CB+SC*CXC/CC)/(WA+WB+WC)
        YDOT=(WA*AYD+WB*BYD+WC*CYD)/(WA+WB+WC)
        YDOT1=C2*(SA*CYA/CA+SB*CYB/CB+SC*CYC/CC)/(WA+WB+WC)
        X=0.5*(AX+BX+CX)
        Y=0.5*(AY+BY+CY)
        IF(K.EQ.2)X=2./3.*X
        IF(K.EQ.2)Y=2./3.*Y
        TH=WA*((AX-X)*AYD-(AY-Y)*AXD)+WB*((BX-X)*BYD-(BY-Y)*
     1  BXD)+WC*((CX-X)*CYD-(CY-Y)*CXD)
```

```
      TI=WA*((AX-X)**2+(AY-Y)**2)+WB*((BX-X)**2+(BY-Y)**2)
     2 +WC*((CX-X)**2+(CY-Y)**2)
      THED=C3*TH/TI
      TT=SA*((AX-X)*CYA/CA-(AY-Y)*CXA/CA)+SB*((BX-X)*CYB/CB-
     3 (BY-Y)*CXB/CB)+SC*((CX-X)*CYC/CC-(CY-Y)*CXC/CC)
      THED1=C2*TT/TI
25    RETURN
30    X=RANF(C)
      Y=RANF(C)
      Z1=0.04*(WB/WA+(1.+B)/(1.+A)+1.+B1)
      Z2=0.04*(WC/WA+(1.+C)/(1.+A)+1.+C1)
      Z3=1.-0.02*(WA/WB+(1.+A)/(1.+B)+1.-B1)
      Z4=1.-0.02*(WA/WC+(1.+A)/(1.+C)+1.-C1)
      IF(X.GT.Z1)GO TO 35
      IC=2
      IO=2
      IC=0


      BXD=0.
      BYD=0.
      AXD=0.
      AYD=0.
      CXD=0.
      CYC=0.
      RETURN
35    IF(X.LT.Z3)GO TO 40
      IC=2
      BXD=0.
      BYD=0.
      WB=0.
      SB=0.
40    IF(Y.GT.Z2)GO TO 45
      ID=2
      IC=0
      IC=2
      AXD=0.
      AYD=0.
      CXD=0.
      CYD=0.
      CXB=0.
      CYB=0.
      RETURN
45    IF(Y.LT.Z4)GO TO 50
      IC=2
      CXD=0.
      CYD=0.
      WC=0.
      SC=0.
```

```
50      IF(IC.EQ.2)K=K-1
        IF(IC.EQ.2)K=K-1
        IF(K.EQ.0)RETURN
        GO TO 20
        END


      SUBROUTINE UPDATE(AX,AY,AXD,AYD,BX,BY,BXD,BYD,IOFF,IDEF,IFREEO,
     1IFREED,IBALL,IMATCH,CXA,CYA,CXB,CYB,L,N,K,C1,C2,C3,CA,CB,WA,WB,
     2SA,SB)
      DIMENSION AX(11),AY(11),AXD(11),AYD(11),BX(11),
     3BY(11),BXD(11),BYD(11),IFREEO(11),IFREED(11),
     4IMATCH(11),CXA(11),CYA(11),CXB(11),CYB(11),CA(11),
     5CB(11),WA(11),WB(11),SA(11),SB(11),AX1(11),AY1(11),
     6BX1(11),BY1(11),X(11),Y(11),THED(11),XDOT(11),
     7YDOT(11),ITIME(11),XDOT1(11),YDOT1(11),AXD1(11),AYD1(11),
     8BXD1(11),BYD1(11),ITACKLE(11),THED1(11)
      DO 45 I=1,IDEF
      ITIME(I)=0
      IF(IFREED(I).NE.1)GO TO 45
      J1=0
      DO 15 J=1,IOFF
      K3=0
      IF(IMATCH(J).NE.1)GO TO 15
      IF(IFREEO(J).NE.1)GO TO 15
      J1=J1+1
      IF(J1.EQ.2)GO TO 10
      I1=J
      GO TO 15
10    I2=J
      GO TO 20
15    CONTINUE
20    Z8=SQRT((AX(I)-BX(IBALL))**2+(AY(I)-BY(IBALL))**2)
      Z9=SQRT((BX(I1)-BX(IBALL))**2+(BY(I1)-BY(IBALL))**2)
      IF(Z8.GT.Z9)GO TO 35
      IF(J1.EQ.1.OR.K3.EQ.1)IFREED(I)=0
      K3=1
      Z9=RANF(C)
      IF(Z9.LT.0.4)GO TO 25
      IFREEO(I1)=2
      BXD(I1)=0.
      BYD(I1)=0.
      GO TO 30
25    IFREEO(I1)=0
      CXB(I1)=0.
```

```
      CYB(I1)=0.
 30   IF(J1.EQ.1)GO TO 45
      I1=I2
      J1=1
      GO TO 20
 35   IF(J1.EQ.1)GO TO 38
      Z9=SQRT((BX(I2)-BX(IBALL))**2+(BY(I2)-BY(IBALL))**2)
      IF(Z8.GT.Z9)GO TO 40
      Z9=RANF(C)
      IF(K3.EQ.1)IFREED(I)=0
      IF(Z9.LT.0.4)GO TO 36
      IFREEO(I2)=2
      BXD(I2)=0.
      BYD(I2)=0.
      GO TO 38
 36   IFREEO(I2)=0
       CXB(I2)=0.
      CYB(I2)=0.
 38   Z9=1.
      J2=0
      CALL BLOCK(IFREEO(I1),J2,IFREED(I),AX(I),AY(I),AXD(I),
     1AYD(I),BX(I1),BY(I1),BXD(I1),BYD(I1),0.,0.,0.,0.,
     2WA(I),WB(I1),0.,SA(I),SB(I1),0.,CA(I),CB(I1),Z9,
     3CXA(I),CYA(I),CXB(I1),CYB(I1),0.,0.,C1,C2,C3,1,X(I),
     4Y(I),THED(I),THED1(I),XDOT(I),YDOT(I),XDOT1(I),YDOT1(I))
      GO TO 45
 40   Z3=WB(I1)
      Z4=WB(I2)
      Z5=SB(I1)
      Z6=SB(I2)
      CALL BLOCK(IFREEO(I1),IFREEO(I2),IFREEO(I),AX(I),AY(I),
     5AXD(I),AYD(I),BX(I1),BY(I1),BXD(I1),BYD(I1),BX(I2),
     6BY(I2),BXD(I2),BYD(I2),WA(I),Z3,Z4,SA(I),Z5,
     7Z6,CA(I),CB(I1),CB(I2),CXA(I),CYA(I),CXB(I1),CYB(I1),
     8CXB(I2),CYB(I2),C1,C2,C3,2,X(I),Y(I),THED(I),THED1(I),XDOT(I),
     9YDOT(I),XDOT1(I),YDOT1(I))
 45   CONTINUE
      K=0
      DO 46 I=1,IDEF
      IF(IFREED(I).NE.3)GO TO 46
       K=K+1
       ITACKLE(K)=I
 46   CONTINUE
      IF(K.EQ.0)GO TO 46
      CALL TACKLE(BX(IBALL),BY(IBALL),BXD(IBALL),BYD(IBALL),
     1AX,AY,AXD,AYD,X,Y,XDOT,YDOT,XDOT1,YDOT1,THED,THED1,L,ITACKLE,
     2K,CXB(IBALL),CYB(IBALL),CXA,CYA,IFREEO(IBALL),IFREED,
     3WB(IBALL),WA,SB(IBALL),SA,C2,C3,CB(IBALL),CA)
      IF(L.NE.1)GO TO 48
      K=3
      RETURN
```

```
 48     DO 125 K=5,N,5
        DO 71 I=1,ICFF
        J=IMATCH(I)
        IF(I.EQ.IBALL)T=K*0.01
        IF(I.NE.IBALL)T=(K-ITIME(J))*0.01
        Z2=EXP(-C1*T)
        Z1=1.-Z2
        IF(IFREEO(I).EQ.0)GO TO 50
        IF(IFREEO(I).EQ.1)GO TO 55
        IF(IFREEO(I).EQ.2)GO TO 60
        IF(IFREEO(I).EQ.3)GO TO 65
        IF(IFREEO(I).EQ.4)GO TO 50
        BX1(I)=50.
        BY1(I)=100.
        BXD1(I)=0.
        BYD1(I)=0.
        GO TO 70
 50     BX1(I)=BX(I)+CXB(I)*T/C1-(CXB(I)-C1*BXD(I))*Z1/C1/C1
        BY1(I)=BY(I)+CYB(I)*T/C1-(CYB(I)-C1*BYD(I))*Z1/C1/C1
        IF(BX1(I).LT.-26.67.OR.BX1(I).GT.26.67)IFREEO(I)=5
        BXD1(I)=CXB(I)*Z1/C1+BXD(I)*Z2
        BYD1(I)=CYB(I)*Z1/C1+BYD(I)*Z2
        GO TO 70
 55     J=IMATCH(I)
 56     XP=BX(I)-X(J)
        YP=BY(I)-Y(J)
        THE=THED1(J)*0.00005*(K-ITIME(J))**2+THED(J)*0.01*(K-ITIME(J))
        BX1(I)=X(J)+XDOT1(J)*0.01*(K-ITIME(J))+XDOT(J)*0.00005*
     3  (K-ITIME(J))**2+XP*COS(THE)-YP*SIN(THE)
        BY1(I)=Y(J)+YDOT1(J)*0.01*(K-ITIME(J))+YDOT(J)*0.00005*
     4  (K-ITIME(J))**2+XP*SIN(THE)+YP*COS(THE)
        IF(BX1(I).LT.-26.67.OR.BX1(I).GT.26.67)IFREEO(I)=5
        BXD1(I)=XDOT(J)*0.01*(K-ITIME(J))+XDOT1(J)+(XP*
     1  SIN(THE)+YP*COS(THE))*(THED1(J)*0.01*(K-ITIME(J))+THED(J))
        BYD1(I)=YDOT(J)*0.01*(K-ITIME(J))+YDOT1(J)+(XP*
     2  COS(THE)-YP*SIN(THE))*(THED1(J)*0.01*(K-ITIME(J))+THED(J))
        IF(IFREEO(IBALL).EQ.5)L=1
        GO TO 70
 60     BX1(I)=BX(I)
        BY1(I)=BY(I)
        GO TO 70
 65     DO 66 J=1,ICEF
        IF(IFREED(J).EQ.3)GO TO 56
 66     CONTINUE
 70     IF(IFREEO(I).NE.2)GO TO 71
        BXD1(I)=0.
        BYD1(I)=0.
 71     CONTINUE
        DO 90 I=1,ICEF
        T=(K-ITIME(I))*0.01
        Z2=EXP(-C1*T)
```

```
      Z1=1.-Z2
      IF(IFREED(I).EQ.0)GO TO 75
      IF(IFREED(I).EQ.1)GO TO 80
      IF(IFREED(I).EQ.2)GO TO 85
      IF(IFREED(I).EQ.3)GO TO 80
      IF(IFREED(I).EQ.4)GO TO 75
      AX1(I)=50.
      AY1(I)=-100.
      AXD1(I)=0.
      AYD1(I)=0.
      GO TO 95
75    AX1(I)=AX(I)+CXA(I)*T/C1-(CXA(I)-C1*AXD(I))*Z1/C1/C1
      AY1(I)=AY(I)+CYA(I)*T/C1-(CYA(I)-C1*AYD(I))*Z1/C1/C1
      IF(AX1(I).LT.-26.67.OR.AX1(I).GT.26.67)IFREED(I)=5
      AXD1(I)=CXA(I)*Z1/C1+AXD(I)*Z2
      AYD1(I)=CYA(I)*Z1/C1+AYD(I)*Z2
      GO TO 95
80    XP=AX(I)-X(I)
      YP=AY(I)-Y(I)
      THE=THED1(I)*0.00005*(K-ITIME(I))**2+THED(I)*0.01*(K-ITIME(I))
      AX1(I)=X(I)+XDOT1(I)*0.01*(K-ITIME(I))+XDOT(I)*0.00005*
     5(K-ITIME(I))**2+XP*COS(THE)-YP*SIN(THE)
      AY1(I)=Y(I)+YDOT1(I)*0.01*(K-ITIME(I))+YDOT(I)*0.00005*
     6(K-ITIME(I))**2+XP*SIN(THE)+YP*COS(THE)
      IF(AX1(I).LT.-26.67.OR.AX1(I).GT.26.67)IFREED(I)=5
      AXD1(I)=XDOT(I)*0.01*(K-ITIME(I))+XDOT1(I)+(XP*
     1SIN(THE)+YP*COS(THE))*(THED1(I)*0.01*(K-ITIME(I))+THED(I))
      AYD1(I)=YDOT(I)*0.01*(K-ITIME(I))+XDOT1(I)+(XP*
     2COS(THE)-YP*SIN(THE))*(THED1(I)*0.01*(K-ITIME(I))+THED(I))
      GO TO 95
85    AX1(I)=AX(I)
      AY1(I)=AY(I)
95    IF(IFREED(I).NE.2)GO TO 96
      AXD1(I)=0.
      AYD1(I)=0.
96    CONTINUE
      IF(L.EQ.1)GO TO 130
      K1=0
      K2=0
      DO 100 I=1,IDEF
      IF(IFREED(I).EQ.3)GO TO 98
      IF(IFREED(I).NE.0)GO TO 100
      D=(AX1(I)-BX1(IBALL))**2+(AY1(I)-BY1(IBALL))**2
      IF(D.GT.1.)GO TO 100
      K2=1
      IFREED(I)=3
98    K1=K1+1
      ITACKLE(K1)=I
100   CONTINUE
      IF(K2.EQ.0)GO TO 104
      IFREED(IBALL)=3
```

```
      BX(IBALL)=BX1(IBALL)
      BY(IBALL)=BY1(IBALL)
      BXD(IBALL)=BXD1(IBALL)
      BYD(IBALL)=BYD1(IBALL)
      CALL TACKLE(BX1(IBALL),BY1(IBALL),BXD1(IBALL),BYD1(IBALL),
     1 AX1,AY1,AXD1,AYD1,X,Y,XDOT,YDOT,XDOT1,YDOT1,THED,THED1,L,
     2 ITACKLE,K1,CXB(IBALL),CYB(IBALL),CXA,CYA,IFREED(IBALL),
     3 IFREED,WB(IBALL),WA,SB(IBALL),SA,C2,C3,CB(IBALL),CA)
      IF(L.EQ.1)GO TO 130
      DO 103 I=1,IDEF
      IF(IFREED(I).EQ.2)GO TO 106
      DO 101 J=1,K1
      IF(ITACKLE(I).EQ.J)GO TO 102
  101 CONTINUE
      GO TO 103
  102 AX(I)=AX1(I)
      AY(I)=AY1(I)
      AXD(I)=AXD1(I)
      AYD(I)=AYD1(I)
      ITIME(I)=K
      GO TO 103
  106 AX(I)=AX1(I)
      AY(I)=AY1(I)
      AXD(I)=0.
      AYD(I)=0.
  103 CONTINUE
  104 DO 120 I=1,IOFF
      IF(I.EQ.IBALL)GO TO 120
      IF(IFREED(I).EQ.1)GO TO 120
      IF(IFREED(I).EQ.5)GO TO 120
      J=IMATCH(I)
      IF(IFREED(J).EQ.3)GO TO 120
      D=(BX1(I)-AX1(J))**2+(BY1(I)-AY1(J))**2
      IF(D.GT.1.)GO TO 120
      K1=0
      DO 105 J1=1,IOFF
      IF(J1.EQ.I)GO TO 105
      IF(IMATCH(J1).NE.J)GO TO 105
      IF(IFREED(J1).NE.1)GO TO 105
      K1=1
      GO TO 110
  105 CONTINUE
  110 IF(K1.EQ.1)GO TO 115
      IFREED(I)=1
      IFREED(J)=1
      K1=0
      Z9=1.
      CALL BLOCK(IFREED(I),K1,IFREED(J),AX1(J),AY1(J),AXD1(J),AYD1(J),
     7 BX1(I),BY1(I),BXD1(I),BYD1(I),0.,0.,0.,0.,WA(J),WB(I),0.,SA(J),
```

```
      8   SB(I),0.,CA(J),CB(I),Z4,CXA(J),CYA(J),CXB(I),CYB(I),0.,0.,C1,
      9   C2,C3,1,X(J),Y(J),THED(J),THED1(J),XDOT(J),YDOT(J),XDOT1(J),
      1   YDOT1(J))
          ITIME(J)=K
          AX(J)=AX1(J)
          AY(J)=AY1(J)
          BX(I)=BX1(I)
          BY(I)=BY1(I)
          BXD(I)=BXD1(I)
          BYD(I)=BYD1(I)
          AXD(I)=AXD1(I)
          AYD(I)=AYD1(I)
          GO TO 120
115       IFREED(I)=1
          AX(J)=AX1(J)
          AY(J)=AY1(J)
          AYD(J)=AYD1(J)
          AXD(J)=AXD1(J)
          BX(I)=BX1(I)
          BY(I)=BY1(I)
          BXD(I)=BXD1(I)
          BYD(I)=BYD1(I)
          BX(J1)=BX1(J1)
          BY(J1)=BY1(J1)
          BXD(J1)=BXD1(J1)
          BYD(J1)=BYD1(J1)
          Z3=SB(I)
          Z4=SB(J1)
          Z5=WB(I)
          Z6=WB(J1)
          CALL BLOCK(IFREED(I),IFREED(J1),IFREED(J),AX(J),AY(J),
      4   AXD(J),AYD(J),BX(I),BY(I),BXD(I),BYD(I),BX(J1),BY(J1),
      1   BXD(J1),BYD(J1),WA(J),Z5,Z6,SA(J),Z3,Z4,
      2   CA(J),CB(I),CB(J1),CXA(J),CYA(J),CXB(I),CYB(I),CXB(J1),
      3   CYB(J1),C1,C2,C3,Z,X(J),Y(J),THED(J),THED1(J),XDOT(J),YDOT(J),
      4   XDOT1(J),YDOT1(J))
          ITIME(J)=K
120       CONTINUE
125       CONTINUE
130       Z2=EXP(-C1*T)
          DO 150 I=1,ICFF
          IF(IFREED(I).EQ.5)GO TO 145
          BX(I)=BX1(I)
          BY(I)=BY1(I)
          BXD(I)=BXD1(I)
          BYD(I)=BYD1(I)
          GO TO 150
145       BXD(I)=0.
          BYD(I)=0.
150       CONTINUE
          DO 170 I=1,IDEF
```

```
      IF(IFREEC(I).EQ.3)GO TO 165
      AX(I)=AX1(I)
      AY(I)=AY1(I)
      AXD(I)=AXD1(I)
      AYD(I)=AYD1(I)
      GO TO 170
165   AXD(I)=0.
      AYD(I)=0.
170   CONTINUE
      IF(K.GT.N)K=N
      RETURN
      END

      SUBROUTINE TACKLE(BX,BY,BXD,BYD,AX,AY,AXD,AYD,X,Y,
     1XDOT,YDOT,XDOT1,YDOT1,THED,THED1,L,ITACKLE,K,CXB,CYB,CXA,
     2CYA,IFREED,IFREED,WB,WA,SB,SA,C2,C3,CE,CA)
      DIMENSION AX(11),AY(11),AXD(11),AYD(11),THED(11),
     3X(11),Y(11),XDOT(11),YDOT(11),XDOT1(11),YDOT1(11),
     4ITACKLE(11),CXA(11),CYA(11),IFREED(11),WA(11),
     5SA(11),CA(11),THED1(11)
      W=0.
      A=0.
      B=SQRT(BXD*BXD+BYD*BYD)
      DO 10 I=1,K
      J=ITACKLE(I)
      A=AXD(J)**2+AYD(J)**2+A
10    W=WA(J)+W
      A=SQRT(A)
      Z1=0.1*(W/WB+(1.+A)/(1.+B))
      Z2=RANF(C)
      IF(Z2.GT.Z1)GO TO 20
      L=1
      RETURN
20    DO 30 I=1,K
      J=ITACKLE(I)
      Z2=RANF(C)
      Z1=0.05*(WB/WA(J)+(1.+B)/(1.+SQRT(AXD(J)**2+AYD(J)**2)))
      IF(Z2.GT.Z1)GO TO 30
      IFREED(J)=2
      AXD(J)=0.
      AYD(J)=0.
      ITACKLE(I)=0
30    CONTINUE
      K1=0
      K2=K
      DO 50 I=1,K
      I1=I-K1
      IF(ITACKLE(I1).NE.0)GO TO 50
      K1=K1+1
      K2=K-K1
      IF(K2.EQ.0)GO TO 55
```

```
      IF(1.GT.K2)GC TO 60
      DO 40 J=I,K2
      J1=J+1
40    ITACKLE(J)=ITACKLE(J1)
50    CONTINUE
      GC TO 60
55    IFFEEO=0
      RETURN
60    K=K2
      Z1=0.
      Z2=0.
      Z3=0.
      Z4=0.
      Z5=0.
      Z6=0.
      Z7=0.
      Z8=0.
      W=0.
      DO 70 I=1,K
      J=ITACKLE(I)
      W=W+WA(J)
      Z1=Z1+AX(J)
      Z2=Z2+AY(J)
      Z3=Z3+SA(J)*CXA(J)/CA(J)
      Z4=Z4+SA(J)*CYA(J)/CA(J)
      Z5=Z5+WA(J)*AXD(J)
      Z6=Z6+WA(J)*AYD(J)
70    Z7=Z7+CA(J)
      Z1=(Z1+BX)/(K+1)
      Z2=(Z2+BY)/(K+1)
      Z3=C2*(Z3+SB*CXB/CB)/(W+WB)
      Z4=C2*(Z4+SB*CYB/CB)/(W+WB)
      Z5=(Z5+WB*BXD)/(W+WB)
      Z6=(Z6+WB*BYD)/(W+WB)
      TI=WB*((BX-Z1)**2+(BY-Z2)**2)
      TH=WB*((BX-Z1)*BYD-(BY-Z2)*BXD)
      TT=SB*((BX-Z1)*CYB/CB-(BY-Z2)*CXB/CB)
      DO 80 I=1,K
      J=ITACKLE(I)
      TH=TH+WA(J)*((AX(J)-Z1)*AYD(J)-(AY(J)-Z2)*AXD(J))
      TI=TI+WA(J)*((AX(J)-Z1)**2+(AY(J)-Z2)**2)
80    TT=TT+SA(J)*((AX(J)-Z1)*CYA(J)/CA(J)-(AY(J)-Z2)*CXA(J)/CA(J))
      DO 90 I=1,K
      J=ITACKLE(I)
      X(J)=Z1
      Y(J)=Z2
      XDOT(J)=Z3
      YDOT(J)=Z4
      XDOT1(J)=Z5
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```
            YDOTI(J)=Z6
            THED(J)=C3*TH/TI
   90       THEDI(J)=C2*TI/TI
            RETURN
            END

            SUBROUTINE OFFPLAY(AX,AY,AXD,AYD,BX,BY,BXD,BYD,CA,CB,
         1  CXB,CYB,IMATCH,IFREEO,IFREED,IBALL,N1,C1,N)
            DIMENSION AX(11),AY(11),AXD(11),AYD(11),BX(11),BY(11),
         2  BXD(11),BYD(11),CA(11),CB(11),CXB(11),CYB(11),IMATCH(11),
         3  IFREEO(11),IFREED(11),I(11),XDUMMY(11),YDUMMY(11)
            DO 10 J=1,11
   10       I(J)=0
            IF(N1.GT.6)GO TO 50
            GO TO (20,20,20,30,30,40),N1
   20       IMATCH(1)=2
            IMATCH(2)=3
            IMATCH(3)=5
            IMATCH(4)=4
            IMATCH(5)=8
            IMATCH(6)=9
            IMATCH(7)=11
            IMATCH(8)=0
            IMATCH(9)=8
            IMATCH(10)=0
            IMATCH(11)=0
            IBALL=8
            D=SQRT((BX(1)-AX(2)+0.5)**2+(BY(1)-AY(2))**2)
            CXB(1)=CB(1)*(AX(2)-BX(1)-0.5)/D
            CYB(1)=CB(1)*(AY(2)-BY(2))/D
            D=SQRT((BX(2)-AX(3)+0.5)**2+(BY(2)-AY(3))**2)
            CXB(2)=CB(2)*(AX(3)-BX(2)-0.5)/D
            CYB(2)=CB(2)*(AY(3)-BY(2))/D
            D=SQRT((BX(3)-AX(5))**2+(BY(3)-AY(5))**2)
            CXB(3)=CB(3)*(AX(5)-BX(3))/D
            CYB(3)=CB(3)*(AY(5)-BY(3))/D
            D=SQRT((BX(4)-AX(4)-1.0)**2+(BY(4)-AY(4))**2)
            CXB(4)=CB(4)*(AX(4)-BX(4)+1.0)/D
            CYB(4)=CB(4)*(AY(4)-BY(4))/D
            D=SQRT((BX(5)-AX(8))**2+(BY(5)-AY(8))**2)
            CXB(5)=CB(5)*(AX(8)-BX(5))/D
            CYB(5)=CB(5)*(AY(8)-BY(5))/D
            D=SQRT((BX(6)-AX(9)-0.5)**2+(BY(6)-AY(9))**2)
            CXB(6)=CB(6)*(AX(9)-BX(6)+0.5)/D
            CYB(6)=CB(6)*(AY(9)-BY(6))/D
            CXB(7)=0.
            CYB(7)=CB(7)
            CXB(8)=SQRT(2.)*CB(8)/4.
```

```
      CYB(8)=-CB(8)/5.
      D=SQRT((BX(9)-AX(8)-1.)**2+(BY(9)-AY(8))**2)
      CXB(9)=CB(9)*(AX(8)-BX(9)+1.)/D
      CYB(9)=CB(9)*(AY(8)-BY(9))/D
      D=SQRT((BX(10)-2.)**2+BY(10)**2)
      CXB(10)=CB(10)*(2.-BX(10))/D
      CYB(10)=-CB(10)*BY(10)/D*0.7
      CXB(11)=-CB(11)
      CYB(11)=0.
      IF(N1.GE.2)GO TO 25
      CXB(10)=CXB(10)/0.7
      CYB(10)=CYB(10)/0.7
      RETURN
25    CXB(8)=0.
      CYB(8)=CB(8)/10.
      RETURN
30    DO 31 J=1,6
      XDUMMY(J)=2.
      YDUMMY(J)=-1.
31    I(J)=1
      I(11)=2
      CXB(11)=-CB(11)
      CYB(11)=0.
       CYB(10)=CB(10)
      I(8)=2
      IF(IFREED(5).LT.2)IFREED(5)=0
      IF(IFREED(8).LT.2)IFREED(8)=0
      IMATCH(5)=7
      I(5)=2
      CXB(3)=CB(3)*0.707
      CYB(3)=CB(3)*0.707
      I(3)=2
      D=SQRT((BX(5)-AX(7)-1.)**2+(BY(5)-AY(7))**2)
      CXB(5)=CB(5)*(AX(7)-BX(5)+1.)/D
      CYB(5)=CB(5)*(AY(7)-AY(5))/D
      D=SQRT((BX(9)-AX(8))**2+(BY(9)-AY(8))**2)
      CXB(9)=CB(9)*(AX(8)-BX(9))/D
      CYB(9)=CB(9)*(AY(8)-BY(9))/D
      GO TO 50
40    IBALL=10
      GO TO 30
50    DO 100 J=1,11
      IF(I(J).EQ.2)GO TO 100
      IF(J.NE.IBALL)GO TO 60
      IF(N1.LT.7)GO TO 100
      CALL OFFBA(CA,CB,C1,CXB(IBALL),CYB(IBALL),AX,AY,AXD,AYD,
     1 BX,BY,BXD,BYD,11,11,IFREED,IFREED,IBALL,IMATCH)
      GO TO 100
60    I1=IMATCH(J)
```

```
            IF(I1.EQ.0)GO TO 80
            Z1=AX(I1)+0.01*N*AXD(I1)
            Z2=AY(I1)+0.01*N*AYD(I1)
            IF(I(J).NE.1)GO TO 70
            Z3=((BX(J)-Z1)**2+(BY(J)-Z2)**2)/((BX(J)-XDUMMY(J))**2+
         2  (BY(J)-YDUMMY(J))**2)
            Z3=SQRT(Z3)
            IF(Z3.GT.0.9)Z3=0.9
            Z4=CA(I1)*(1.-Z3)
            CALL OFFLIN(Z4,CB(J),C1,CXB(J),CYB(J),Z1,Z2,AXD(I1),AYD(I1),
         3  BX(J),BY(J),BXD(J),BYD(J),XDUMMY(J),YDUMMY(J))
            GO TO 100
 70         Z3=BX(IBALL)+0.01*BXD(IBALL)*N
            Z4=BY(IBALL)+0.01*BYD(IBALL)*N
            Z5=((BX(J)-Z1)**2+(BY(J)-Z2)**2)/((BX(J)-Z3)**2+(BY(J)-
         4  Z4)**2)
            Z5=SQRT(Z5)
            IF(Z5.GT.0.9)Z5=0.9
            Z6=CA(I1)*(1.-Z5)
            CALL OFFLIN(Z6,CB(J),C1,CXB(J),CYB(J),Z1,Z2,AXD(I1),AYD(I1),
         5  BX(J),BY(J),BXD(J),BYD(J),Z3,Z4)
            GO TO 100
 80         CXB(J)=0.
            CYB(J)=0.
 100        CONTINUE
            IF(N1.GE.4.AND.N1.LT.7)CYB(10)=CB(10)
            IF(N1.EQ.5)IBALL=10
            RETURN
            END
```

```
            SUBROUTINE DEFPLAY(AX,AY,AXD,AYD,BX,BY,BXD,BYD,CA,CB,
         1  CXA,CYA,IMATCH,IFREEO,IFREED,IBALL,N1,C1,N)
            DIMENSION AX(11),AY(11),AXD(11),AYD(11),BX(11),BY(11),
         2  BXD(11),BYD(11),CA(11),CB(11),CXA(11),CYA(11),IMATCH(11),
         3  IFREEO(11),IFREED(11)
            DO 10 J=1,11
            CXA(J)=0.
 10         CYA(J)=0.
            IF(N1.GE.6)GO TO 50
            GO TO (20,30,30,40,40),N1
 20         CYA(10)=-CA(10)
            CALL DEFBA(CA(1),CB(10),C1,AX(1),AY(1),AXD(1),AYD(1),
         3  BX(10),BY(10),BXD(10),BYD(10),CXA(1),CYA(1),1,91)
            CYA(1)=0.
            RETURN
 30         DO 31 J=1,3
            I1=1
            I2=91
```

```
          IF(J.GT.1)I1=41
          IF(J.EQ.2)I2=51
          CALL DEFBA(CA(J),CB(10),C1,AX(J),AY(J),AXD(J),AYD(J),
     4    BX(10),BY(10),BXD(10),BYD(10),CXA(J),CYA(J),I1,I2)
 31       CONTINUE
          IF(N1.EQ.2)CYA(4)=-CA(4)*0.5
          CALL DEFBA(CA(5),Z1,C1,AX(5),AY(5),AXD(5),AYD(5),
     6    BX(10),BY(10),BXD(10),BYD(10),CXA(5),CYA(5),71,111)
          DO 32 J=7,8
          IF(J.EQ.7)I1=81
          IF(J.EQ.7)I2=181
          IF(J.EQ.8)I1=91
          IF(J.EQ.8)I2=151
          CALL DEFBA(CA(J),CB(9),C1,AX(J),AY(J),AXD(J),AYD(J),
     7    BX(9),BY(9),BXD(9),BYD(9),CXA(J),CYA(J),I1,I2)
 32       CONTINUE
          IF(N1.EQ.2)CYA(7)=0.
          CYA(10)=-CA(10)*0.75
          CXA(10)=CA(10)*0.3
          Z3=BX(7)+0.5
          Z1=0.8*CB(7)
          Z2=BY(7)+8.
          CALL DEFBA(CA(11),Z1,C1,AX(11),AY(11),AXD(11),AYD(11),
     8    Z3,Z2,BXD(7),BYD(7),CXA(11),CYA(11),1,181)
          CXA(11)=CXA(11)*0.15
          IF(N1.EQ.2)RETURN
          CALL DEFBA(CA(6),CB(10),C1,AX(6),AY(6),AXD(6),AYD(6),
     9    BX(10),BY(10),BXD(10),BYD(10),CXA(6),CYA(6),46,136)
          CYA(6)=CYA(6)*0.3
          IF(N1.EQ.2)RETURN
          CALL DEFBA(CA(4),CB(10),C1,AX(4),AY(4),AXD(4),AYD(4),
     1    BX(10),BY(10),BXD(10),BYD(10),CXA(4),CYA(4),61,121)
          IF(N1.EQ.3)GO TO 52
          RETURN
 40       DO 41 J=1,6
          I1=1
          I2=91
          IF(J.EQ.1)I1=46
          IF(J.EQ.2)I2=51
          IF(J.GT.3)I1=46
          IF(J.GT.3)I2=136
          IF(J.EQ.6)I1=81
          IF(J.EQ.6)I2=181
          CALL DEFBA(CA(J),CB(10),C1,AX(J),AY(J),AXD(J),AYD(J),
     2    BX(10),BY(10),BXD(10),BYD(10),CXA(J),CYA(J),I1,I2)
 41       CONTINUE
          CYA(6)=CYA(6)*0.1
          DO 42 J=7,8
          I1=101
```

```
      I2=181
      IF(J.EQ.8)I1=151
      IF(J.EQ.8)I2=181
      CALL DEFBA(CA(J),CB(9),C1,AX(J),AY(J),AXD(J),AYD(J),
     3 BX(9),BY(9),BXD(9),BYD(9),CXA(J),CYA(J),I1,I2)
42    CONTINUE
      CYA(10)=-CA(10)*0.75
      CXA(10)=CA(10)*0.3
      Z1=BY(7)+7.
      Z2=0.8*CB(7)
      Z3=BX(7)+0.5
      CALL DEFBA(CA(11),Z2,C1,AX(11),AY(11),AXD(11),AYD(11),
     4 Z3,Z1,BXD(7),BYD(7),CXA(11),CYA(11),1,181)
44    DO 43 J=1,11
      IF(IFREED(J).NE.3)GO TO 43
      CYA(J)=-CA(J)
      CXA(J)=0.
43    CONTINUE
      RETURN
50    Z1=0.4*CB(10)
      IF(IFREED(10).EQ.3)Z1=0.1*CB(10)
      DO 51 J=2,10
      CALL DEFBA(CA(J),Z1,C1,AX(J),AY(J),AXD(J),AYD(J),
     5 BX(10),BY(10),BXD(10),BYD(10),CXA(J),CYA(J),1,181)
51    CONTINUE
      Z1=CB(10)*0.8
      IF(IFREED(10).EQ.3)Z1=0.1*Z1
      CALL DEFBA(CA(1),Z1,C1,AX(1),AY(1),AXD(1),AYD(1),
     6 BX(10),BY(10),BXD(10),BYD(10),CXA(1),CYA(1),1,91)
      CALL DEFBA(CA(6),Z1,C1,AX(6),AY(6),AXD(6),AYD(6),
     7 BX(10),BY(10),BXD(10),BYD(10),CXA(6),CYA(6),46,136)
52    CALL DEFBA(CA(11),Z1,C1,AX(11),AY(11),AXD(11),AYD(11),
     8 BX(10),BY(10),BXD(10),BYD(10),CXA(11),CYA(11),91,181)
      GO TO 44
      END
```

```
      SUBROUTINE OFFPLAY(AX,AY,AXD,AYD,BX,BY,BXD,BYD,CA,CB,
     1 CXB,CYB,IMATCH,IFREEO,IFREED,IBALL,N1,C1,N)
      DIMENSION AX(11),AY(11),AXD(11),AYD(11),BX(11),BY(11),
     2 BXD(11),BYD(11),CA(11),CB(11),CXB(11),CYB(11),IMATCH(11),
     3 IFREEO(11),IFREED(11),I(11),XDUMMY(11),YDUMMY(11)
      DO 10 J=1,11
10    I(J)=0
      IF(N1.GT.4)GO TO 40
      GO TO (20,20,20,30,30,40),N1
20    IMATCH(1)=2
      IMATCH(2)=3
      IMATCH(3)=5
```

```
      IMATCH(4)=4
      IMATCH(5)=8
      IMATCH(6)=9
      IMATCH(7)=11
      IMATCH(8)=0
      IMATCH(9)=0
      IMATCH(10)=0
      IMATCH(11)=0
      IBALL=8
      D=SQRT((BX(1)-AX(2)+0.5)**2+(BY(1)-AY(2))**2)
      CXB(1)=CB(1)*(AX(2)-BX(1)-0.5)/D
      CYB(1)=CB(1)*(AY(2)-BY(2))/D
      D=SQRT((BX(2)-AX(3)+1.0)**2+(BY(2)-AY(3))**2)
      CXB(2)=CB(2)*(AX(3)-BX(2)-1.0)/D
      CYB(2)=CB(2)*(AY(3)-BY(2))/D
      D=SQRT((BX(3)-AX(5)+0.5)**2+(BY(3)-AY(5))**2)
      CXB(3)=CB(3)*(AX(5)-BX(3)-0.5)/D
      CYB(3)=CB(3)*(AY(5)-BY(3))/D
      D=SQRT((BX(4)-AX(4)+1.0)**2+(BY(4)-AY(4))**2)
      CXB(4)=CB(4)*(AX(4)-BX(4)-2.0)/D
      CYB(4)=CB(4)*(AY(4)-BY(4))/D
      D=SQRT((BX(5)-AX(8))**2+(BY(5)-AY(8))**2)
      CXB(5)=CB(5)*(AX(8)-BX(5))/D
      CYB(5)=CB(5)*(AY(8)-BY(5))/D
      D=SQRT((BX(6)-AX(9)-0.5)**2+(BY(6)-AY(9))**2)
      CXB(6)=CB(6)*(AX(9)-BX(6)+0.5)/D
      CYB(6)=CB(6)*(AY(9)-BY(6))/D
      CXB(7)=0.
      CYB(7)=CB(7)
      CXB(8)=SQRT(2.)*CB(8)/4.
      CYB(8)=-CB(8)/5.
      D=SQRT((BX(9)-AX(8)-1.)**2+(BY(9)-AY(8))**2)
      CXB(9)=CB(9)*(AX(8)-BX(3)+1.)/D
      CYB(9)=CB(9)*(AY(8)-BY(9))/D
      CYB(10)=-CB(10)*BY(10)/D*0.7
      CXB(10)=-CB(10)
      CYB(10)=0.
      CXB(11)=-CB(11)
      CYB(11)=0.
      IF(N1.GE.2)GO TO 25
      CXB(10)=CXB(10)/0.7
      CYB(10)=CYB(10)/0.7
      RETURN
25    CXB(8)=0.
      CYB(8)=CB(8)/10.
```

```
      IF(N1.NE.3)RETURN
      D=SQRT((BX(4)-AX(4))**2+(BY(4)-AY(4))**2)
      CXB(4)=CB(4)*(AX(4)-BX(4))/D
      CYB(4)=CB(4)*(AY(4)-BY(4))/D
      RETURN
30    DO 31 J=1,6
      XDUMMY(J)=-2.
      YDUMMY(J)=-1.
31    I(J)=1
32    I(10)=2
      I(11)=2
      CXB(11)=-CB(11)
      CYB(11)=0.
       CYB(10)=0.
      CXB(10)=-CB(10)
      IF(N1.GT.8)CXB(10)=0.
      IF(N1.GT.8)CXB(11)=0.
      CXB(9)=0.
      CYB(9)=CB(9)
      I(8)=2
      I(5)=2
      IF(N1.NE.4)GO TO 50
      I(4)=2
      D=SQRT((BX(4)-AX(4)-0.5)**2+(BY(4)-AY(4))**2)
      CXB(4)=CB(4)*(AX(4)-BX(4)+0.5)/D
      CYB(4)=CB(4)*(AY(4)-BY(4))/D
      IF(N1.EQ.4)IBALL=9
      CXB(5)=CB(5)*0.707
      CYB(5)=CB(5)*0.707
      I(5)=2
      GO TO 50
40    IBALL=9
      IF(N1.GT.6)GO TO 32
      GO TO 30
50    DO 100 J=1,11
      IF(I(J).EQ.2)GO TO 100
      IF(J.NE.IBALL)GO TO 60
      IF(N1.LT.4)GO TO 100
      CALL OFFBA(CA,CB,CI,CXB(IBALL),CYB(IBALL),AX,AY,AXD,AYD,
     1 BX,BY,BXD,BYD,11,11,IFREEO,IFREED,IBALL,IMATCH)
      GO TO 100
60    I1=IMATCH(J)
      IF(J.EQ.9)GO TO 100
      IF(I1.EQ.0)GO TO 80
      Z1=AX(I1)+0.01*N*AXD(I1)
      Z2=AY(I1)+0.01*N*AYD(I1)
      IF(I(J).NE.1)GO TO 70
      Z3=((BX(J)-Z1)**2+(BY(J)-Z2)**2)/((BX(J)-XDUMMY(J))**2+
     2 (BY(J)-YDUMMY(J))**2)
      Z3=SQRT(Z3)
```

```
      IF(Z3.GT.0.9)Z3=0.9
      Z4=CA(I1)*(1.-Z3)
      CALL OFFLIN(Z4,CB(J),C1,CXB(J),CYB(J),Z1,Z2,AXD(I1),AYD(I1),
     3 BX(J),BY(J),BXD(J),BYD(J),XDUMMY(J),YDUMMY(J))
      GO TO 100
70    Z3=BX(IBALL)+0.01*BXD(IBALL)*N
      Z4=BY(IBALL)+0.01*BYD(IBALL)*N
      Z5=((BX(J)-Z1)**2+(BY(J)-Z2)**2)/((BX(J)-Z3)**2+(BY(J)-
     4 Z4)**2)
      Z5=SQRT(Z5)
      IF(Z5.GT.0.9)Z5=0.9
      Z6=CA(I1)*(1.-Z5)
      CALL OFFLIN(Z6,CB(J),C1,CXB(J),CYB(J),Z1,Z2,AXD(I1),AYD(I1),
     5 BX(J),BY(J),BXD(J),BYD(J),Z3,Z4)
      GO TO 100
60    CXB(J)=0.
      CYB(J)=0.
100   CONTINUE
      RETURN
      END

      SUBROUTINE DEFPLAY(AX,AY,AXD,AYD,BX,BY,BXD,BYD,CA,CB,
     1 CXA,CYA,IMATCH,IFREEO,IFREED,IBALL,N1,C1,N)
      DIMENSION AX(11),AY(11),AXD(11),AYD(11),BX(11),BY(11),
     2 BXD(11),BYD(11),CA(11),CB(11),CXA(11),CYA(11),IMATCH(11),
     3 IFREEO(11),IFREED(11)
      DO 10 J=1,11
      CXA(J)=0.
10    CYA(J)=0.
      IF(N1.GE.6)GO TO 50
      GO TO (20,30,30,40,40),N1
20    CYA(10)=-CA(10)
      CALL DEFBA(CA(1),CB(9),C1,AX(1),AY(1),AXD(1),AYD(1),
     3 BX(9),BY(9),BXD(9),BYD(9),CXA(1),CYA(1),1,91)
      CYA(1)=0.
      RETURN
30    DO 31 J=1,3
      I1=1
      I2=51
      IF(J.EQ.1)I1=41
      IF(J.EQ.1)I2=91
      CALL DEFBA(CA(J),CB(9),C1,AX(J),AY(J),AXD(J),AYD(J),
     4 BX(9),BY(9),BXD(9),BYD(9),CXA(J),CYA(J),I1,I2)
31    CONTINUE
      IF(N1.EQ.2)CXA(4)=-CA(4)*0.5
      Z1=CB(9)
      CALL DEFBA(CA(5),Z1,C1,AX(5),AY(5),AXD(5),AYD(5),
```

```
      6 BX(9),BY(9),BXD(9),BYD(9),CXA(5),CYA(5),1,91)
        DO 32 J=7,9
        I2=161
        I1=151
        IF(J.EQ.7)I1=91
        IF(J.EQ.7)I2=161
        IF(J.EQ.8)I1=91
        IF(J.EQ.8)I2=111
        CALL DEFBA(CA(J),CB(11),C1,AX(J),AY(J),AXD(J),AYD(J),
      7 BX(11),BY(11),BXD(11),BYD(11),CXA(J),CYA(J),I1,I2)
 32     CONTINUE
        IF(N1.EQ.2)CYA(7)=0.
        CYA(10)=-CA(10)*0.75
        CXA(10)=CA(10)*0.3
        Z3=BX(7)+0.5
        Z1=0.8*CB(7)
        Z2=BY(7)+8.
        CALL DEFBA(CA(11),Z1,C1,AX(11),AY(11),AXD(11),AYD(11),
      8 Z3,Z2,BXD(7),BYD(7),CXA(11),CYA(11),1,181)
        CXA(11)=CXA(11)*0.15
        IF(N1.EQ.2)RETURN
        CALL DEFBA(CA(6),CB(11),C1,AX(6),AY(6),AXD(6),AYD(6),
      9 BX(11),BY(11),BXD(11),BYD(11),CXA(6),CYA(6),46,136)
        CYA(6)=CYA(6)*0.3
        IF(N1.EQ.2)RETURN
        CALL DEFBA(CA(4),CB(10),C1,AX(4),AY(4),AXD(4),AYD(4),
      1 BX(10),BY(10),BXD(10),BYD(10),CXA(4),CYA(4),61,121)
        IF(N1.EQ.3)GO TO 52
        RETURN
 40     DO 41 J=1,5
        I1=1
        I2=91
        IF(J.EQ.4)I1=46
        IF(J.EQ.4)I2=136
        CALL DEFBA(CA(J),CB(9),C1,AX(J),AY(J),AXD(J),AYD(J),
      2 BX(9),BY(9),BXD(9),BYD(9),CXA(J),CYA(J),I1,I2)
 41     CONTINUE
        CYA(6)=CYA(6)*0.1
        DO 42 J=7,8
        I1=81
        I2=141
        IF(J.EQ.8)I2=91
        CALL DEFBA(CA(J),CB(9),C1,AX(J),AY(J),AXD(J),AYD(J),
      3 BX(9),BY(9),BXD(9),BYD(9),CXA(J),CYA(J),I1,I2)
 42     CONTINUE
        DO 45 J=9,10
        CALL DEFBA(CA(J),CB(10),C1,AX(J),AY(J),AXD(J),AYD(J),
      1 BX(10),BY(10),BXD(10),BYD(10),CXA(J),CYA(J),91,181)
 45     CONTINUE
```

```
            Z1=BY(7)+7.
            Z2=0.8*CB(7)
            Z3=BX(7)+0.5
            CALL DEFBA(CA(11),Z2,C1,AX(11),AY(11),AXD(11),AYD(11),
         4  Z3,Z1,BXD(7),BYD(7),CXA(11),CYA(11),1,181)
  44        DO 43 J=1,11
            IF(IFREED(J).NE.3)GO TO 43
            CYA(J)=-CA(J)
            CXA(J)=0.
  43        CONTINUE
            IF(N1.NE.4)RETURN
            CXA(7)=0.
            CYA(7)=0.
            RETURN
  50        Z1=0.4*CB(9)
            IF(IFREEO(9).EQ.3)Z1=0.1*CB(9)
            DO 51 J=2,10
            CALL DEFBA(CA(J),Z1,C1,AX(J),AY(J),AXD(J),AYD(J),
         5  BX(9),BY(9),BXD(9),BYD(9),CXA(J),CYA(J),1,181)
  51        CONTINUE
            Z1=CB(9)*0.8
            IF(IFREEO(9).EQ.3)Z1=0.1*Z1
            CALL DEFBA(CA(1),Z1,C1,AX(1),AY(1),AXD(1),AYD(1),
         6  BX(9),BY(9),BXD(9),BYD(9),CXA(1),CYA(1),1,91)
            CALL DEFBA(CA(6),Z1,C1,AX(6),AY(6),AXD(6),AYD(6),
         7  BX(9),BY(9),BXD(9),BYD(9),CXA(6),CYA(6),46,136)
  52        CALL DEFBA(CA(11),Z1,C1,AX(11),AY(11),AXD(11),AYD(11),
         8  BX(9),BY(9),BXD(9),BYD(9),CXA(11),CYA(11),91,181)
            GO TO 44
            END
```

```
            SUBROUTINE OFFPLAY(AX,AY,AXD,AYD,BX,BY,BXD,BYD,CA,CB,
         1  CXB,CYB,IMATCH,IFREEO,IFREED,IBALL,N1,C1,N)
            DIMENSION AX(11),AY(11),AXD(11),AYD(11),BX(11),BY(11),
         2  BXD(11),BYD(11),CA(11),CB(11),CXB(11),CYB(11),IMATCH(11),
         3  IFREEO(11),IFREED(11),I(11),XDUMMY(11),YDUMMY(11)
            DO 10 J=1,11
  10        I(J)=0
            IF(N1.GT.4)GO TO 40
            GO TO (20,20,20,30,30,40),N1
  20        IMATCH(1)=2
            IMATCH(2)=3
            IMATCH(3)=4
            IMATCH(4)=5
            IMATCH(5)=7
            IMATCH(6)=8
            IMATCH(7)=11
            IMATCH(8)=0
            IMATCH(9)=0
```

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      IMATCH(10)=0
      IMATCH(11)=0
      IBALL=8
      D=SQRT((BX(1)-AX(2)+0.5)**2+(BY(1)-AY(2))**2)
      CXB(1)=CB(1)*(AX(2)-BX(1)-0.5)/D
      CYB(1)=CB(1)*(AY(2)-BY(1))/D
      D=SQRT((BX(2)-AX(3)+1.0)**2+(BY(2)-AY(3))**2)
      CXB(2)=CB(2)*(AX(3)-BX(2)-1.0)/D
      CYB(2)=CB(2)*(AY(3)-BY(2))/D
      D=SQRT((BX(3)-AX(4)+2.0)**2+(BY(3)-AY(4))**2)
      CXB(3)=CB(3)*(AX(4)-BX(3)-2.0)/D
      CYB(3)=CB(3)*(AY(4)-BY(3))/D
      D=SQRT((BX(4)-AX(5))**2+(BY(4)-AY(5))**2)
      CXB(4)=CB(4)*(AX(5)-BX(4))/D
      CYB(4)=CB(4)*(AY(5)-BY(4))/D
      D=SQRT((BX(5)-AX(7))**2+(BY(5)-AY(7))**2)
      CXB(5)=CB(5)*(AX(7)-BX(5))/D
      CYB(5)=CB(5)*(AY(7)-BY(5))/D
      D=SQRT((BX(6)-AX(8)-0.5)**2+(BY(6)-AY(8))**2)
      CXB(6)=CB(6)*(AX(8)-BX(6)+0.5)/D
      CYB(6)=CB(6)*(AY(8)-BY(6))/D
      CXB(7)=0.
      CYB(7)=CB(7)
      CXB(8)=SQRT(2.)*CB(8)/4.
      CYB(8)=-CB(8)/5.
      D=SQRT((BX(9)-AX(8)-1.)**2+(BY(9)-AY(8))**2)
      CXB(9)=CB(9)*(AX(8)-BX(9)+1.)/D
      CYB(9)=CB(9)*(AY(8)-BY(9))/D
      CYB(10)=-CB(10)*BY(10)/D*0.7
      CXB(10)=-CB(10)
      CYB(10)=0.
      CXB(11)=-CB(11)
      CYB(11)=0.
      IF(N1.GE.2)GO TO 25
      CXB(2)=-CB(2)
      CYB(2)=0.
      CXB(10)=CXB(10)/0.7
      CYB(10)=CYB(10)/0.7
      RETURN
25    CXB(8)=0.
      CYB(8)=CB(8)/10.
      RETURN
30    DO 31 J=1,8
      XDUMMY(J)=2.
      YDUMMY(J)=-1.
31    I(J)=1
32    I(10)=2
      I(11)=2
      CXB(11)=-CB(11)
```

```
      CYB(11)=0.
      CYB(10)=0.
      CXB(10)=-CB(10)
      IF(N1.GT.8)CXB(10)=0.
      IF(N1.GT.8)CXB(11)=0.
      CXB(9)=0.
      CYB(9)=CB(9)
      I(8)=2
      I(5)=2
      IF(N1.EQ.4)IBALL=9
      GO TO 50
40    IBALL=3
      IF(N1.GT.6)GO TO 32
      GO TO 30
50    DO 100 J=1,11
      IF(I(J).EQ.2)GO TO 100
      IF(J.NE.IBALL)GO TO 60
      IF(N1.LT.5)GO TO 100
      CALL OFFBA(CA,CB,C1,CXB(IBALL),CYB(IBALL),AX,AY,AXD,AYD,
     1 BX,BY,BXD,BYD,11,11,IFREED,IFREED,IBALL,IMATCH)
      GO TO 100
60    I1=IMATCH(J)
      IF(J.EQ.9)GO TO 100
      IF(I1.EQ.0)GO TO 80
      Z1=AX(I1)+0.01*N*AXD(I1)
      Z2=AY(I1)+0.01*N*AYD(I1)
      IF(I(J).NE.1)GO TO 70
      Z3=((BX(J)-Z1)**2+(BY(J)-Z2)**2)/((BX(J)-XDUMMY(J))**2+
     2 (BY(J)-YDUMMY(J))**2)
      Z3=SQRT(Z3)
      IF(Z3.GT.0.9)Z3=0.9
      Z4=CA(I1)*(1.-Z3)
      CALL OFFLIN(Z4,CB(J),C1,CXB(J),CYB(J),Z1,Z2,AXD(I1),AYD(I1),
     3 BX(J),BY(J),BXD(J),BYD(J),XDUMMY(J),YDUMMY(J))
      GO TO 100
70    Z3=BX(IBALL)+0.01*BXD(IBALL)*N
      Z4=BY(IBALL)+0.01*BYD(IBALL)*N
      Z5=((BX(J)-Z1)**2+(BY(J)-Z2)**2)/((BX(J)-Z3)**2+(BY(J)-
     4 Z4)**2)
      Z5=SQRT(Z5)
      IF(Z5.GT.0.9)Z5=0.9
      Z6=CA(I1)*(1.-Z5)
      CALL OFFLIN(Z6,CB(J),C1,CXB(J),CYB(J),Z1,Z2,AXD(I1),AYD(I1),
     5 BX(J),BY(J),BXD(J),BYD(J),Z3,Z4)
      GO TO 100
80    CXB(J)=0.
      CYB(J)=0.
```

```
100   CONTINUE
      IF(N1.EQ.4)CXB(9)=CB(9)
      IF(N1.EQ.4)CYB(9)=0.
      RETURN
      END

      SUBROUTINE DEFPLAY(AX,AY,AXD,AYD,BX,BY,BXD,BYD,CA,CB,
     1CXA,CYA,IMATCH,IFREEO,IFREED,IBALL,N1,C1,N)
      DIMENSION AX(11),AY(11),AXD(11),AYD(11),BX(11),BY(11),
     2BXD(11),BYD(11),CA(11),CB(11),CXA(11),CYA(11),IMATCH(11),
     3IFREEO(11),IFREED(11)
      DO 10 J=1,11
      CXA(J)=0.
10    CYA(J)=0.
      IF(N1.GE.6)GO TO 50
      GO TO (20,30,30,40,40),N1
20    CYA(10)=-CA(10)
      CALL DEFBA(CA(1),CB(9),C1,AX(1),AY(1),AXD(1),AYD(1),
     3BX(9),BY(9),BXD(9),BYD(9),CXA(1),CYA(1),1,91)
      CYA(1)=0.
      RETURN
30    DO 31 J=1,3
       I1=1
      I2=51
      IF(J.EQ.1)I1=41
      IF(J.EQ.1)I2=91
      CALL DEFBA(CA(J),CB(9),C1,AX(J),AY(J),AXD(J),AYD(J),
     4BX(9),BY(9),BXD(9),BYD(9),CXA(J),CYA(J),I1,I2)
31    CONTINUE
      IF(N1.EQ.2)CXA(4)=-CA(4)*0.5
      Z1=CB(9)
      CALL DEFBA(CA(5),Z1,C1,AX(5),AY(5),AXD(5),AYD(5),
     6BX(9),BY(9),BXD(9),BYD(9),CXA(5),CYA(5),1,91)
      DO 32 J=7,9
      I2=181
      I1=151
      IF(J.EQ.7)I1=91
      IF(J.EQ.7)I2=151
      IF(J.EQ.8)I1=91
      IF(J.EQ.8)I2=111
      CALL DEFBA(CA(J),CB(11),C1,AX(J),AY(J),AXD(J),AYD(J),
     7BX(11),BY(11),BXD(11),BYD(11),CXA(J),CYA(J),I1,I2)
32    CONTINUE
      IF(N1.EQ.2)CYA(7)=0.
      CYA(10)=-CA(10)*0.75
      CXA(10)=CA(10)*0.3
```

```
      Z3=BX(7)+0.5
      Z1=0.5*CB(7)
      Z2=BY(7)+8.
      CALL DEFBA(CA(11),Z1,C1,AX(11),AY(11),AXC(11),AYD(11),
     8Z3,Z2,BXD(7),BYD(7),CXA(11),CYA(11),1,181)
      CXA(11)=CXA(11)*0.15
      IF(N1.EQ.2)RETURN
      CALL DEFBA(CA(6),CB(11),C1,AX(6),AY(6),AXD(6),AYD(6),
     9BX(11),BY(11),BXD(11),BYD(11),CXA(6),CYA(6),46,136)
      CYA(6)=CYA(6)*0.3
      IF(N1.EQ.2)RETURN
      CALL DEFBA(CA(4),CB(10),C1,AX(4),AY(4),AXD(4),AYD(4),
     1BX(10),BY(10),BXD(10),BYD(10),CXA(4),CYA(4),61,121)
      IF(N1.EQ.3)GO TO 52
      RETURN
   40 DO 41 J=1,5
      I1=1
      I2=91
      IF(J.EQ.4)I1=46
      IF(J.EQ.4)I2=136
      CALL DEFBA(CA(J),CB(9),C1,AX(J),AY(J),AXD(J),AYD(J),
     2BX(9),BY(9),BXD(9),BYD(9),CXA(J),CYA(J),I1,I2)
   41 CONTINUE
      CYA(6)=CYA(6)*0.1
      DO 42 J=7,8
      I1=61
      I2=141
      IF(J.EQ.8)I2=91
      CALL DEFBA(CA(J),CB(9),C1,AX(J),AY(J),AXD(J),AYD(J),
     3BX(9),BY(9),BXD(9),BYD(9),CXA(J),CYA(J),I1,I2)
   42 CONTINUE
      DO 45 J=9,10
      CALL DEFBA(CA(J),CB(10),C1,AX(J),AY(J),AXD(J),AYD(J),
     1BX(10),BY(10),BXD(10),BYD(10),CXA(J),CYA(J),91,181)
   45 CONTINUE
      Z1=BY(7)+7.
      Z2=0.8*CB(7)
      Z3=BX(7)+0.5
      CALL DEFBA(CA(11),Z2,C1,AX(11),AY(11),AXC(11),AYD(11),
     4Z3,Z1,BXD(7),BYD(7),CXA(11),CYA(11),1,181)
   44 DO 43 J=1,11
      IF(IFREED(J).NE.3)GO TO 43
      CYA(J)=-CA(J)
      CXA(J)=0.
   43 CONTINUE
      IF(N1.NE.4)RETURN
      CXA(7)=0.
      CYA(7)=0.
      RETURN
```

```
50        Z1=0.4*CB(9)
          IF(IFREEQ(9).EQ.3)Z1=0.1*CB(9)
          DO 51 J=2,10
          CALL DEFBA(CA(J),Z1,C1,AX(J),AY(J),AXD(J),AYD(J),
     5    BX(9),BY(9),BXD(9),BYD(9),CXA(J),CYA(J),1,181)
51        CONTINUE
          Z1=CB(9)*0.8
          IF(IFREEQ(9).EQ.3)Z1=0.1*Z1
          CALL DEFBA(CA(1),Z1,C1,AX(1),AY(1),AXD(1),AYD(1),
     6    BX(9),BY(9),BXD(9),BYD(9),CXA(1),CYA(1),1,91)
          CALL DEFBA(CA(6),Z1,C1,AX(6),AY(6),AXD(6),AYD(6),
     7    BX(9),BY(9),BXD(9),BYD(9),CXA(6),CYA(6),46,136)
52        CALL DEFBA(CA(11),Z1,C1,AX(11),AY(11),AXD(11),AYD(11),
     8    BX(9),BY(9),BXD(9),BYD(9),CXA(11),CYA(11),91,181)
          GO TO 44
          END
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